Separating Laguerre-Gaussian Radial Modes Via Projective Measurements

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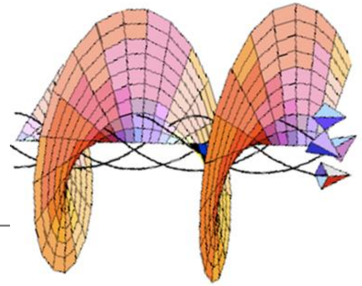
Laguerre-Gaussian (LG) Beams

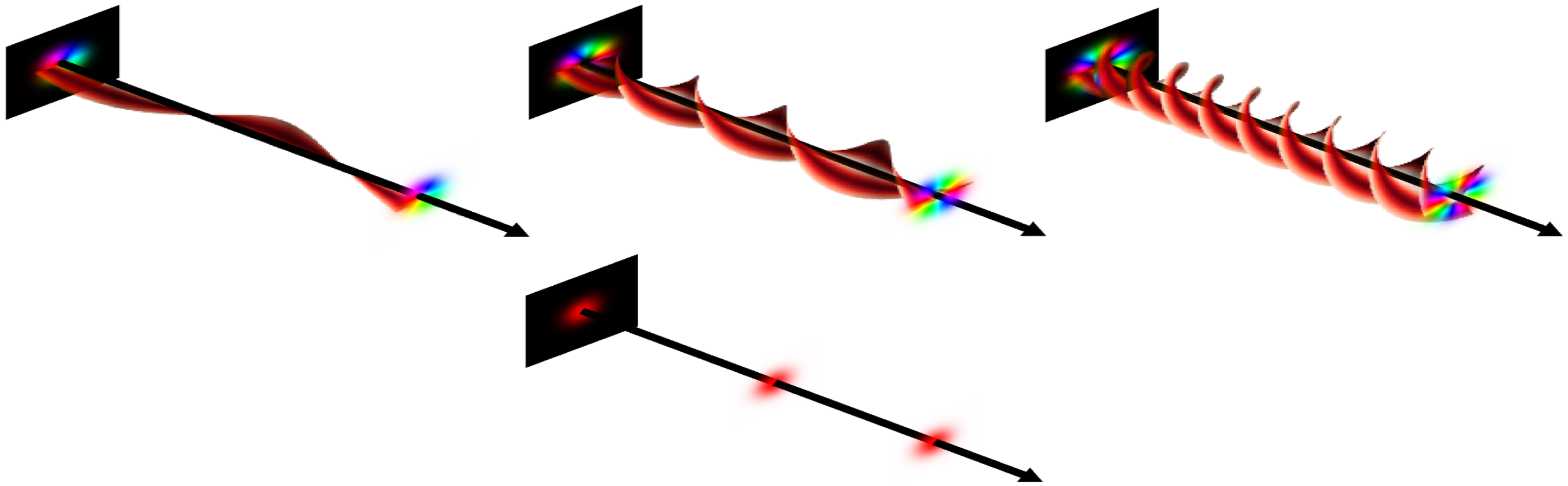


- Cylindrically symmetric and form a basis set for paraxial light beams
- Properties of beams defined by two indices:
  - Azimuthal number (l) – number of spirals in 2π, related to the orbital angular momentum (OAM) by $L_z = l\hbar$
  - Radial index (p) - number of rings plus one

$$U_{p,\ell}(r,\phi,z) = \frac{C_{\ell p}^{LG}}{w(z)}\left(\frac{\sqrt{2}r}{w(z)}\right)^{|\ell|} L_p^{|\ell|}\left(\frac{2r^2}{w(z)^2}\right)\exp\left(\frac{-r^2}{w(z)^2}\right)\exp\left(ik\frac{r^2}{2R(z)}\right)\exp[i(2p+|\ell|+1)\xi(z)]\exp(i\ell\phi)$$

