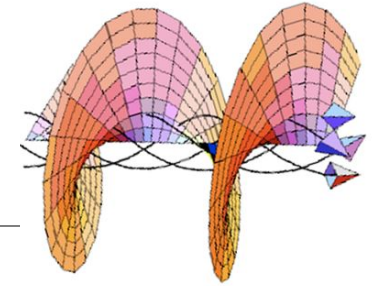


Separating Laguerre-Gaussian Radial Modes Via Projective Measurements

RACHEL SAMPSON

Laguerre-Gaussian (LG) Beams



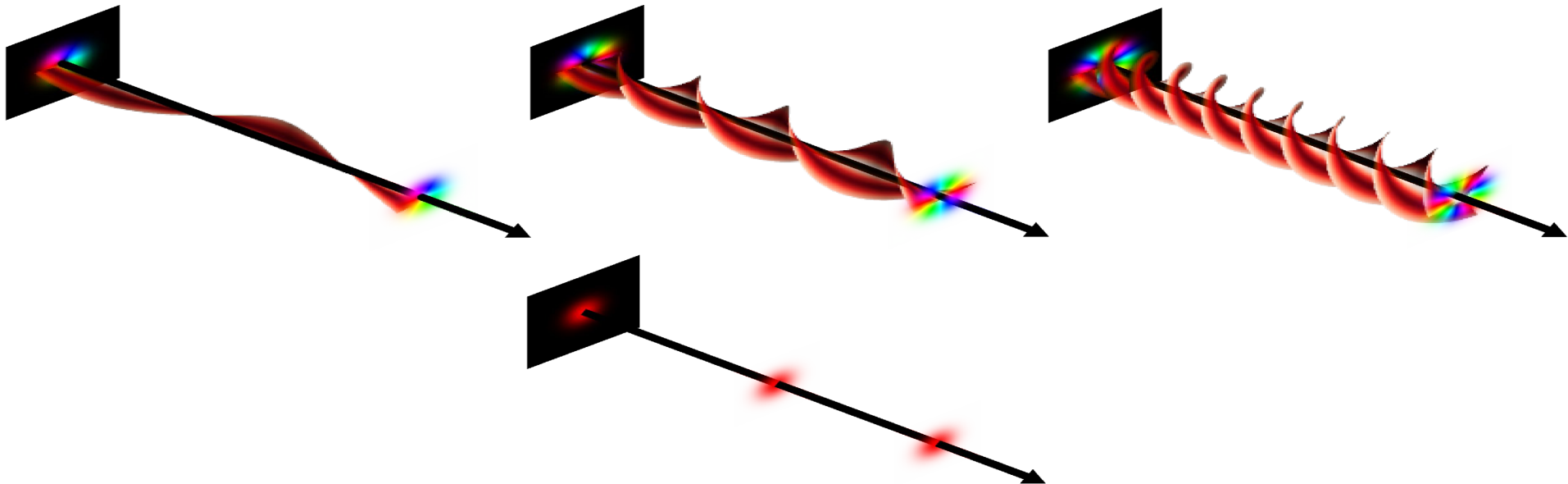
- Cylindrically symmetric and form a basis set for paraxial light beams
- Properties of beams defined by two indices:
 - Azimuthal number (l) – number of spirals in 2π , related to the orbital angular momentum (OAM) by $L_z = l \hbar$
 - Radial index (p) - number of rings plus one

$$U_{p,\ell}(r, \phi, z) = \frac{C_{\ell p}^{LG}}{w(z)} \left(\frac{\sqrt{2}r}{w(z)} \right)^{|\ell|} L_p^{|\ell|} \left(\frac{2r^2}{w(z)^2} \right) \exp \left(\frac{-r^2}{w(z)^2} \right) \exp \left(ik \frac{r^2}{2R(z)} \right) \exp[i(2p+|\ell|+1)\xi(z)] \exp(i\ell\phi)$$



