Magnifying the Irradiance of Sunlight

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1 Introduction

This experiment began from a small discussion about a book, which was later turned into a film, called "Lord of the Flies." In one scene in the film a character uses his glasses to start a small fire by focusing the Sun's rays, which the stranded boys on the island needed for heat, cooking and other things. We questioned if it was actually possible for regular glasses to burn something.

There were two kinds of experiments, those done indoors in the lab and those done outside in bright sunlight. The main purpose of the indoor experiments was to verify the Thin-Lens Equation and use it to determine the focal lengths of several lenses as precisely as possible.

2 Theory

A key formula when working with lenses is the Thin-Lens Equation:

$$\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o}, \tag{1}$$

where f is the focal length of the lens, d_o is the object distance and d_i is the image distance. From the Thin-Lens Equation we are able to mathematically see and understand many interesting and valuable situations that arise when working with lenses. When the object is the same distance from the lens as the image, for instance, we can easily verify that the focal length must equal half the image (object) distance or if the object is very far away from the lens (at infinity) the focal length will equal the image distance, just to mention a few.

3 Indoor Measurements

I used a work table to precisely measure the focal lengths of the magnifying lenses and the reading glasses by placing a flashlight at one end of the table, a screen at the other and the lenses in between them. The light was covered with black tape and had a small hole poked in it, thus making the light a point source. I moved the lenses at different points between the screen and light and by recording data at the spots when the light came to a focus on the screen, from the Thin-Lens Equation the focal length of the lenses could be calculated. Varying the distance between the screen and source also allowed us to varify the properties of the Thin-Lens Equation. We see that when $d_o = d_i$, the focal length f is equal to one-half the image or the object distance (or the distance between the lens and the object or image is twice the focal length). If the object moves closer than this point (twice the focal length) we can predict that we will not be able to view the image on the screen.

Shown below is the data for a 7 cm diameter magnifying lens and the reading glasses:

Magnifying Lens		
Image Distance	Object Distance	Focal Length
24.3 cm	218.9 cm	21.9 cm
25.1 cm	197 cm	22.3 cm
25.0 cm	181 cm	22.0 cm
26.0 cm	141 cm	22.0 cm
27.7 cm	112 cm	22.2 cm
32.2 cm	71.7 cm	22.2 cm
44.6 cm	44.3 cm	22.0 cm

Reading Glasses		
Image Distance	Object Distance	Focal Length
118 cm	229 cm	77.9 cm
119 cm	216 cm	76.7 cm
122 cm	210 cm	77.2 cm
149 cm	151 cm	75.0 cm
159 cm	159 cm	79.5 cm

From the data it is clear that when the object is far away from the lens the focal length does not equal the image distance. Why is that? A major limiting factor in this experiment is the size of the table that the flash light and the lenses were on. It is true that at infinity the focal length will equal the image distance, however, in our case, the object is not at infinity but at a distance much closer (a few hundred centimeters). When d_o is not infinity but a smaller number (as in our case), the term $\frac{1}{d_o}$ is not zero and the value of the focal length is not solely determined by the image distance.

4 Outdoor Measurements

We brought four types of lenses outside on a sunny day, three magnifying lenses and the reading glasses. We placed our screen (a piece of paper) on the ground and held a lens above it, varying the distance until the Sun's light came to a focus; when this point was reached the paper began to burn. The Sun's energy flux density or irradiance is what was responsible for this burning.

4.1 Solar Irradiance

Irradiance is defined as the rate of energy emission per unit area. The standard and accepted value for the Sun's irradiance at the surface of the Earth is 1 kW/m^2. Knowing the solar irradiance we can find the power through a specific area, such as the power through the lenses. At a point before the Sun's light hits the lens, the irradiance is simply 1 kW/m^2. Since the area is not specified the power is zero before the light enters the lens. Once it enters the lens, though, we can find the power because the area is known. At the lens, the irradiance is also 1 kW/m^2, but the power is now the area of the lens multiplied by the irradiance entering it.