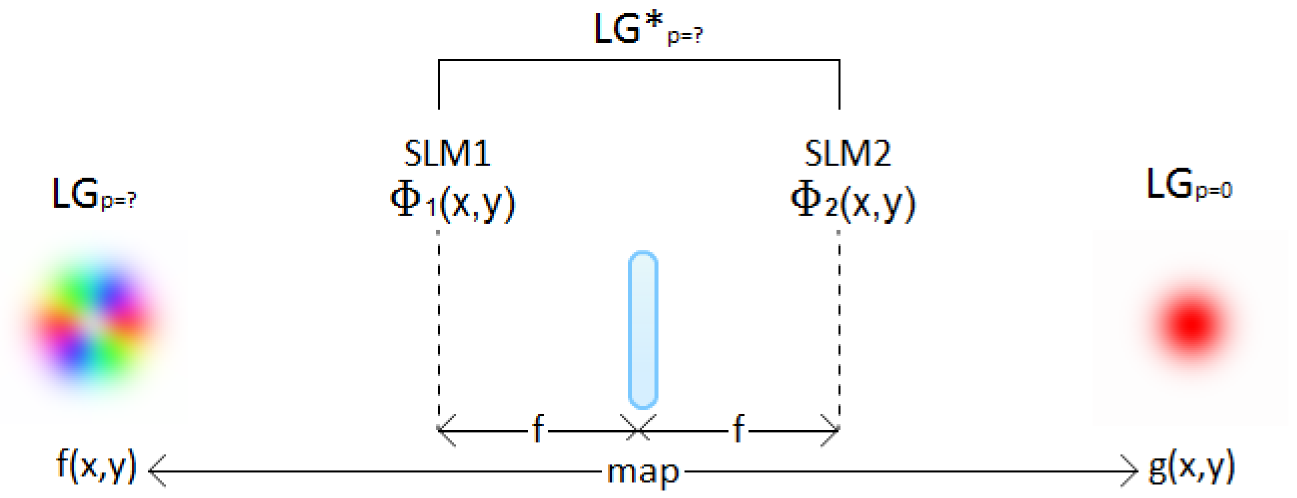
LG OAM Modes

- Applications in:
  - Free space communication (increase channel capacity)
  - QKD (increase tolerance to eavesdropping attacks)
- Radial modes have received less attention



Projective Measurements

- Mode is imaged onto a conjugate mode and the resulting field is propagated and coupled into a single mode optical fiber (SMOF)
- Method gives yes or no, requires scanning through the radial indices to determine the mode

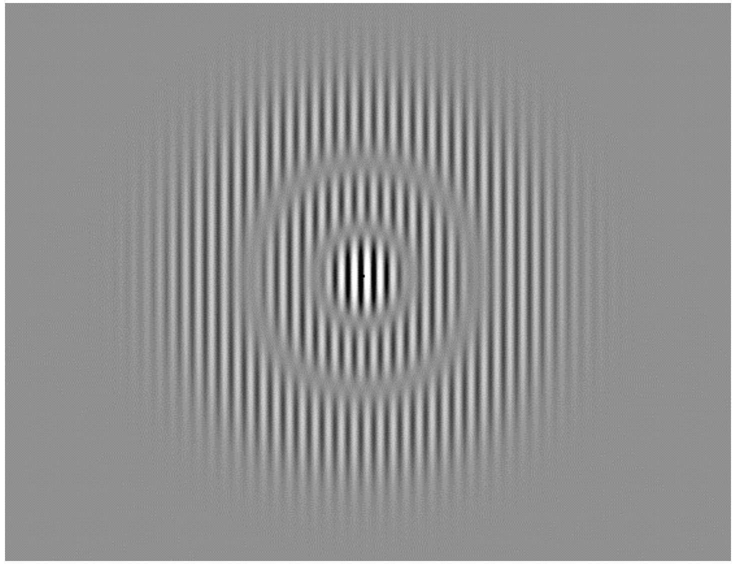


TASKS

- Generate radial modes
- Transform radial modes into Gaussians
- Couple into fiber

Creating Radial Modes

- Used computer generated holograms placed on a spatial light modulator
  - Essentially phase mask with blazed diffraction grating on top
  - Can be implemented using spatial light modulator (SLM) with reduced phase domain