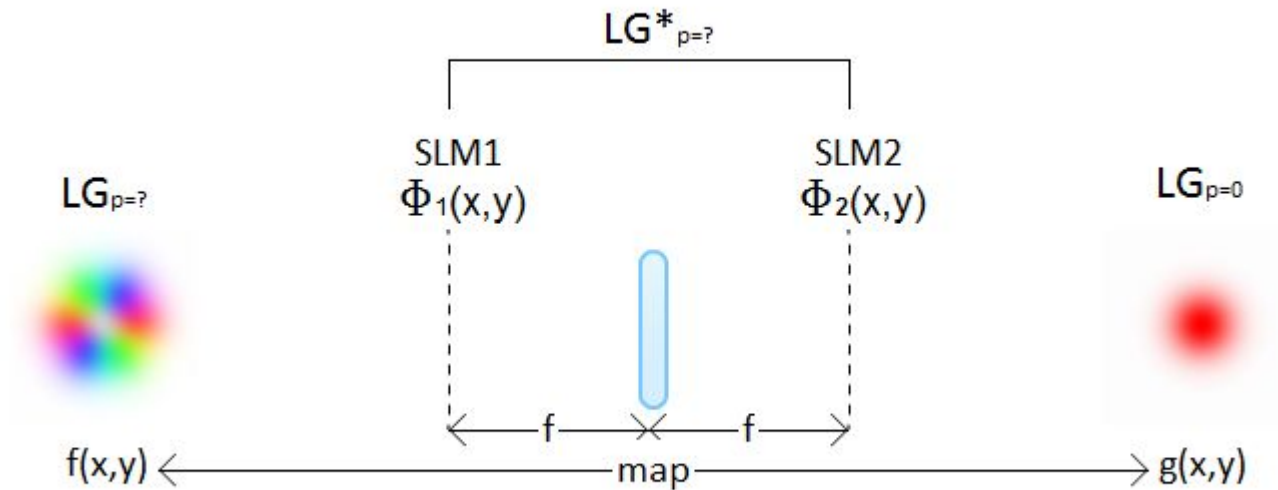
LG OAM Modes

- Applications in:
 - Free space communication (increase channel capacity)
 - QKD (increase tolerance to eavesdropping attacks)
- Radial modes have received less attention



Projective Measurements

- Mode is imaged onto a conjugate mode and the resulting field is propagated and coupled into a single mode optical fiber (SMOF)
- Method gives yes or no, requires scanning through the radial indices to determine the mode

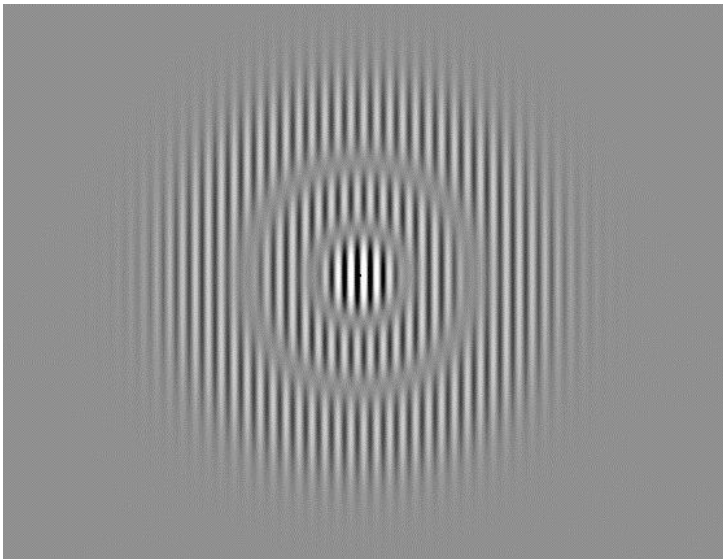


TASKS

- ❖ Generate radial modes
- ❖ Transform radial modes into Gaussians
- ❖ Couple into fiber