When the lens brings the sunlight to a focus on the screen, it begins to burn. One might reason that in order for a hole to burn through the screen the Sun's power hitting the screen must be amplified. Therefore, the bright spot at the screen must contain more power than the light entering the lens. This line of reasoning is on the right track, however it is incorrect. In actuality the power at the lens and the power at the spot is exactly the same. The only thing that differs between them is their irradiances. That is, while the power at the spot is the same as at the lens, the irradiance at the spot is much greater here than at the lens.

The reason why the irradiance changes in going from the lens to the spot is the two different areas that the light goes through. It's concept is similar to water moving through a funnel. The rate at which water enters the funnel is relatively slow, depending on how fast or slow someone pours it. In contrast, at the end or tip of the funnel, the rate that the water leaves is fast, however the amount of water that is poured in and that leaves is the same (amount of water in = amount of water out, but rate differs). To find the irradiance at the spot we simply multiply the irradiance at the lens by the ratio of the lenses area to spot's area:

$$\text{Irradiance at Spot} = (\text{Irradiance at Lens}) \left(\frac{\text{Area at Lens}}{\text{Area at Spot}} \right). \tag{2}$$

Below is the data that was collected for each lens:

	Magnifying Lens 1	Magnifying Lens 2	Magnifying Lens 3	Reading Glasses
Focal Length	15 cm	20 cm	26 cm	77 cm
Diameter of Lens	4.7 cm	7.0 cm	8.5 cm	4.7 cm
Area of Lens	17.3 cm^2	38.5 cm^2	56.7 cm^2	17.3 cm^2
Spot Size (Measured)	0.2 cm	0.3 cm	0.4 cm	0.8 cm
Spot Size (Predicted)	0.139 cm	0.185 cm	0.24 cm	0.71 cm
Area of Spot	0.0314 cm^2	0.0707 cm^2	0.126 cm^2	0.636 cm^2
Irradiance at Lens	1 kW/m^2	1 kW/m^2	1kW/m^2	1 kW/m^2
Power at Lens	1.73 W	3.85 W	5.67 W	1.73 W
Irradiance at Spot	551 kW/m^2	545 kW/m^2	450 kW/m^2	27.2 kW/m^2
Power at Spot	1.73 W	3.85 W	5.67 W	1.73 W

While taking measurements outside I noticed that while all three magnifying lenses were able to burn the paper the reading glasses were not. Comparing the data for lens 1 with the reading glasses it is clear that both lenses deliver the same power to the spot, however their irradiances are different. The irradiance for lens 1 is about 20 times larger than the reading glasses! Yet why does the magnifying lens burn the paper and the reading glasses do not? After all, the glasses magnify the Sun's irradiance by about 27 times! At first glance it would seem that 27,000 W/m^2 would be sufficient to burn a tiny hole into a piece of paper, however from experimentation we know that it is not. There must be a minimum irradiance, then, in order for the paper to burn.

4.1.1 Minimum Irradiance

Bringing a magnifying lens outside to find the minimum distance and spot size for the paper to burn, we can find the minimum irradiance needed for burning to occur by moving the lens from a distance above the screen to a point where burning just begins to occur. Using magnifying lens 3 we measured the minimum spot size to be 1.3 cm. Using (2) we found that the minimum irradiance for burning to occur was 43.6 kW/m^2. Even though the spot size that we measured is larger than that of the reading glasses, it is the ratio of *lens* area to *spot* area that is important; the magnifying lenses area is larger than the reading glasses). The reason, then, that the magnifying lenses easily burned the paper and the reading glasses did not was because the ratio of the lens area to the spot area for the glasses did not increase the irradiance by the minimum factor of about 44 and the magnifying lenses did.