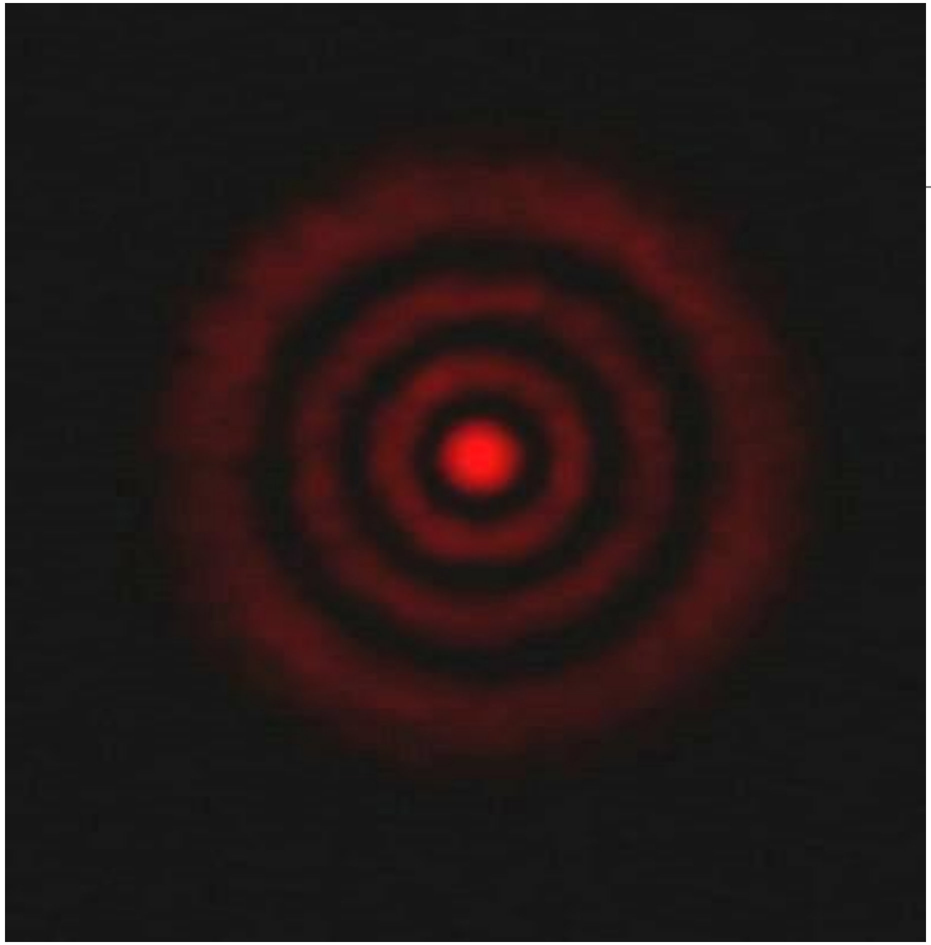


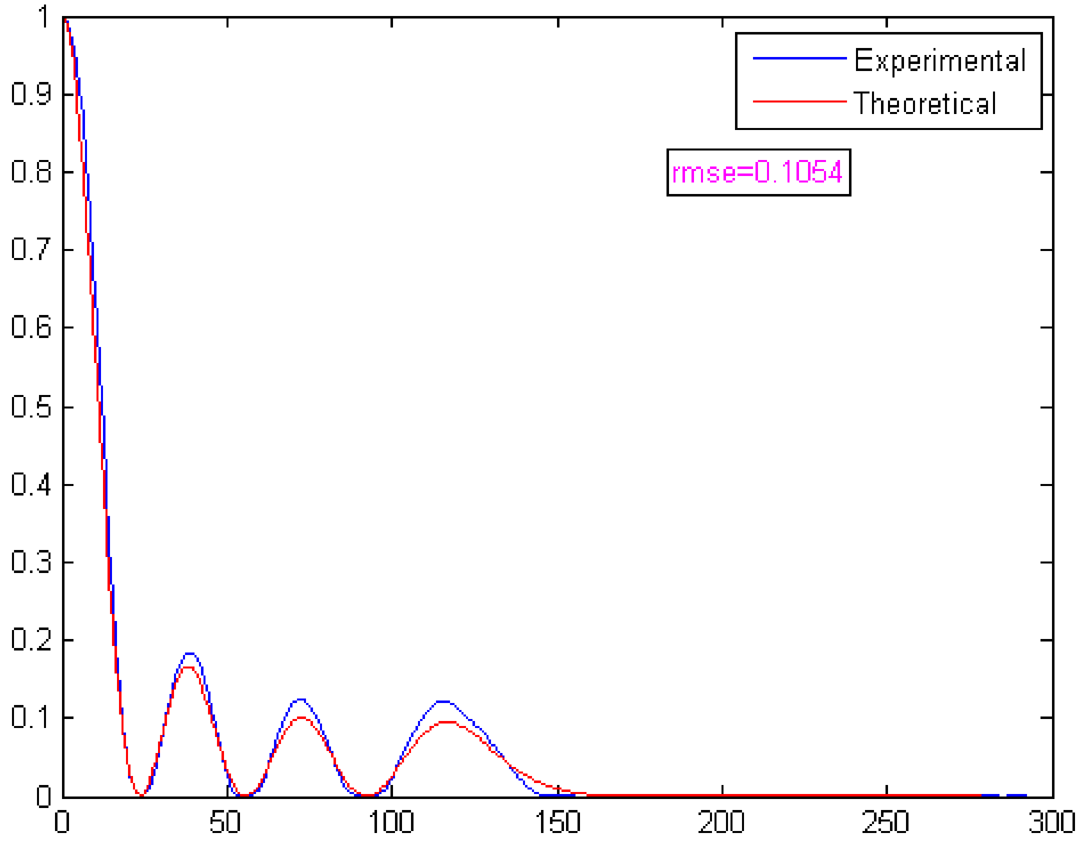
$$\Psi(\phi, a) = f(a) sin(\phi)$$

$$C_q^a = J_q[f(a)]$$

$$J_1[f(a)] = Aa$$

V. Arrizón, U. Ruiz, R. Carrada, and L. A. González. "Pixelated Phase Computer Holograms for the Accurate Encoding of Scalar Complex Fields." *Journal of the Optical Society of America A* 24.11 (2007): 3500. Web.

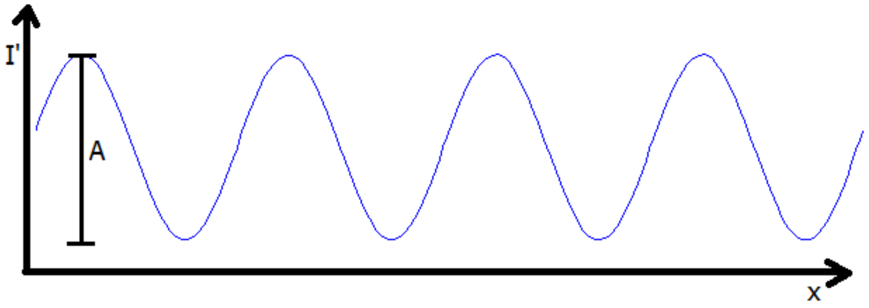




Experimental
Theoretical
rmse=0.1054
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
50
100
150
200
250
300

Calibrating the SLM

