Creating Dark Lines in Space with Linear Zone Plates

Intel Science Talent Search

(Physics)

November 2010

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1 Introduction

This project was inspired by a general fascination with zone plates and the interesting work on zone plates containing a π-phase jump done by Vinas et al. [1]. Such specialized optical devices create a series of dark focal points in space which can be used for precision alignment. Zone plates are optical elements which focus light by diffraction instead of refraction as in conventional lenses [8]. A Fresnel-type zone plate has many concentric circular regions which are alternately opaque and transparent. The widths and radii of the zones are such that light diffracted from the center of every transparent region reaches a focal point on the axis of the plate in phase [7]. The result is constructive interference, and the creation of a bright spot. Such Fresnel zone plates also have minor bright spots along the beam axis, which occur when the phase shift from adjacent transparent regions is an integer multiple of 2π.

Linear zone plates have one-dimensional patterns and act like cylindrical lenses to create line foci [10]. Introducing a π-phase jump across the center of such a zone plate effectively inverts the focus, creating a "dark line in space" between two bright lines. A π-phase jump occurs when one half of the zone plate has transparent regions where the other half has opaque regions. The phase jump causes half of the light passing through the zone plate to be completely out of phase with the other half of the light, thus causing fully destructive interference along the dark line [1]. *Sinusoidal* zone plates have smooth transitions from completely transparent regions to completely opaque regions, unlike binary zone plates, which are strictly opaque or transparent. The smooth transitions lead to sharper focal lines, and eliminate the minor focal lines that are present with the use of binary zone plates [3, 9]. Zone plate patterns were generated by using Mathematica. Transmittance equations were derived for each of the four zone plates, and Mathematica's ContourPlot function was used to plot these equations which yielded the zone plate designs. Several 8 mm square patterns generated in this way were imaged on to 35 mm black and white film by photographer Gene Lewis [6].

The goal of our project was to create and evaluate four linear zone plates. There were two binary zone plates, a conventional one and one with a π-phase jump. Likewise, there are two sinusoidal zone plates, a conventional one and one with a π-phase jump. The zone plates were illuminated by a HeNe laser. The interference patterns from the zone plates were projected directly onto a CCD camera without a lens. The greatest challenge throughout this project was making sure that the zone plate designs were of a high quality despite their complexity.

2 Background

Zone plates were first invented by Lord Rayleigh in April 11, 1871 [7]. A zone plate is made of alternating rings of opaque and transparent rings. Traditional zone plates are made such that the alternating transparent rings only allow light that constructively interferes to pass through. This causes the light coming from the transparent zones to be out of phase by no more than π. Zone plates have two unique configurations, even and odd. Odd zone plates have a transparent zone at the center, whereas even zone plates have an opaque zone at the center [7]. Despite this difference, both even and odd zone plates produce the same interference pattern. But, the light coming from an odd zone plate is out of phase by π with the light coming from an even zone plate. This concept is especially important to the idea of π-phase jumps.

2.1 Zone Plate Basics

In order to achieve constructive interference at the focus, the alternating zones must have specific widths such that the difference in path length from the distance and the center of the zone plate to the focus is an integer multiple of the wavelength of the light [7]. This ensures that the light coming from a zone is not out of phase by more than π with the light coming from another zone. Therefore, if one has an odd circular Fresnel zone plate,

then the path length of the light coming from the center of the zone plate is f. If the light coming from each transparent zone can be out of phase by no more than π, then the path difference between the light from the center of the first zone and the light from the edge of the first zone must be $\frac{\lambda}{2}$. Therefore, the path length for the light from the edge of the first transparent zone to the focus is $f + \frac{\lambda}{2}$. Furthermore, the light traveling from one edge must travel an additional $\frac{\lambda}{2}$ compared to the light coming from the previous edge [7]. Therefore, the path length of any light coming from the edge of any boundary, which is an n number of boundaries away from the center is

$$d_n = f + \frac{n\lambda}{2}$$

Furthermore, if the radius of the nth boundary is r_n, then by using the Pythagorean theorem,

$$d_n = \sqrt{f^2 + r_n^2}$$

Then setting the two equations equal to each other and solving for r_n, the distance of the zone boundaries from the center of the zone plate is described by the equation,

$$r_n = \sqrt{fn\lambda + \frac{n^2\lambda^2}{4}}$$

2.2 The π-Phase Jump

For the importance of analyzing the interference pattern of the zone plate, one must analyze the wave equation in relation to the zone plate. The wave equation is,

$$\Phi(d,t) = \frac{A}{d}e^{i(kd-\omega t)}$$

Then by considering the light illuminating the zone plate to be uniform, one can ignore the

time dependence portion of the wave equation, thus yielding,

$$\Phi(d) = \frac{A}{d} e^{ikd}$$

It is important to note why two wavefronts that arrive at the same point, out of phase by π, destructively interfere. This can be accomplished by examining the sum of the wave equations of the two different wavefronts,

$$\Phi_1 + \Phi_2 = \frac{A}{d} e^{ikd} + \frac{A}{d} e^{i(kd+\pi)}$$

Using Euler's formula, this can be rewritten as,

$$\begin{aligned} \Phi_1 + \Phi_2 &= \frac{A}{d}(\cos kd + i \sin kd + \cos kd + \pi + i \sin kd + \pi) \\ &= \frac{A}{d}(\cos kd + i \sin kd - \cos kd - i \sin kd) \\ &= 0 \end{aligned}$$

This concept is important to understand, because, the zone plate creates destructive interference by using this property of electromagnetic radiation. By combining an even zone plate and an odd zone plate it is possible to create destructive interference because the light that comes through the even side of the zone plate will be out of phase by π in relation to the odd zone plate.

