A Jones-Matrix Model of Color Patterns in Polarized-Light Art

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Abstract

The purpose of this project was to create a mathematical model which could be used to understand the color patterns in "Polage." Polage ("polarized collage") uses layers of birefringent cellophane to modify the electric field of light, so when viewed between two linear polarizers, beautiful colors are created.

Our experiment was comprised of both theoretical and experimental parts. Jones Calculus was used to calculate the final intensity of a light vector transmitted through a sandwich of two linear polarizers with cellophane in between. Experimentally, the setup was designed to characterize the retardance of various cellophane films, and included a HeNe laser, and the aforementioned sandwich, and a photodetector. Data were plotted and compared to a theoretical model and it was determined that our sample of cellophane had a retardance of 145°, being close to that of a half-wave plate (180°).

In the future, we plan to use a spectrophotometer and lasers of different wavelengths to futher understand the relationship between color and transmitted light intensity.

Research Topic Selection

This project was developed after seven full weeks of exploring the science of optics, and searching for a project we wanted to devote our passion and time to. We became interested in different aspects of polarized light, and eventually our interests combined to form one project. One of us became interested in experimentation, and making precise measurements using a laser, cellophane, and polarizers. The other became interested in calculations, and the use of Jones Calculus, initially aiming to derive a matrix representation of the cellophane. As a result, we became a team of an experimentalist and a theorist. We hope to continue our experiment as student researchers to expand our knowledge and experience in the beauties of physics and mathematics.

1 Introduction

While it is well known that polarized light has many important applications in diverse fields of science, technology, and even consumer products (polarized sunglasses), it is less well known that polarized light is the basis for a unique and fascinating art form, Polage ("polarized collage"), pioneered by Austine Wood Comarow in 1967 [1]. The objective of this research is to create a mathematical model that can be used to simulate and analyze the striking color effects of the Polage images.

Light is a transverse electromagnetic wave comprised of rapidly oscillating orthogonal electric and magnetic fields. If the orientation of the electric field vector $\vec{E}$ is constant in time then the light has a linear polarization, which is the most familiar form of polarization. It is also possible to have a circular polarization, in which the electric field vector rotates around the direction of propagation and traces out a spiral in space. The most general form of polarized light, elliptically polarized light, is a combination of these.

Polarized light is created and altered with the help of dichroic and birefringent materials. Dichroic materials include the familiar Polaroid$^{\text{TM}}$ films, which absorb one component of the electric field vector, thus creating linearly polarized (LP) light. Birefringent materials include crystals like quartz or calcite (calcium carbonate) and anisotropic polymers like cellophane, all of which have two perpendicular axes with different indices of refraction [2]. The differing indices cause phase shifts which influence the magnitude and the orientation of the $\vec{E}$ vector. Optical devices made from birefringent materials are called retarders due to their variable phase shifts. The color effects in Polage come about because the phase shifts depend on the wavelength of the incident light. Polage consists of carefully arranged layers of birefringent cellophane sandwiched between linear polarizers and is illuminated from behind.

Our research had both theoretical and experimental components. Numerical models that describe the polarization changes as light passes through various linear polarizers (P) and retarders (R) were created. The description employs 2×2 matrices with complex-valued components, the Jones matrices [3]. One set of experiments tested various predictions of the model, while another characterized the retardance of various samples of cellophane. In addition, qualitative observations were made of the visual (color) effects when a white light source is observed through a polarizer-retarder-polarizer (PRP) sandwich.

2 Polarized Light

2.1 Characteristics of polarized light

Although it is commonly believed that polarization is a property of light not detectable by human eyes, humans actually have a slight sensitivity to polarization, through a phenomenon known as "Haidinger's brush" [4]. Some insects and animals such as cuttlefish, bees, and squid not only have a sensitivity to polarized light but even depend on this adaptation for their survival [5]. Humans cannot easily observe polarized light, but polarized light is ubiquitous in nature; for example light from a clear blue sky or light reflected from water can be highly polarized.

Natural light is unpolarized, which means that the polarization is changing so rapidly that all polarization effects cancel out. In other words, light is unpolarized when human technology cannot detect its polarization [6]. To simplify the visualization, unpolarized light can be thought to have all polarizations. Imagine a circle of light vectors pointing in all directions from the center. When this mixture of vectors enters a polarizer, only one oscillation direction will be allowed to pass and all other vectors will be blocked.