- Create a lookup table between grayscale value and induced phase delay
- Varied amplitude on of grating on the SLM and measured the corresponding relative 1st diffraction order intensities
- Assumed flat-field (uniform) illumination
  - Corrected by creating inverse normalized intensity phase mask



$$\Psi = J^{-1}[Aa]sin(\phi)$$

$$\phi' = gsvalue = [\phi * \frac{255}{2\pi}]$$

$$\phi' = \gamma^{-1}[\phi * \frac{255}{2\pi}]$$

$$\Psi = J^{-1}[Aa]sin(\gamma^{-1}[\phi * \frac{255}{2\pi}])$$

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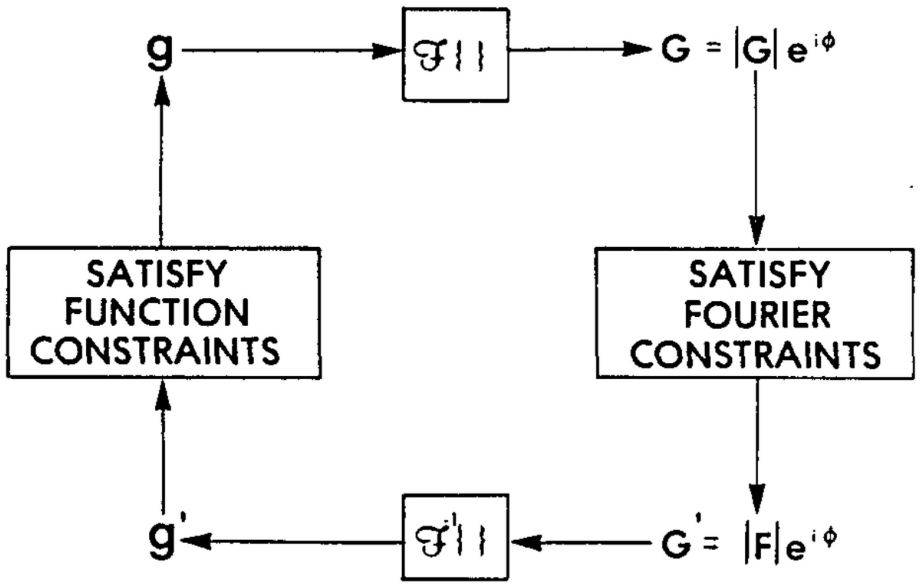
Phase Retrieval