Creating a zone plate that produces two seperate wavefronts that are out of phase by π is known as introducing a π-phase jump to a zone plate. This destructive interference at the focus of the zone plate is what results in the dark focal spots.

2.3 Sinusoidal Zone Plates

Sinusoidal zone plates differ from traditional binary zone plates in that the transparency of the zone plate varies sinusoidally from the center of the zone plate, unlike binary zone plates which are strictly transparent or opaque [3, 9]. The phase of the light coming from the transparent regions of the zone plate is not uniform, so binary zone plates produce extraneous background interference patterns. They also produce minor foci, which form at fractional distances of the focal length, because of their non-uniformity. However, sinusoidal zone plates reduce any such background interference as well as minor foci, which helps to further improve the quality of the dark lines, thus creating finer dark lines [3, 9].

The binary transmittance values for a zone plate which alternate from 0 to 1 are described by the equation,

$$t(r^2) = \frac{1}{2} + (\frac{1}{i\pi}) \sum_{p=-\infty}^{\infty} e^{-\frac{ip\pi r^2}{r_1^2}}$$

However, this has to be simplified to a sinusoidal variation, which then yields an equation of the form,

$$t(r) = \frac{1 \pm \cos kr^2}{2},$$

where k is a constant.

The value of k can be determined by recognizing that although different, both sinusoidal and binray zone plates possess many of the same characteristics. Therefore, the equations that describe a binary zone plate can still be used to describe sinusoidal zone plates, which can then be used to find the constant k. It was important in this project to make the sinusoidal and binary zone plates as consistent as possible, so it became necessary to derive an equation describing the transmittance values of binary zone plates in a simple manner as well. This can easily be done, by recognizing that the transmittance values have to be either 0 or 1, thus resulting in the equation,

$$t(r) = \frac{1 \pm \mathrm{sgn}(\cos kr^2)}{2}$$

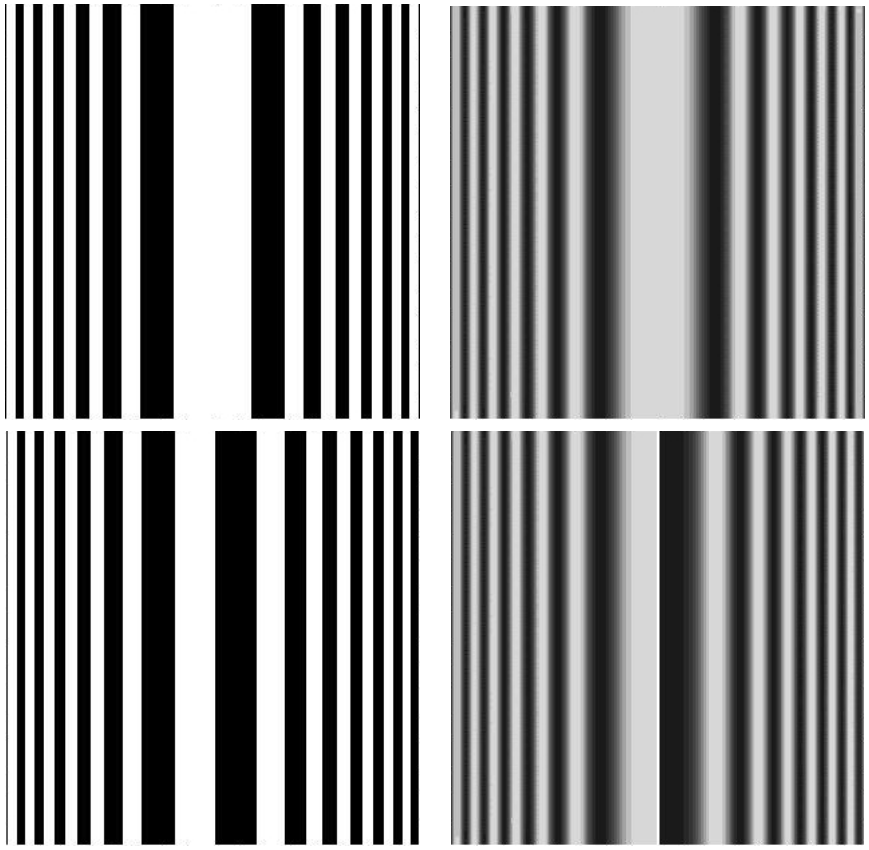


Figure 1: Zone plate designs: binary zone plate (top left), sinusoidal zone plate (top right), binary zone plate with π-phase jump (bottom left), and sinusoidal zone plate with π-phase jump (bottom right).

