

# Projective Measurement of LG Modes via Refractive Beam Shaping

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We describe an experimental technique for separating the orthonormal basis set of Laguerre-Gaussian (LG) radial modes. In this method, we exploit the orthogonality of the modes to use projective measurements to identify the radial modes. Results suggest this method is capable of correctly identifying LG radial modes,  $p=1-5$ , with about 80% efficiency and results in low cross-talk between modes. The ability to efficiently and accurately identify LG radial modes could have applications in increasing channel capacity in free-space communication and tolerance to eavesdropping attacks in quantum key distribution.

## I. INTRODUCTION

Laguerre-Gaussian (LG) orbital angular momentum (OAM) modes have received considerable attention recently because of their applications in free-space communication and quantum key distribution [1–5]. Radial LG modes have been largely overlooked though despite also being an orthogonal basis set making them potentially useful for the same applications as OAM modes. Many of the technologies that utilize LG OAM modes require the different modes be properly sorted and identified.

In this paper, we propose a method for separating LG radial modes via projective measurements. Utilizing the properties of projection measurements, an LG radial mode incident onto its conjugate mode should produce a Gaussian beam. This beam can then be coupled into a single mode optical fiber. LG radial modes and the phase masks to transform the beam can be produced using a hologram realized by a spatial light modulator. Using the Gerchberg-Saxton method of phase retrieval, we calculated theoretical identification efficiencies of approximately 80% for  $p=1-5$ .

## II. THEORETICAL

LG radial modes form an orthonormal set. Using the orthogonality principle,

$$\int_{-\infty}^{\infty} \psi_m^* \psi_n dx = \begin{cases} 0, & m \neq n \\ 1, & m = n \end{cases}, \quad (1)$$

we can see that an LG radial mode projected onto its conjugate will produce a non-zero output, whereas a radial mode projected onto an orthogonal mode's conjugate will result in no output.

Experimentally this can be demonstrated by an input laser beam being shone onto a spatial light modulator

(SLM) which has a radial mode's complex conjugate displayed on it. If the hologram on the SLM is the incident radial mode's complex conjugate, then a Gaussian will be output; if the hologram on the the SLM is the complex conjugate of another radial mode, no light will be output. The output of this procedure will then be coupled into a single mode fiber. This method of using projection measurements to identify the mode has been shown to distinguish between LG OAM modes with high-accuracy [1, 6].

The complex conjugates of the radial modes were calculated using Gerchberg-Saxton's method of phase retrieval with a Hankel transform used in place of the usual Fourier transform. The Gerchberg-Saxton method of phase retrieval is an iterative Fourier transform where constraints are applied in each domain [7]. A Hankel transform was used in place of the usual Fourier transform because the Hankel transform forces the calculated mask to be radially symmetric [8].

An LG radial mode served as the input for the phase retrieval, while a Gaussian served as the output. A flat phase was used as the initial guess for the phase. Using this method, a coupling efficiency of approximately 80% was found for  $p=1-5$ , where the coupling efficiency is defined as the probability of detecting a radial mode correctly [Fig. 1].

We also found that cross-talk between modes was relatively low. We defined cross-talk as the probability of misidentifying a radial mode [Fig. 2].

## III. EXPERIMENTAL

In our experiment, LG radial modes were produced using a SLM [9]. The radial mode then propagated through a 4-f lens system and was incident on a second SLM, as shown in Fig. 3. The two halves of the second SLM display holograms, which form the complex conjugate of a radial

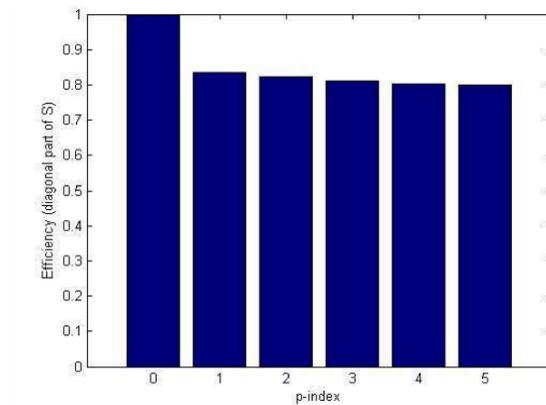


FIG. 1. **Calculated coupling efficiency for LG radial modes  $p=0-5$ .** Calculations found that LG radial modes,  $p=1-5$ , incident on their complex conjugate should form a Gaussian on the output with approximately 80% efficiency