- Used to recover the phase when only intensity information is known
- Calculate phase mask needed to convert radial modes to Gaussians
- Used Gerchberg-Saxton method of phase retrieval



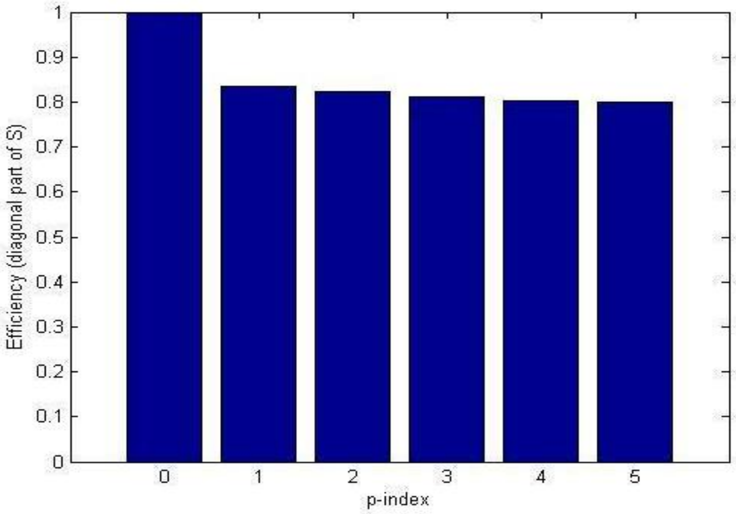
J. Fienup, "Phase retrieval algorithms: a comparison," Appl. Opt. 21, 2758-2769 (1982).

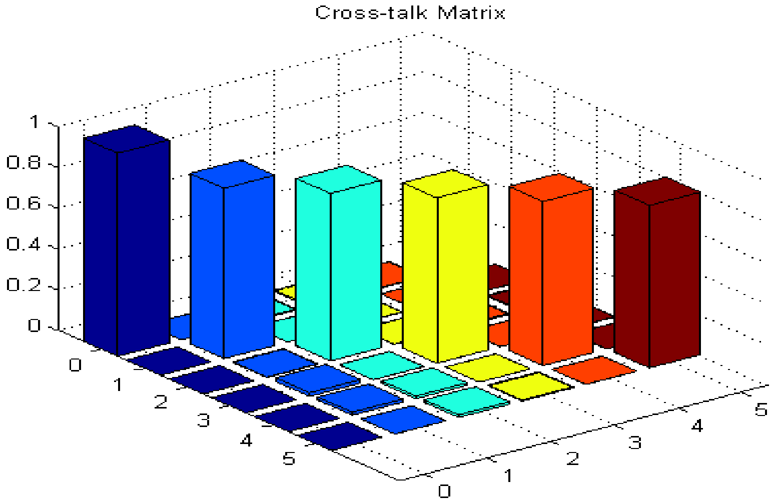
Phase Retrieval Contd…

- Used Hankel transform because it forces phase masks to be radially symmetric

$$g_R(r) = 2\pi \int_0^\infty \rho G_0(\rho) J_0(2\rho) d\rho$$
$$G_0(\rho) = 2\pi \int_0^\infty r g_R(r) J_0(2\rho) dr$$