By using the equation,

$$r_n = \sqrt{fn\lambda + \frac{n^2\lambda^2}{4}}$$

it was possible to find the constant k, by recognizing that r_n, indicates the distance at which a binary zone plate changes transmittance, which is the same as when $t(r)$ changes from 1 to 0 or vice versa. This occurs when $\cos kr^2$ is equal to 0. So, assuming $r = r_n$ and that $n = 1$,

$$\cos k(fn\lambda + \frac{n^2\lambda^2}{4}) = 0$$

$$k(fn\lambda + \frac{n^2\lambda^2}{4}) = \frac{\pi}{2}$$

$$k = \frac{\pi}{2fn\lambda + \frac{n^2\lambda^2}{2}}$$

$$k = \frac{\pi}{2f\lambda + \frac{\lambda^2}{2}}$$

Now the equation for the transmittance of a sinusoidal zone plate becomes,

$$t(r) = \frac{1}{2}(1 \pm \cos \frac{\pi r^2}{2f\lambda + \frac{\lambda^2}{2}})$$

2.4 Interference Patterns

When zone plates are illuminated they produce distinct interference patterns, based on the shape of the zones, and whether or not the zone plate is binary or sinusoidal. To analyze the results of this paper, it is important to gain an understanding of the interference patterns produced by zone plates.

The interference patterns produced by zone plates are the result of all the rays of light being diffracted across all the Fresnel zones. The equation that describes the amplitude distribution in the focal plane for a linear zone plate is,

$$U(x_0, f) = -\frac{C_1}{\sqrt{\lambda f}} \int P_t(x) e^{-2\pi i x f_x} \, dx$$

where C_1 is the constant phase factor,

$$C_1 = e^{i(\frac{kx_0^2}{2f + kf - \frac{\pi}{4}})}$$

and for a normal binary linear zone plate,

$$P_t = \text{rect}(\frac{x}{A})$$

given that A is the width of the zone plate. In this case the intensity distribution in the focal plane is,

$$I(x_0, f) = I_0 \text{sinc}^2(t)$$

where $I_0 = \frac{A^2}{\lambda f}$, $t = \frac{Ax_0}{\lambda f}$, and $\text{sinc}(t) = \frac{\sin x}{x}$ [1].

Because sinusoidal zone plates do not have the minor foci like binary zone plates, and an axially continuous dark line is desirable, it is important to analyze the axial amplitude distribution as well. The equation describing the axial amplitude distribution of a regular binary linear zone plate is,

$$U(0,z) = \frac{C_2}{\sqrt{\lambda f}} \int P_t(x) e^{\frac{ikx^2}{2}(\frac{1}{f}-\frac{1}{z})} dx$$

where C_2 is another constant phse factor,

$$C_2 = e^{i(kz-\frac{\pi}{4})}$$

Again assuming $P_t = rect(\frac{x}{A})$ the axial intensity distribution becomes,

$$I(0,z) = \frac{2f}{|f-z|}[S^2(\gamma) + C^2(\gamma)]$$

where $\gamma = \frac{A}{2}\sqrt{\frac{k}{2|\frac{1}{f}-\frac{1}{z}|}}$, $S(x) = \sqrt{\frac{2}{\pi}} \int \sin t^2 dt$ and $C(x) = \sqrt{\frac{2}{\pi}} \int \cos t^2 dt$ and $S(x)$ and $C(x)$ are Fresnel integrals [1].

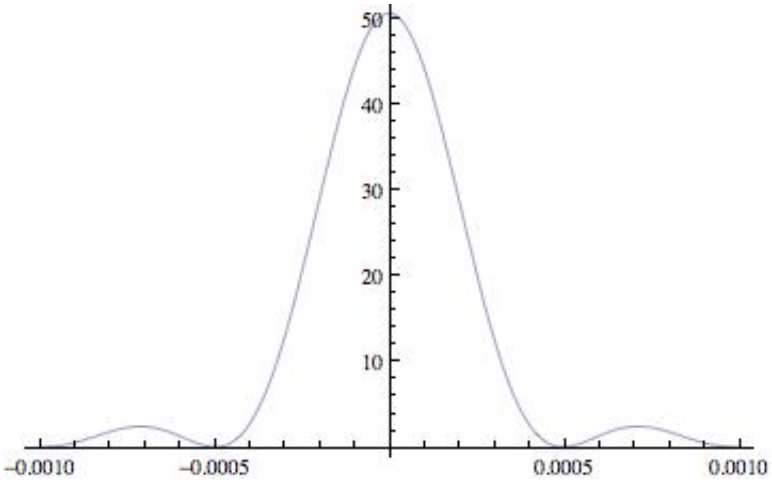


Figure 2: The intensity distribution in the focal plane of a regular binary linear zone plate

Now, in order to analyze the intensity distribution for a zone plate with a π-phase jump, a new P_t must be introduced, which takes into account the π-phase jump. For complete destructive interference to occur, half the light must be out of phase with the other half of the light. This can by splitting the zone plate in half and making one side an even configuration and the other side an odd configuration. This means that the light coming from the even side will be exactly out of phase by π with other half, hence giving rise to the name π-phase jump.