4.2 Spot Size

So far we have learned how to find the irradiance and power entering a lens and at a spot on a screen and also the minimum irradiance needed to burn a hole in the screen, yet we have not really discussed much about the spot's size that is viewed on the screen. Looking at the data for the three magnifying lenses and the reading glasses we see that the size of the spots change for each lens. What do these different spot sizes represent, do they effect the irradiance or anything we have observed so far and given a specific lens can we actually predict the spot size we will see? By utilizing (1) we can find the magnification of a lens by:

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}. \tag{3}$$

In our experiment d_o is the distance from the Earth to the Sun (1.50 x 10^{11} m), d_i is the lens to screen distance, h_o is the size or diameter of the Sun (1.39 x 10^9 m) and h_i is the spot size. From (3) we see that if an object is upright then the image we see from the lens will be inverted. Since our object is the Sun, a big "circle" so to speak, we can negate the minus sign in the formula because the orientation of its image does not matter. Since we know the Earth to Sun distance, the Sun's size and the distance from the lens to the screen we can find the spot size by:

$$h_i = \left(\frac{d_i}{d_o}\right)(h_o) \textbf{ or } h_i = \left(\frac{h_o}{d_o}\right)(d_i). \tag{4}$$

Here, $\frac{h_o}{d_o}$ is the angular size of the Sun, which is about 0.53^o, or 92.7 radians. Viewing table 2 it is apparent that the measured spot size and theoretical values differ slightly. These indiscretions can be explained by a few different factors. When we were recording data obviously it was a very bright, sunny day. To measure the size of the spot accurately I had to use two polarizing sheets in order to cut down the immense amount of bright light in the spot. Even with the polarizers, though, it was still rather difficult to view the spot with absolute clarity and sharpness. Besides viewing the spot, it was also complicated to measure the diameter of it.

While a partner held the lens at roughly a focal length away from the screen I marked two opposite sides of the spot while holding the polarizing sheets in the other hand. Needless to say, even though we tried our best to keep things straight and steady I'm sure some error was introduced in the process. Another source of contrariety in our data is spherical aberration. When light rays pass through a lens the focal length actually varies depending where on the lens the rays enter, such as directly through the center or at the very top or bottom of it. The focal length of the lens, therefore, does not actually come to one exact point, but is rather spread out over a very small distance. Spherical aberration is dependent on a lenses curvature and is something that cannot be avoided, however it can be minimized by use of a multi-lens system. Considering all these sources for error, then, a one or one and a half millimeter difference between predicted spot size and actual size is not very bad.

5 Conclusion

In this experiment we started with a basic question; if glasses could actually burn something as depicted in "Lord of the Flies." We learned about the basic properties and principles of lenses and useful mathematical relations that helped us visualize and understand how lenses functioned. With this basic understanding we brought lenses outside and went about using what we knew to understand what we observed and recorded information to analyze later. Using the known solar irradiance at the Earth's surface, we found the energy flux within the spots, and with the relationship between the Sun's size, the Earth-Sun distance and the focal length we learned that it was possible to predict spot size.

So can glasses really focus sunlight to start a fire? There are two answers to this question; glasses can focus sunlight to start the burning process if the irradiance at the spot is a minimum of at least $43.6\ \mathrm{kW/m^2}$. This is one of two requirements of the lens for burning to happen. To focus light at a point, the lens *must* be convex. In "Lord of the Flies," a character named Piggy focused the Sun's light onto a pile of wood and started a fire. One thing that the film maker neglected to realize was that Piggy was nearsighted. Nearsighted people's eyes are not able to focus things at a distance because the focal point falls in front of the retina (where the image is actually seen). To correct this, a convex lens is used to make the light coming from a distance, where the rays come in parallel, fall on the retina. Someone that is farsighted on the other hand is able to focus on images far away, however cannot clearly view objects that are close because the image falls behind the retina. A concave lens placed in front of the eye does the trick, allowing the images of nearby objects to fall cleanly on the retina.