- Theoretical coupling efficiencies of around 80% for p=1-5

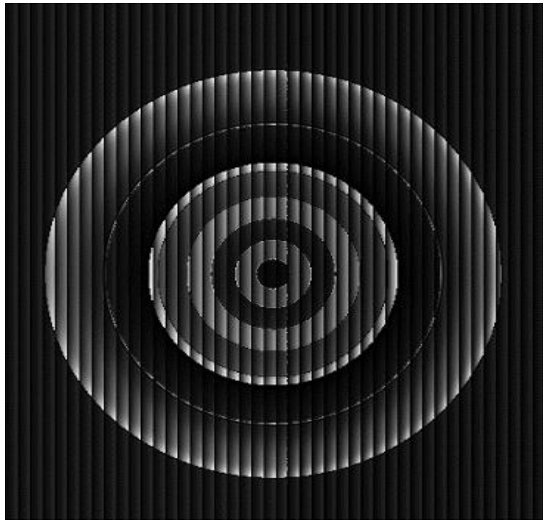




J. W. Goodman. *Introduction to Fourier Optics*. San Francisco: McGraw-Hill, 1968. Print.

Holograms for Second SLM

- No amplitude modulation, only phase modulation
  - Allows for usage of a binary diffraction grating
- Each half of SLM controlled separately
  - Variable tip, tilt, lens, and centering can be added in real-time



$$\Psi(\phi, a) = f(a)$$

$$C_q^a = sinc[q - f(a)]$$

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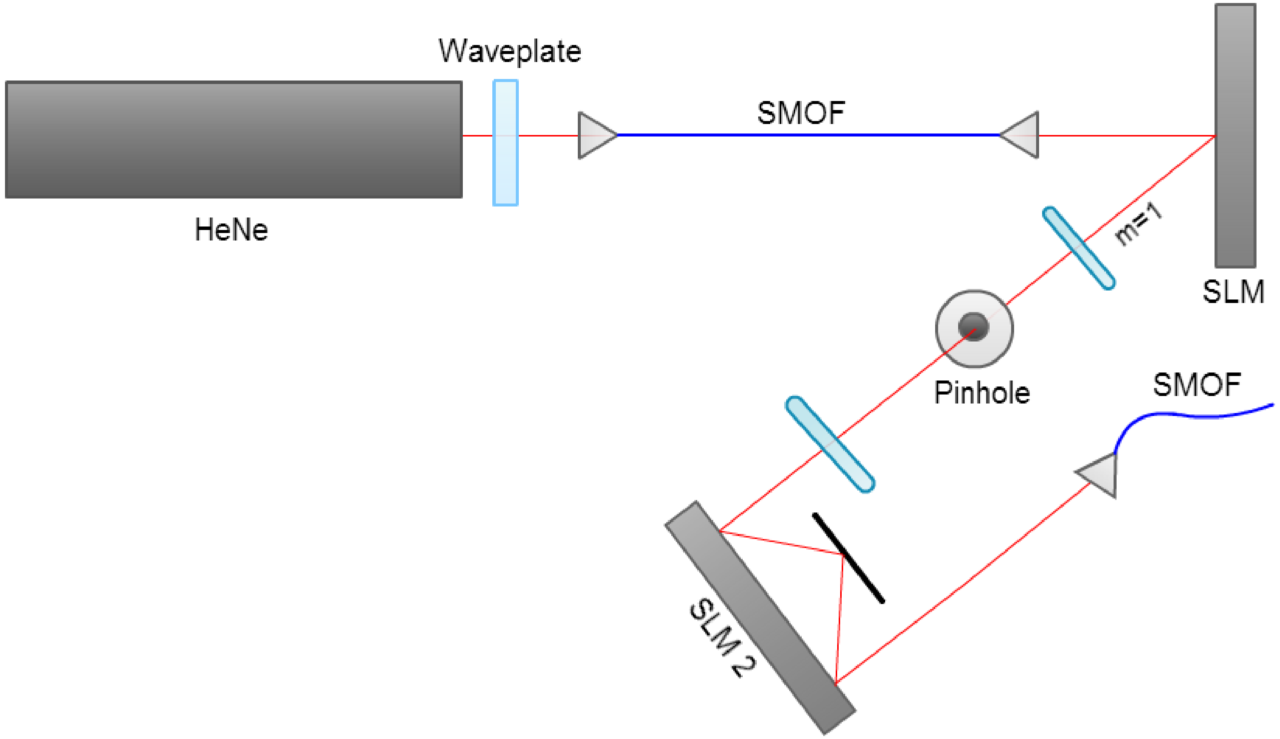
Experimental Set-up



Future Work

- Finish aligning second half of set up
- Take coupling efficiency data

Acknowledgements

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- Everyone in the Boyd Group

References

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