The P_t for this type of zone plate is

$$P_t(x) = \text{rect}(\frac{x + \frac{A}{4}}{\frac{A}{2}}) - \text{rect}(\frac{x - \frac{A}{4}}{\frac{A}{2}})$$

After substituting this function into $U(x_0, f)$ and $U(0, z)$, the intensity distributions in the focal plane and along the axis become,

$$I(x_0, f) = I_0 \text{sinc}^2(\frac{t}{2}) \sin^2(\frac{\pi t}{2}),$$

$$I(0, z) = 0$$

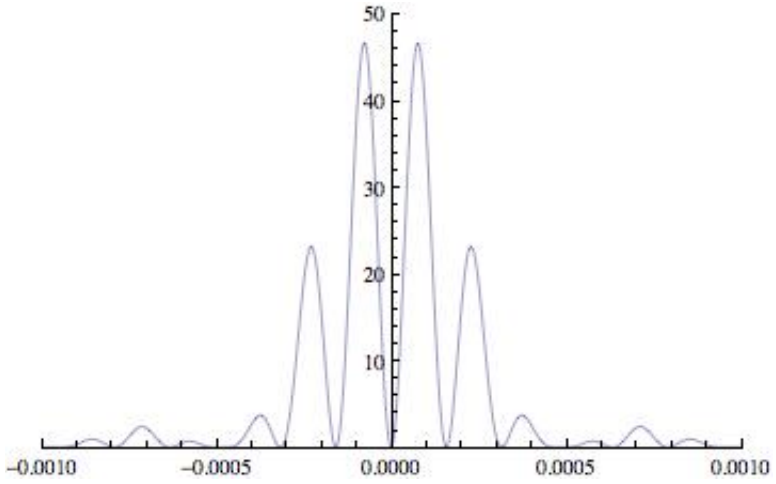


Figure 3: The intensity distribution in the focal plane of a binary zone plate with a π-phase jump

However, all of these equations represent the intensity distributions of binary zone plates. In this paper the same zone plate designs are rendered in a sinusoidal configuration as well. Sinusoidal zone plates drastically reduce the amount of background interference occuring in the focal plane, thus creating a much finer dark focal line. One of the primary applications for dark focal lines created by zone plates are their applications in optical alignment, therefore for more precision alignment a finer focal line is preferable.

The goal of this experiment is create sharper and finer dark lines through the use of sinusoidal zone plates, at the same time we are looking to eliminate the unnecessary background radiation. So, it was expected that the data from the sinusoidal zone plates would show that the dark region in the middle of the interference pattern is thinner than that of the dark line produced by the use of binary zone plates.

3 Creating the Zone Plates

Wolfram Mathematica's ContourPlot Program was used to generate the designs of the zone plates. The transmittance equations for both the binary and sinusoidal zone plates were plotted using the ContourPlot function, which generated false color images of the zone plates. Then the sinusoidal zone plates were placed in a grayscale color space, and the binary zone plates were placed in a binary color space. The equation

$$t(r) = \frac{1}{2}(1 + \cos\frac{\pi r^2}{2f\lambda + \frac{\lambda^2}{2}})$$

was plotted in Mathematica, where $\lambda = 632.8$nm, and $f = 2m$, to obtain the design for the sinusoidal zone plate. The value for f is a result of the fact that having a focal length significantly larger than 2, would result in a loss of quality in the image generated by Mathematica. On the other hand, if the focal length is significantly smaller, there would not be enough zones in the design to create any significant interference pattern. On the other hand, the equation used for the binary zone plate differs only in its use of the function $\text{sgn}(x)$ to

make the transmittance values binary,

$$t(r) = \frac{1}{2}(1 + \text{sgn}(\cos\frac{\pi r^2}{2f\lambda + \frac{\lambda^2}{2}}))$$

For the sinusoidal zone plate with a π-phase jumps, the piecewise equation,

$$t(r) = \begin{cases} \frac{1}{2}(1 + \cos\frac{\pi r^2}{2f\lambda + \frac{\lambda^2}{2}}) & : r \geq 0 \\ \frac{1}{2}(1 - \cos\frac{\pi r^2}{2f\lambda + \frac{\lambda^2}{2}}) & : r < 0 \end{cases}$$

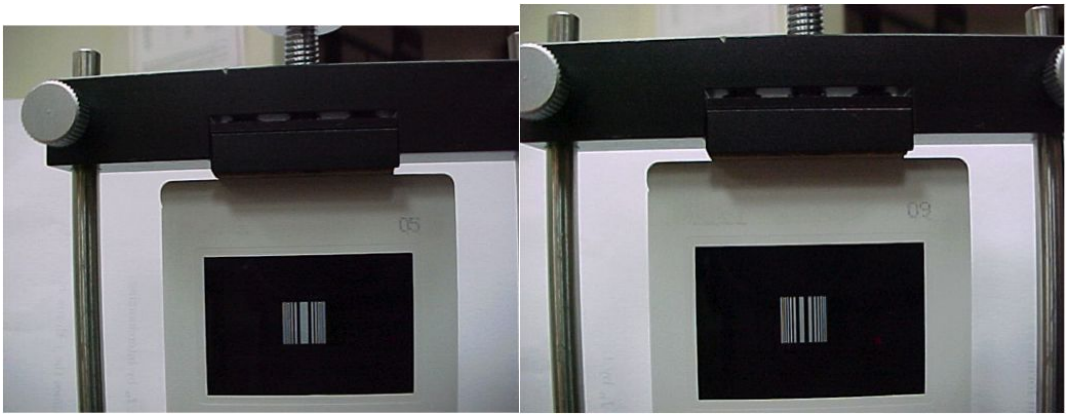
was used. Whereas, for the binary zone plate with a π-phase jump, the $\text{sgn}(x)$ function has to be introduced again to make the transmittance values binary,