Creating Radial Modes

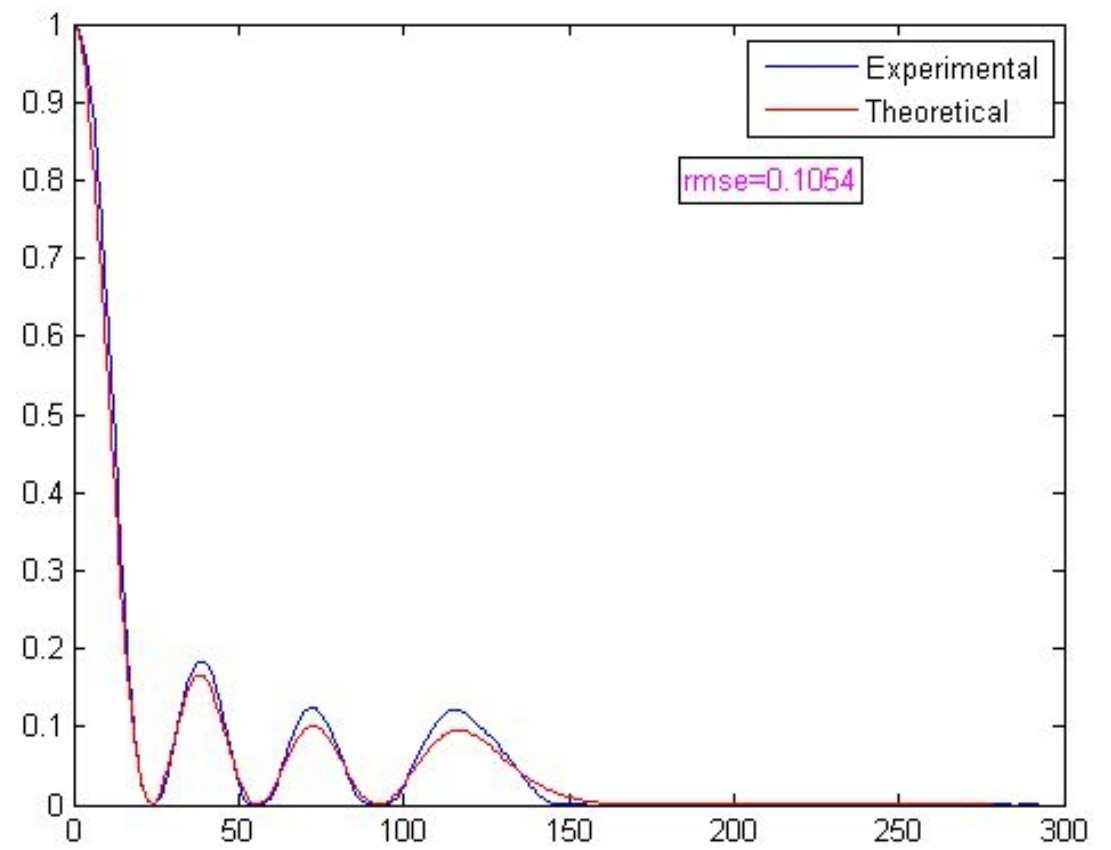
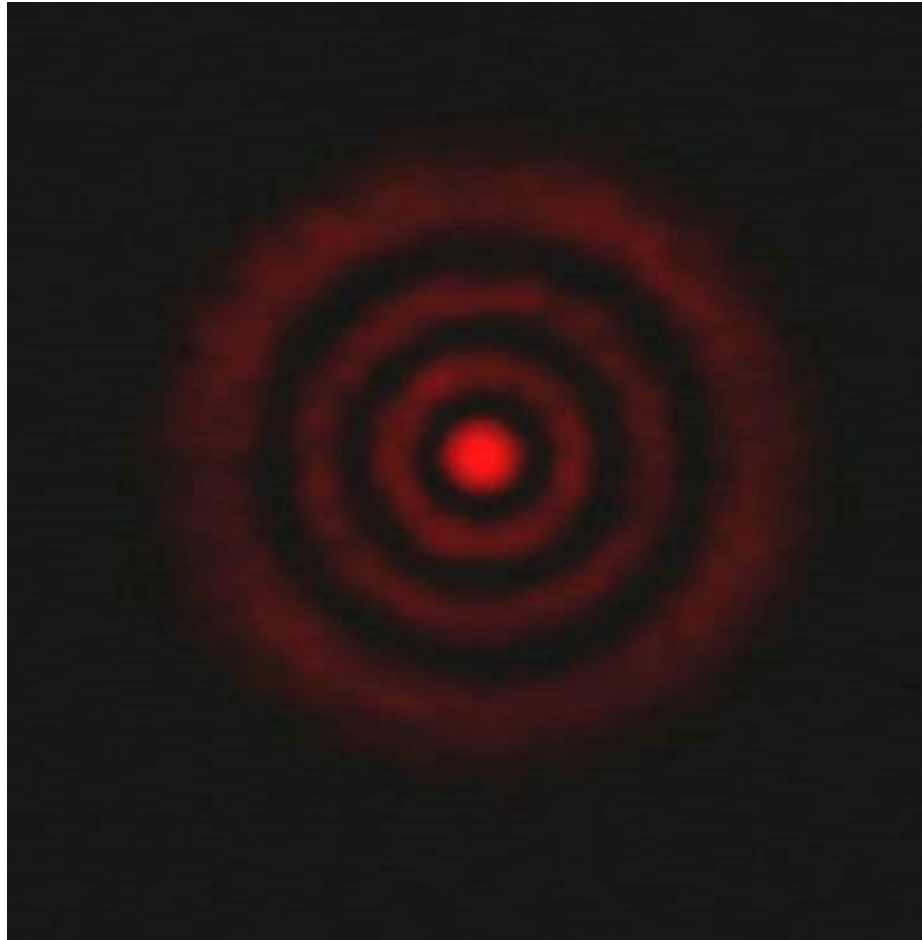
- Used computer generated holograms placed on a spatial light modulator
 - Essentially phase mask with blazed diffraction grating on top
 - Can be implemented using spatial light modulator (SLM) with reduced phase domain