Polarization describes the magnitude and the direction of the transverse electromagnetic light wave. The vector nature of the electromagnetic waves allows the phenomenon of polarization. Polarization can be thought of as just the shape that the tip of the time-variant electric field vector traces as it propagates. Polarization state can be divided into three categories: linear, circular, and elliptical. Essentially, linear and circular polarizations are just special cases of elliptical polarization. Linear polarized light has a phase shift of 0 or $\pm m2\pi$ (m is any integer), or no phase shift between its orthogonal components. One way to imagine a linear and a circular polarized light as a type of an elliptical polarized light is to visualize ellipse getting flatter and flatter until it becomes a line segment, and imagining ellipse getting more circular for the latter. A circle is an ellipse with congruent x and y components. Thus, circularly polarized light has a phase shift of $\pm\pi/2$ with x and y components being equal in magnitude. Polarization can be produced by reflection, scattering, use of dichroic or birefringent materials, or by commerical products such as Polaroids.

2.2 Birefringent materials

Birefringence, also referred to as “double refraction,” is a property of materials whose atoms are bound more tightly in one axis than the other. A birefringent material has two separate indices of refraction, n_{slow} and n_{fast}, with n_{slow} being greater than n_{fast} for two perpendicular axes, x and y (relationship $v = c/n$ indicates that, greater the index of refraction n, smaller the velocity v). Due two different indices of refraction, x and y components of the E-field will travel at different speeds through the medium. This results in the phase difference of the components when they recombine. Due to the phase difference, elliptically polarized light will be produced.

Retarders are used to alter the polarization state of light. Retarders have a carefully chosen thickness to allow the desired manipulation of light's polarization. Waveplates, especially quarter-wave plates and half-wave plates, are the most common commercial forms of retarders. Lesser known forms of retarders are some minerals such as quartz and cellophane. Quarter-wave plates can convert circularly-polarized light to linearly-polarized light and *vice versa*. Quarter-wave plates shift the phase of light by 90°. Half-wave plates simply rotate the $\vec{E}$ vector of the linearly-polarized light. For every θ, the angle between the axis of polarization and the axis of a half-wave plate, polarization is rotated by 2θ.

2.3 Polage

One does not usually think of art in terms of mathematics, but math can be used to model the color patterns created by a unique form of artwork,“Polage.” Most people are not awared of polage's existence, but it is significant to understand polage, because polage demonstrates how two different, commonly non-related, or even opposite fields, math and art, can be mixed. Understanding polage will not only enhance the understanding of polarized light, or physics behind the art, but it will also allow more interactions between art and math. Polage is a sandwich of two linear polarizers with a cellophane artwork in between. It is lightened from behind and the final polarizer is rotated to create fascinating color patterns. Also turning the polarizer not only changes the colors, but also the appearing shapes. Polage contains different layers of cellophane to create desired colors, which are created from a wavelength dependent phase shifts. Not only that, Austine Wood Comarow uses her artistic skills to create shapes using these color effects. Art is very sensitive to colors; depending on what color the artwork is depicted as, it can generate differnt moods from people. Thus, a polage be referred to as one family of artwork.

3 The Jones Calculus

3.1 What is Jones Calculus

Jones Calculus was invented by American physicist R. Clark Jones in 1941 [7]. It is extremely useful in optics to describe completely polarized light. Jones Calculus uses two by two matrices to represent optical instruments that modify polarized light, including linear polarizers and retarders. Any state of fully polarized light can be described by a two by one vector. The elements of the vectors and matrices can be complex numbers, which are used to represent phase shifts but have no physical meaning. In fact, complex numbers are used to represent elliptically or circularly polarized light vectors only. However, the final result of a Jones matrix calculation, such as a light intensity, is always a real number.

Matrix calculation is extremely useful in polarized light applications. Like many other topics in science where visualization is difficult, understanding the mathematics behind the physics, or mastering the matrix treatment of polarized light in this case, will serve as to precisely explain what polarized light actually is. The matrices representing optical instruments are applied to a light vector to calculate the transmitted light vector. When a sequence of matrices are applied to a vector, calculation is done from left to right. With matrix calculations, the order of multiplication is significant; matrix multiplication is not commutative. Another way to reach the same solution is to multiply the product of all the matrices to the vector in the end.