$$t(r) = \begin{cases} \frac{1}{2}(1 + \text{sgn}(\cos\frac{\pi r^2}{2f\lambda + \frac{\lambda^2}{2}})) & : r \geq 0 \\ \frac{1}{2}(1 - \text{sgn}(\cos\frac{\pi r^2}{2f\lambda + \frac{\lambda^2}{2}})) & : r < 0 \end{cases}$$

The zone plate designs were sent to Darkroom Specialties LLC, Eugene, OR, which transferred the designs onto trasparent slides. They downloaded the images into Adobe Photoshop, and placed against a background. Then the images were converted to a grayscale, RGB color space. After which the images were sent to a Raster Image Processor, which broke them up into red, green, and blue imaging files. Each of these color channels were sent to Lasermaster 35mm film recorder, which recorded each color channel's luminosity to ensure that the best quality image could be obtained. After exposing each of the films for 6.5 minutes, they were processed in an E-6 slide processor. Then after allowing them to dry, the films were placed inside plastic frames [6].

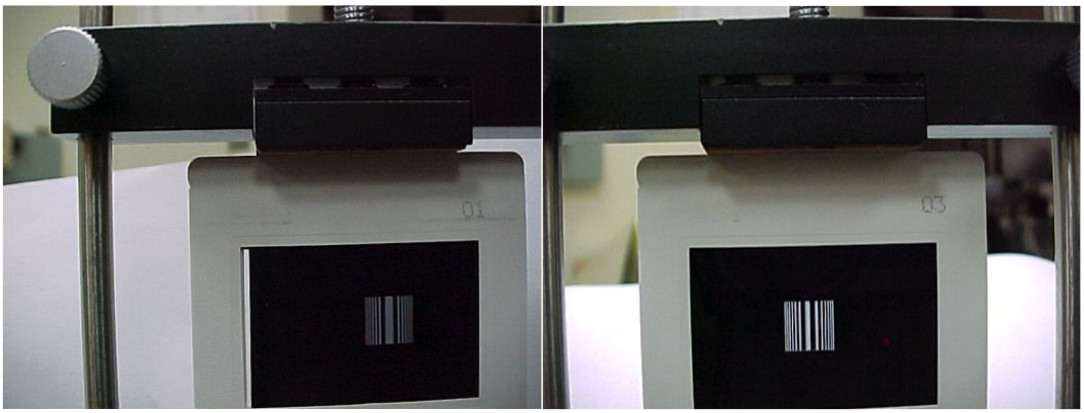
4 Testing the Zone Plates

In order to obtain data about the intensity distributions in the focal planes of the zone plates, it was important to illuminate the entire zone plate pattern, and capture the resulting interference pattern. For the purpose of illuminating the zone plate, a 632.8nm HeNe laser



(a) Binary zone plate (b) Binary zone plate with π-phase jump

Figure 4: Binary zone plate slides



(a) Sinusoidal zone plate (b) Sinusoidal zone plate with π-phase jump

Figure 5: Sinusoidal zone plate slides

was used. However, the width of the beam was far too small to illuminate the zone plates, which were each 8mm by 8mm. Therefore, the beam had to be significantly magnified. However, another problem with using the light directly from the laser was that a laser beam diverges. If a diverging beam were to illuminate the zone plate, there would be major distortions in terms of the focal length and the size of the focal line.

So, to address this problem, a telescope system was used to both magnify and approximately collimate the beam. A telescope system uses two converging lenses, placed apart from each other by the sum of their focal lengths. The first lens in the telescope system brings the laser beam down to a focus and then causes it diverge at a much higher rate to

ensure the beam width is large enough to illuminate the entire zone plate. Then the second lens collimates the diverging beam by refracting the light back towards the axis.

At first a floppy disk camera was used to capture the interference pattern. Later the interference pattern was captured by a CCD(Charged Coupled Device) camera connected to a computer. The CCD stored the images of the interference pattern, so that the intensity distribution for the zone plate can be acquired later. The CCD camera was not able to process the intense laser light, so an attenuator became necessary. Two nearly crossed polarizers were used for this pupose.