$$\Psi(\phi, a) = f(a)\sin(\phi)$$

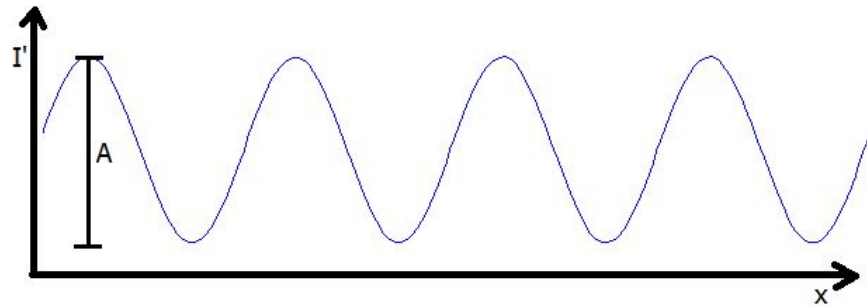
$$C_q^a = J_q[f(a)]$$

$$J_1[f(a)] = Aa$$



Calibrating the SLM

- Create a lookup table between grayscale value and induced phase delay
- Varied amplitude on of grating on the SLM and measured the corresponding relative 1st diffraction order intensities
- Assumed flat-field (uniform) illumination
 - Corrected by creating inverse normalized intensity phase mask



$$\Psi = J^{-1}[Aa] \sin(\phi)$$

$$\phi' = g\text{value} = \left[\phi * \frac{255}{2\pi} \right]$$

$$\phi' = \gamma^{-1} \left[\phi * \frac{255}{2\pi} \right]$$

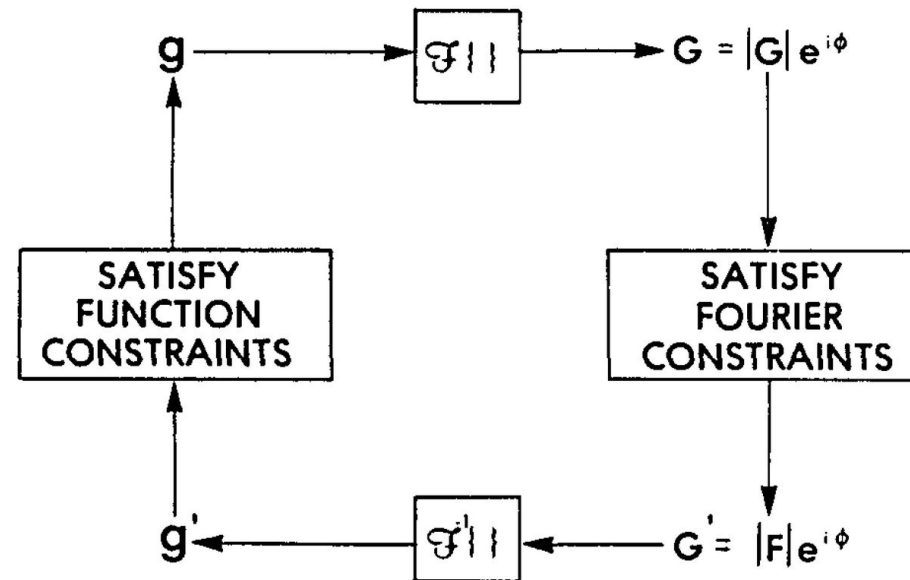
$$\Psi = J^{-1}[Aa] \sin(\gamma^{-1} \left[\phi * \frac{255}{2\pi} \right])$$

TASKS

- ❖ Generate radial modes
- ❖ Transform radial modes into Gaussians
- ❖ Couple into fiber