Jones vectors are most often normalized, which means that the sum of the squares of the components equals one. Normalization is used to simplify the calculations. The Jones vector $\vec{E}$ is

$$\vec{E} = \begin{pmatrix} E_x(t) \\ E_y(t) \end{pmatrix} = \begin{pmatrix} E_0 x e^{i\psi_x} \\ E_0 y e^{i\psi_y} \end{pmatrix}$$

Normalized vector obeys the relationship

$$|E_x|^2 + |E_y|^2 = 1$$

Optical Element	Jones Matrix	Jones Vector
Linear polarizer (horizontal)	$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
Linear polarizer (vertical)	$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$
Linear polarizer ($\pm 45°$)	$\frac{1}{2}\begin{pmatrix} 1 & \pm 1 \\ \pm 1 & 1 \end{pmatrix}$	$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ \pm 1 \end{pmatrix}$
Linear polarizer at ψ	$\begin{pmatrix} \cos^2(\psi) & \cos(\psi)\sin(\psi) \\ \cos(\psi)\sin(\psi) & \sin^2(\psi) \end{pmatrix}$	$\begin{pmatrix} \cos(\psi) \\ \sin(\psi) \end{pmatrix}$
Left or right circular polarizer	$\frac{1}{2}\begin{pmatrix} 1 & \mp i \\ \pm i & 1 \end{pmatrix}$	$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ \pm i \end{pmatrix}$

Table 1: Examples of common Jones matrices and vectors [8]

Table 1 above shows the Jones matrices for some optical elements and the corresponding Jones vectors.

Any retarder with fast axis along x direction is given by [9]

$$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{-i\Gamma} \end{pmatrix}$$

where the phase denoted by Γ is given by

$$\Gamma = 2\pi(\Delta n)L/\lambda$$

In the equation above, Γ is the phase shift in terms of radians or degrees, Δn is the birefringence or the difference between two indices of refraction, L is the thickness of the retarder, and λ is the wavelength of light.

3.2 Calculations

Various calculations of the final intensity were made using the setup of two linear polarizers with a cellophane film in between. First, the Jones vector for the transmitted light was calculated. Then the intensity was determined from the relationship $I = A_x A_x* + A_y A_y*$, where A_x is the x component of the electric field of the light and A_y is the y-component and A_x* and A_y* are corresponding complex conjugates. To find the complex conjugates, i was simply negated. When one or more optical elements were rotated, rotation matrices were used to depict the transformation.

First Calculation

In the first calculation, two linear polarizers were kept crossed, and retarder T was rotated. The first polarizer acted as a horizontal polarizer and the second as a vertical polarizer. Below is the general equation that was solved.

$$\begin{pmatrix} A_x \\ A_y \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} R(\theta)T(\Gamma)R(-\theta) \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

where

$$R(\pm\theta) = \begin{pmatrix} \cos(\theta) & \pm\sin(\theta) \\ \mp\sin(\theta) & \cos(\theta) \end{pmatrix}$$

and the solution is given by

$$I = 2sin^2(\theta)cos^2(\theta)(1 - cos(\Gamma))$$

Second Calculation

In the second calculation, two linear polarizers had their transmission axes parallely aligned, and the retarder T was rotated. In comparison with the first calculation, the final analyzer was simply turned 90° to act as a vertical polarizer. The general equation is

$$\begin{pmatrix} A_x \\ A_y \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} R(\theta) \begin{pmatrix} 1 & 0 \\ 0 & e^{-i\Gamma} \end{pmatrix} R(-\theta) \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

followed by its solution

$$I = sin^4(\theta) + cos^4(\theta) + 2sin^2(\theta)cos^2(\theta)(cos(\Gamma))$$

The first two calculations, in which analyzers were at fixed angles and a retarder was rotated, were done by hand. Afterwards, calculations were confirmed using Mathematica.