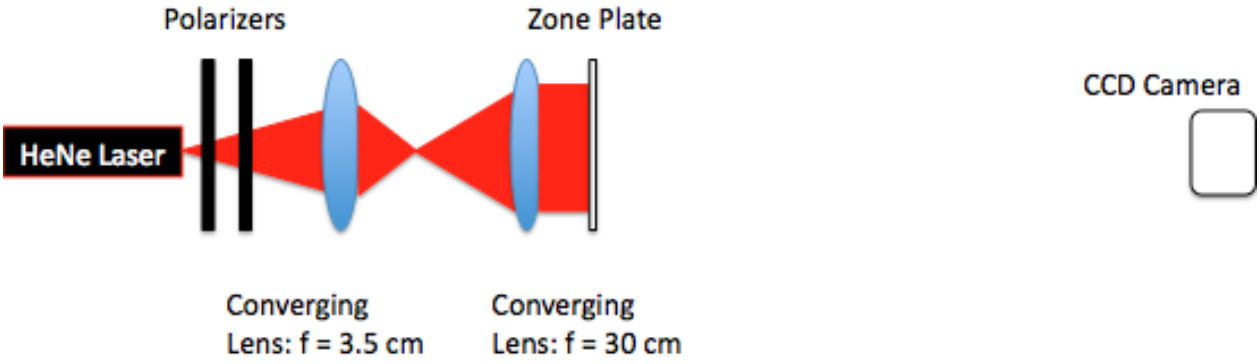


Figure 6: The experimental setup for illuminating the zone plates. Note: not to scale.

5 Results

After the pictures were taken using the CCD camera, they were compressed to a thickness of one pixel in XV image editor. Then these new images were converted to .pgm files, and later .dat files. The intensity values were used to create plots of the intensity distributions in the focal planes of the zone plates.

These .dat files first needed to formatted to exclude extraneous numbers that did not

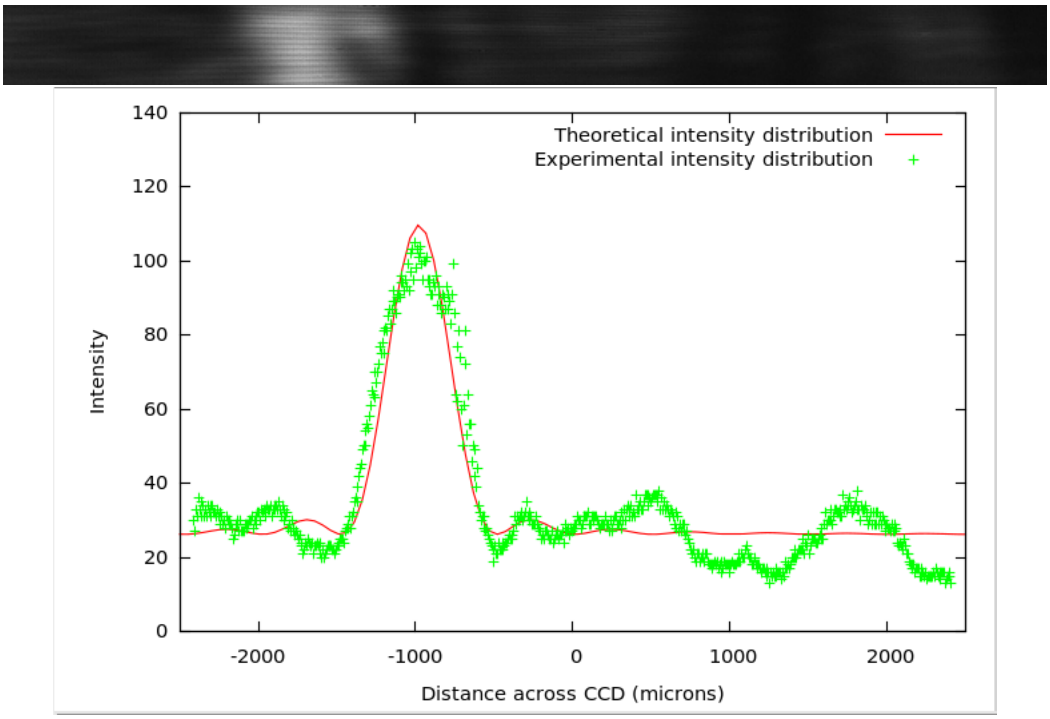


Figure 7: The intensity distribution in the focal plane of a binary linear zone plate.

represent data. So, a simple loop was used to comment out the first 11 lines of every files. After this each of the data files were plotted in GNUplot, to yield the experimental intensity distribution. Then for the normal binary and sinusoidal zone plates the equation,

$$I(x_0, f) = m(I_0 \text{sinc}^2(\frac{A(x - x_0)}{f\lambda})) + i_0$$

where m is the magnitude of the peak of the distribution, x_0 is the value of x at which the peak appears, and i_0 is intensity value for the background radiation. On the other hand, for the zone plates with a π-phase jump, the equation,

$$I(x_0, f) = m(I_0 \text{sinc}^2(\frac{A(x - x_0)/(f\lambda)}{2}) \sin^2(\frac{\pi A(x - x_0)/(f\lambda)}{2})) + i_0$$

was used to create a theoretical curve that fit the experimental values.

After plotting the intensity distributions obtained from these pictures, the data was fitted to the theoretical curves obtained earlier.