Phase Retrieval

- Used to recover the phase when only intensity information is known
- Calculate phase mask needed to convert radial modes to Gaussians
- Used Gerchberg-Saxton method of phase retrieval

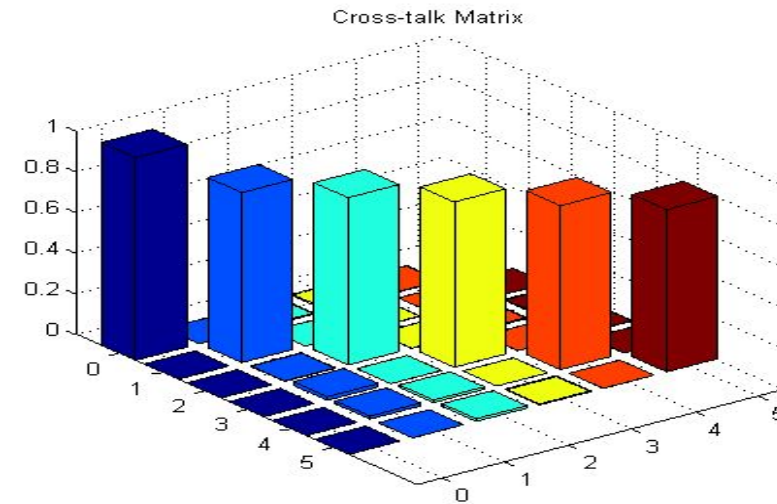
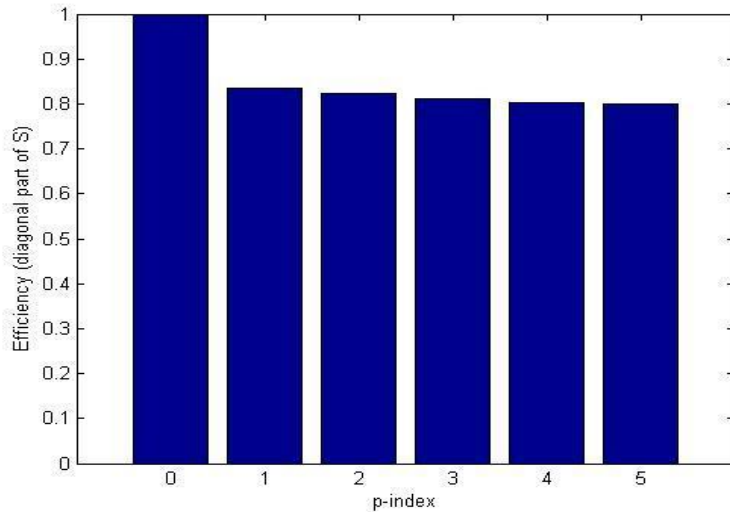


Phase Retrieval Contd...

- Used Hankel transform because it forces phase masks to be radially symmetric

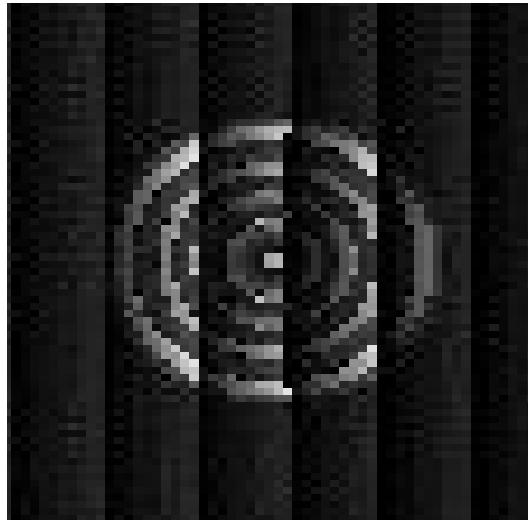
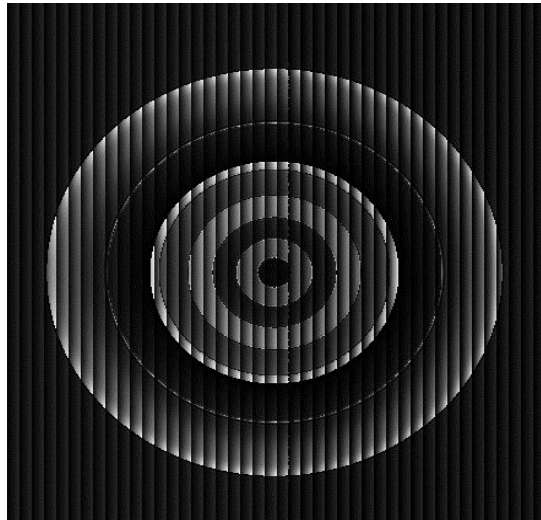
$$g_R(r) = 2\pi \int_0^\infty \rho G_0(\rho) J_0(2\rho) d\rho$$
$$G_0(\rho) = 2\pi \int_0^\infty r g_R(r) J_0(2\rho) dr$$