Third Calculation

In the third calculation, both the analyzer and the retarder were rotated. The general equation for these two optical instruments were calculated. Ultimately, the aim was to experiment with different values of θ and Γ to understand their effects on the transmitted intensity. The equation that was solved is

$$\begin{pmatrix} A_x \\ A_y \end{pmatrix} = LP(\psi)R(\theta)\begin{pmatrix} 1 & 0 \\ 0 & e^{-i\Gamma} \end{pmatrix}R(-\theta)\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

The third calculation was initially attempted by hand but was completed using Mathematica when it was realized that it involved an un-manageablly large number of terms.

4 Methods and Procedures

4.1 Intensity measurements

Apparatus

The experimental apparatus consisted of a $10mW$ red HeNe ($\lambda = 632.8nm$) unpolarized laser, whose beam was then projected through several layers of material. The types of materials in, as well as the number of, layers varied depending on the experiment being conducted. These materials were mounted on standard optical mountings, on Thorlabs rotators which were oriented vertically, and perpendicular to the direction of the beam. The material was mounted to the rotators using double-sided foam tape, and, once secured, the rotator was centered with respect to the beam using a cross-hairs drawn on paper. This is essential because of the uneven nature of cellophane; it's imperfections can cause large differences in thickness from one region of the cellophane to another, which can distort data. Centering the rotator ensures that as the cellophane is rotated, the beam will be passing through the same area of cellophane, allowing for consistent measurements.

Measuring the output of the laser after it had passed through the material was a Thorlabs Model DET-110 photodetector connected by BNC cable to a Tektronix TX3 True RMS digital multimeter. This particular photodetector model had a large photodetection surface, allowing for the entire beam to fall within the the detectable area, ensuring higher-quality data. Measurements of the current generated by the photodetector were measured in milli- or microamps; whichever was appropriate given the amount of light being transmitted. These measurements were taken in steps of 10° of rotation of the second polarizer (analyzer) relative to the orientation of the preceding polarizer. Each data point was recorded by hand in a notebook, and later entered into the graphing program QuattroPro that is discussed later in this section. While this was the setup designed for the warm-up experiments, it is consistent throughout the whole of our experimentation, with only the materials and their degrees of rotation changing from each experiment.

Preliminary Experiments: Two or Three Polarizers

Before experimenting with cellophane, we conducted two preliminary experiments with only linear polarizers. These experiments served several purposes, beyond simply providing some basic introductory experience with polarization. The two experiments presented an opportunity for us to develop a suitable apparatus for our line of experimentation, through measurement of the extinction ratios of our polarizers, and through the development of a procedure which assured high-quality data, as confirmed by the near-perfect agreement with theoretical expectations. In the first experiment light was passed through two linear polarizers (one stationary and one rotating). In the second experiment three polarizers were employed; the two outer ones were stationary while the middle one rotated.

Experiment with PRP Sandwich, with Rotating Cellophane

From the two experiments above, it was an easy transition to experimentation on actual cellophane. This began with an experiment in which we replaced the middle polarizer of the aforementioned three polarizer setup with a piece of cellophane, allowing for a striking comparison between the transmission properties of a polarizer versus a piece of cellophane.

Great pains were taken to ensure that the cellophane and the two linear polarizers were aligned with one another, ensuring that all 0-degree positions of the rotators achieved extinction of the laser beam. Having orthogonally aligned the polarizers, and aligned the cellophane to preserve extinction, the cellophane was then rotated in steps of 5 degrees relative to the initial polarizer, in the same fashion as the preliminary experiments. The readings from the photodetector were used to create the curve seen in Figure 2 of section 5.

Experiment with PRP Sandwich, with offset cellophane and rotating analyzer

The next iteration of this experiment maintained the same physical apparatus, while performing the more complex task of characterizing the retardance of the cellophane, which was one of the goals of this project. In order to successfully characterize the cellophane, it was necessary to collect data at several different rotations of the cellophane relative to the initial polarizer, in order to derive information about how the light was transmitted through the two different axes.

At the start of the experiment, the polarizers were crossed so that extinction of the laser beam would occur. Then the cellophane was inserted at a rotation that would preserve the extinction, which was called 0°. The second polarizer (also called the analyzer) was then rotated in intervals of 5 degrees for a full 360°, a measurement of the intensity at each interval being recorded each time. After this curve had been taken, the cellophane was set to 15 degrees, and another full curve was recorded by rotating the analyzer. This procedure was repeated with the cellophane being rotated in 15 degrees intervals for a full 360°, yielding 7 curves, displayed in Figure 3 of section 5.