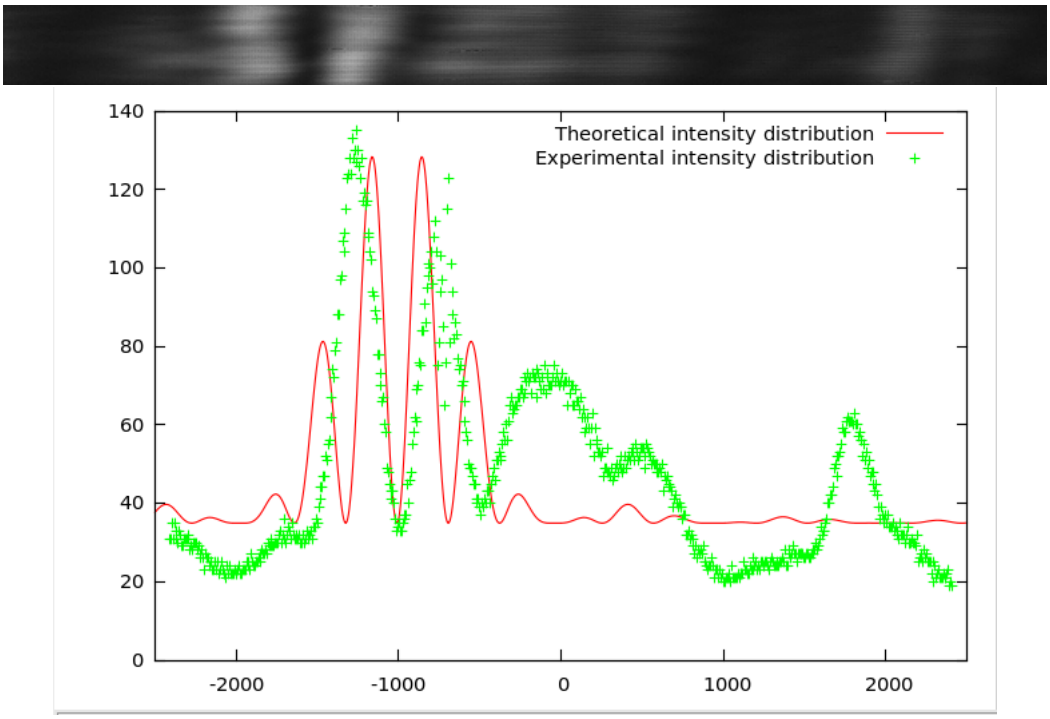


Figure 8: The intensity distribution in the focal plane of a binary linear zone plate with a π-phase jump.

It is clear that the normal binary zone plate behaved as expected, although the focus was slightly of center, likely due to the fact that the laser beam was not entirely uniform. The data obtained from this interference pattern matches the theoretical curve rather well. The intensity distribution for the binary zone plate with a π-phase jump matches the general shape of the curve, although the width of the dark line is much higher than expected.

The intensity distribution for the sinusoidal zone plate agrees with the theoretical curve for the binary zone plate rather well, although it is clear from the data that the binary zone plate produced a much brighter focal line. The intensity distribution for the sinusoidal zone plate with a π-phase jump on the other hand does not match any of the theoretical curves well. It seems to be approaching a dark line, but in the middle of dip where the dark line is expected appear, there is a sudden sharp peak. This could suggest that perhaps the laser was not uniform enough or even that there were some irregularities in the zone plate.

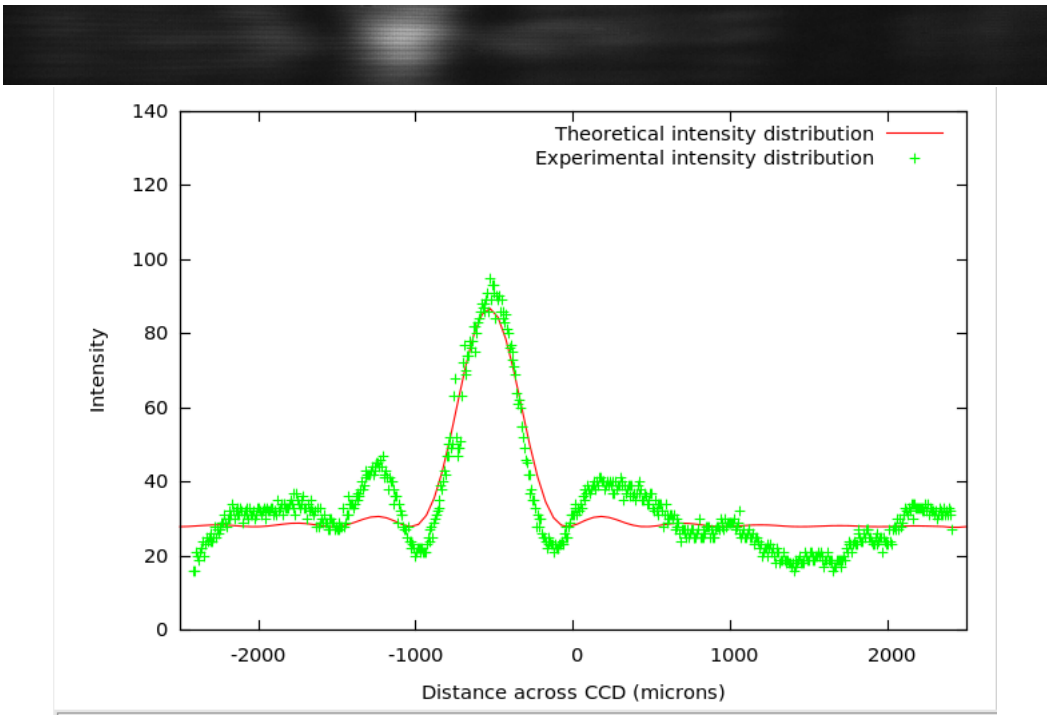


Figure 9: The intensity distribution in the focal plane of a sinusoidal linear zone plate.