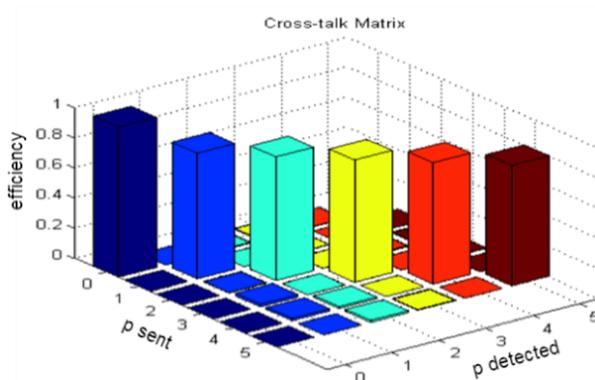


FIG. 2. **Calculated coupling efficiency matrix for LG radial modes  $p=0-5$ .** Calculated coupling efficiencies are shown on diagonal, while cross-talk is shown off diagonal

mode. The two halves of the hologram also have a lens encoded onto them, so that additional optical elements do not have to be used.

#### IV. DISCUSSION

This experiment serves as a proof-of-principal experiment and shows that LG radial modes can be detected with relatively high accuracy. The ability to accurately identify LG radial modes is necessary in order for LG radial modes to enhance classical and quantum communications. LG radial and OAM modes are mutually-unbiased

bases, therefore radial modes are an attractive degree of freedom to encode additional information in LG beams.

One drawback of the method outlined in this paper is that projection measurements give a "yes-or-no" answer, therefore projection measurements are limited by a success rate of  $1/N$ , where  $N$  is the number of the modes. Future work would hopefully focus on sorting LG radial modes in an efficient manner.

#### V. ACKNOWLEDGMENTS

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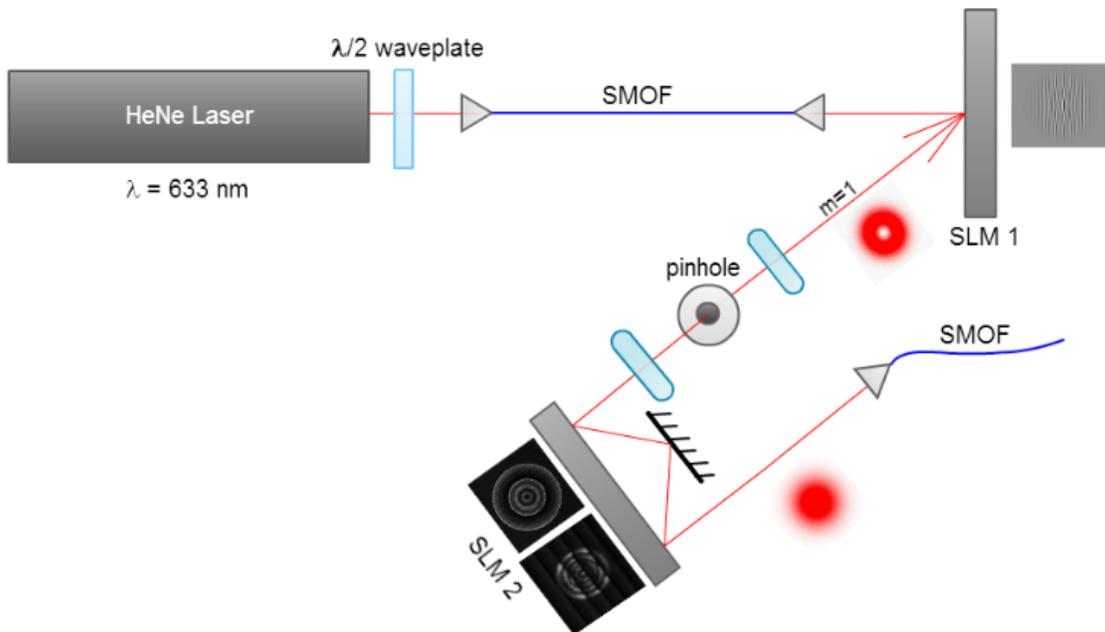


FIG. 3. **Schematic of the mode sorting setup.** SLM 1 converts the incident beam into a radial mode. The second SLM and the 4-f lens system perform the projection measurement. The complex conjugate of one of the radial modes is realized using the two halves of SLM 2.

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