Properties of several varieties of cellophane

In the preceding experiments, we used the same piece of cellophane consistently, so as to exclude the many variables that would be introduced through experimenting with different pieces of cellophane. This cellophane, known in the lab as Sample #4, was one of four different kinds of cellophane we had received from local florists, and was arbitrarily chosen to the be the subject of our experimentation.

However, to ensure that the results of our experimentation were not based on the properties of an extraordinary piece of cellophane, we decided to test the transmission properties of the other three samples of cellophane, to obtain an understanding of how much different pieces of cellophane can vary. To do this, we performed the same experiment as above, in which the laser is projected through a linear polarizer, a piece of cellophane aligned with the polarizer, and an analyzer. The analyzer is turned in steps of 45 degrees, to measure the maximum and minimum transmission of the cellophane. This procedure was repeated three more times, each time with a different sample of cellophane.

4.2 Computer Modeling and Graphing

Theoretical simulations

Calculations shown in the Jones sections were graphed either using the QuattroPro spreadsheet in the DOS environment, or Mathematica on Windows XP.

For the first two calculations done by hand, the general equations derived were used to produce families of curves using the QuattroPro spreadsheets. Equations were entered and various Γ values and angles were explored to see the effects of those on the transmitted intensity. Many graphs were produced, all of which had the transmitted intensity as the dependent variable and retarder angle or wavelength as the independent variable.

For the third calculation, Mathematica was utilized to perform the much more complex calculations and plot the resulting equations. Each vector and matrix that was needed in the calculation was first specified by its components and assigned a variable name. These elements were then multiplied in the appropriate order. Then, taking the advantages of cool tricks Mathematica allows, a family of colorful curves were created.

Different Γ values were experimented with to best model the actual data.

Data plotting

Data recorded during these experiments were entered into QuattroPro, and graphed. The readings from the photodetector were recorded by hand and entered manually into this program. After a graph was produced, a mathematical function was used to calculate points in a best fit curve. The quality of this curve was achieved by readjusting various parameters in order to achieve a minimum value calculated by measuring the deviation of the line of best fit from each data point.

5 Analysis and Results

5.1 Preliminary Experiments: Two or Three polarizers

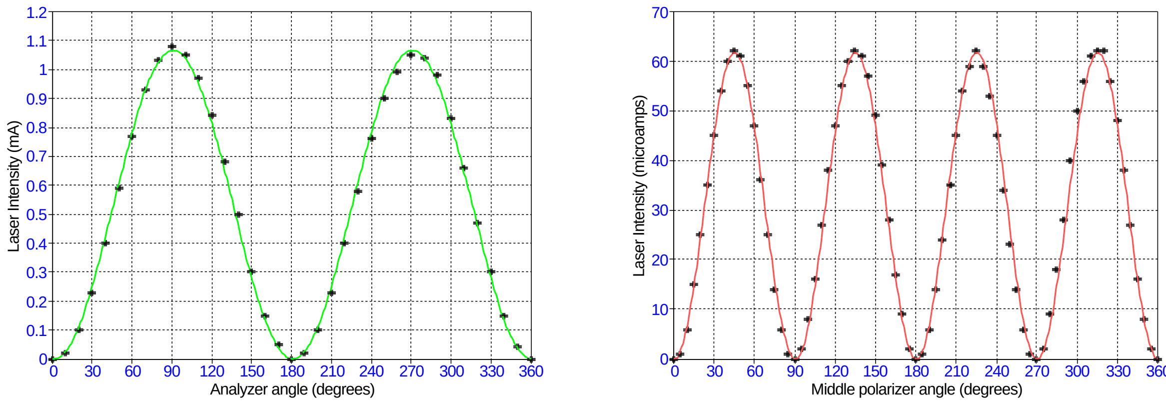

Figure 1: Measured transmittance curves for two or three polarizers.

The preliminary experiments discussed in Section 4 were reproductions of classic experiments describing the relationships between two (or three) polarizers, and inasmuch as they were simply tests to measure the accuracy of our setup, they were reproduced very accurately. The graph of two polarizers (Figure 1, left) displays the basic effect of polarization in an beautifully simple manner: when the polarizers are orthogonally opposed (at 0 and 180 degrees) all light is blocked, but when the analyzer is rotated 90 degrees, and is parallel to the orientation of the first polarizer, maximum transmission occurs.