- Theoretical coupling efficiencies of around 80% for p=1-5



Holograms for Second SLM

- No amplitude modulation, only phase modulation
 - Allows for usage of a binary diffraction grating
- Each half of SLM controlled separately
 - Variable tip, tilt, lens, and centering can be added in real-time



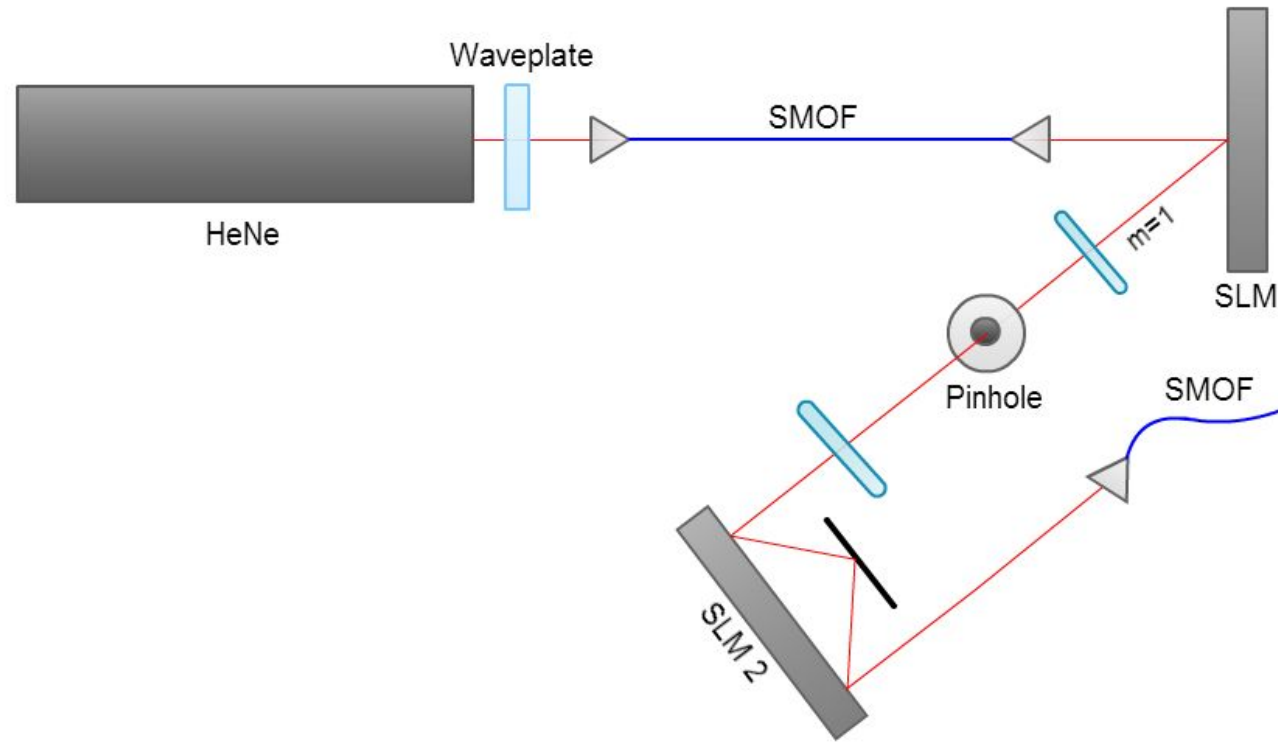
$$\Psi(\phi, a) = f(a)$$

$$C_q^a = \text{sinc}[q - f(a)]$$

TASKS

- ❖ Generate radial modes
- ❖ Transform radial modes into Gaussians
- ❖ Couple into fiber

Experimental Set-up



Future Work

- Finish aligning second half of set up
- Take coupling efficiency data

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