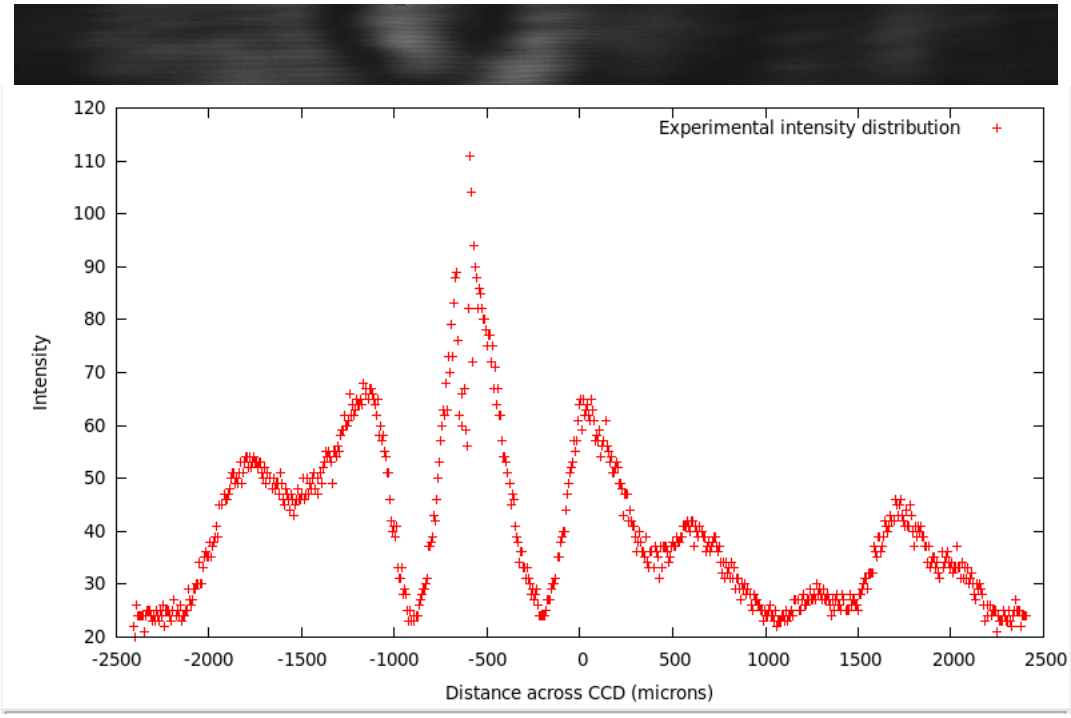


Figure 10: The intensity distribution in the focal plane of a sinusoidal linear zone plate with a π-phase jump.

6 Conclusion

This project was accomplished with simple tools, a basic HeNe, standard lenses, an Electrim EDC-1000N CCD camera, and some freely available computer programs. However, a major difficulty in this project was the complexity involved in creating the zone plates. Not only were there π-phase jumps introduced, but they were introduced onto sinusoidal zone plates.

The results for the normal binary and sinusoidal zone plates were excellent. The data obtained from these two zone plates matched the theoretical curve of the intensity distribution quite well. The results for the binary zone plate with a π-phase jump were not quite as good, but they were still decent, although the dark line created by this zone plate was significantly wider than the theoretical curve had predicted. The sinusoidal zone plate with the π-phase jump on the other hand did not produce the expected results. This could have resulted from shortcomings in the quality of the laser beam and the design of the zone plate.

In the future, it would be absolutely fascinating to create these dark lines while paying closer attention to the quality of the beam by eliminating the presence of dust particles that cause minor diffraction patterns. It would also be interesting to attempt this experiment using a spatial light modulator to see whether or not the actual design or printing played a factor. If any high-quality dark lines are obtained it would be interesting to attempt to align a laser using these dark lines and then compare that to past alignment experiments done using optical vortices. And, to add another level of complexity it would be fascinating to introduce the π-phase jump onto a circular zone plate, which would result in a dark circle, rather than a dark line.

Zone plates hold many possibilities for the future. One important application is a zone plate's ability to focus electromagnetic radiation of any wavelength, unlike conventional lenses. For this reason, zone plates have been prevalent in the area of x-ray microscopy. Regular lenses would fail to focus x-rays, because the indices of refraction for electromagnetic radiation outside the visible spectrum are extremely low [7]. The dark lines created by the π-phase jump zone plates also have some applications in optical alignment [1]. This is a new

type of alignment technique that is different from the established techniques used with optical vortices. Optical vortices are circularly polarized beams of light with a phase singularity in their center, which is used in the same way that the dark line created by zone plate is used.

The application of these zone plates in the field of optical alignment can someday have a profound impact anywhere from nuclear fusion to simple laboratory experiments. Nonetheless, an important step in accomplishing this would be understanding the properties of sinusoidal zone plates on a much deeper level.

7 References

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