In the graph of the three polarizers, we see how the introduction of a middle polarizer can resolve vertically polarized light into both vertical and horizontal components, allowing a small amount of light to pass through the horizontally oriented analyzer. This phenomenon, due to the introduction of the third polarizer, is responsible for the two additional crests not seen in the two-polarizer experiment.

In conducting these two experiments, we noticed very different amounts of current coming from the photodetector. In the two-polarizer experiment, our maximum current was measured at about 1.1 milliamps, while in the three-polarizer experiment, our maximum current was measured at about 65 microamps. This is a tremendous decrease, and is due to the introduction of the third polarizer, which, since it is translucent gray (not clear), absorbs a great amount of light.

5.2 Experiment with PRP sandwich, with rotating cellophane

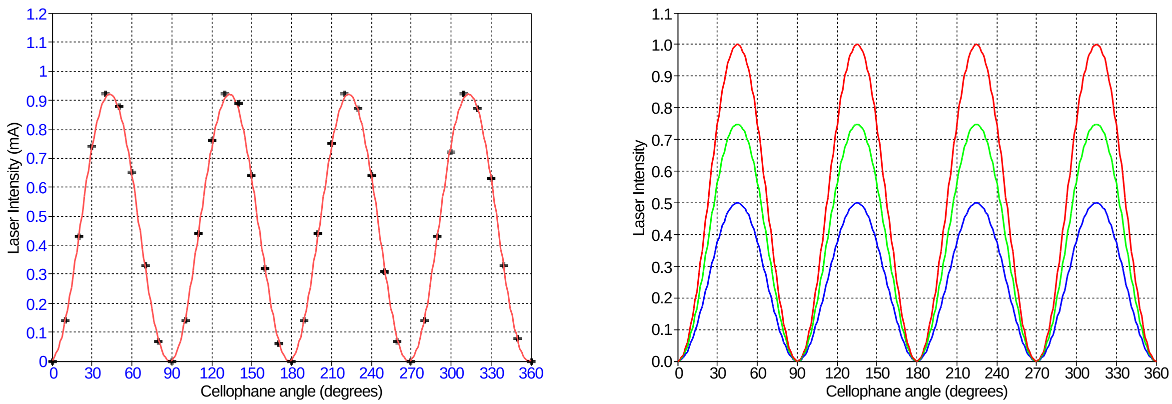

Figure 2: Crossed polarizers and cellophane turning.

Figure 1 displays both experimental and theoretical graphs of transmission of light through cellophane. On the left is a graph displaying the rotation of cellophane between two orthogonally opposed polarizers, which corresponds beautifully with the graph on the right in Figure 1, showing that cellophane works quite effectively as a polarizer. However, unlike that graph, transmission is much higher: 0.9 milliamps, as oposed to the 65 microamps. This highlights an advantage of using cellophane as a polarizer: its clear color allows much more light to pass through than typical colored polarizers.

The graph on the right is a theoretical model of how the cellophane should have acted in the experiment depicted on the left. The four points of extinction represent when one of the axes of cellophane align with the axis of either initial or final polarizer. The four points of maximum intensity is the angle in between the extinctions, or 45° from any of the axes. The graph on the right contains three curves for different values of Γ. Blue, green, and red curves correspond to Γ values of 90, 120, and 180 degrees respectively. Retarder with a phase shift of 90° is a half-wave plate, and that of 180° is a quarter-wave plate. 120° is just one example of retardance in between two waveplates. When Γ value is at 360, the graph will be a straight line at $y = 0$, because phase shift of 360 ($\pm m360$) creates a full-wave plate.

In this case, birefringence nor the thickness of the cellophane is changed, so it is clear that transmitted light intensity is dependent upon wavelength, the only other variable that Γ is influenced by. With two polarizers parallel, same graphs as crossed polarizers-setup will be produced, but the effects of wavelength on the transmitted intensity will be the opposite.

5.3 Experiment with PRP sandwich, with offset cellophane and rotating analyzer

With the preceding experiments completed and a comprehensive understanding of polarization achieved, we were able to move on to the rigorous analysis of the properties of the cellophane, as our goal for this experiment was to measure the the retardance of the cellophane sample and compare its retardance to that of a half-wave plate (HWP).

Given that a half-wave plate rotates polarization at twice the rate it itself is rotated, we theorized that (if cellophane was a half-wave plate) that for every 15° the cellophane was rotated, the analyzer would have to be turned 30° in order to achieve extinction. We also theorized that as the cellophane was rotated, it would transmit less and less light at its maximum, while allowing more and more light through at its minimum.

In order to test whether or not this theory applied to our cellophane, we collected data that resulted in the graph of Figure 3. In certain respects, the data matched our expectations; for every 15°, the polarization was rotated 30°. However, we found that even if the analyzer was turned 30° to compensate for the polarization rotation, extinction was never achieved. Instead, a minimum was achieved, and this minimum grew further from zero as the polarizer rotation came closer to 90°, as can be seen in Fig. 3. Thus, it became clear that the cellophane was not a HWP. Since the retardance was clearly not that of a HWP, we created a graph in Mathematica that used Jones calculus to predict the transmitted intensity for each 15° rotation of the cellophane, in order to calculate the actual retardance. At first, we produced a graph that reflected the curves that would be generated by a perfect HWP, which was defined as a material with a retardance value of 180°. After several attempts at adjusting the graph, we found that when the retardance was defined as 145° or 215° (35° away from 180°) a graph was generated that matched our data very well. (Figure 4) This 35° difference meant that while the cellophane was not truly a HWP, it was very close to one.

The graph's first curve (dark red) represents cellophane parallel to the initial polarizer, or 0° of rotation. Each successive curve represents a 15° rotation of the cellophane, and accordingly, each curve's minimum shifts 30°. As would be expected from a birefringent material, the graph repeats after 180°. Additionally, as the cellophane's rotation approaches 45°, the minimum intensity grows further from extinction, just as the data showed.

An observation of the effects of increased cellophane thickness the showed that when the cellophane was tilted in the beam (and the thickness is effectively increased) the amount of light transmitted decreased. This means that the retardance of the cellophane is actually smaller than 180°, meaning that of the two possible retardance values (145° and 215°) we can be certain that the retardance value of the cellophane is the smaller one, and is therefore 145°.

However, currently we are unsure what order waveplate the cellophane is; that is, wether it is a half wave plate, or a three-halves wave plate, a five-halves wave plate, or so on. Utilizng more accurate measurements of the effect on light transmission as the thickness is increased, and the graph depicted in Figure 5, we hope to soon be able to characterize what order of a waveplate the cellophane actually is.

5.4 Calculations versus wavelength

In Figure 4, the red curve represents crossed polarizers and blue curve represents parallel polarizers. The two graphs are complementary, one reaching its highest peak while the other is demonstrating its extinction [10]. The family of three curves on the left is for the retarder angle of 30° and those on the right is for 45°. The first row represents one layer of the retarder, the second row, two layers, and the third row, ten layers. Figure 4 shows that at 45°, the biggest contrast can be viewed through the PRP sandwich. Also, Figure 4 shows that as layers of retarders or cellophane are increased, light intensity varies more rapidly. Eventually, too many layers will create no noticable difference to the human eye.

5.5 Visual observations with white light

In addition, various qualitative observations were made using the white light from an overhead projector instead of a laser. These involved a sandwich of two linear polarizers with some transparent materials, some of which were birefringent, in between.

Cellophane

Four cellophane films obtained from different florists were observed. The three observations correpond to the three calculations respectively.

First and second observation: There were four points of most intense or saturated color, separated by 90°. There were four transparent points, which were also separated by 90°. These four points made 45° angle with the most intense color points. It was observed that rotating the cellophane changes intensity of the transmitted light only, and not the color. Second observations were same as the first except for the color, which was complementary to the color in the first observation.

Third Observation: It was observed that color saturation was dependent upon the cellophane angle. We used 45° as the angle for the cellophane, because it produced most color. As the analyzer was rotated, it was observed that every 90° rotation changed colors. There were four perpendicular angles at which cellophane did not produce any colors; these spots existed between abrupt changes in two colors.

Other materials

Some examples of other transparent materials were: plastic cutlery, cups and corn syrup. Some displayed photoelasticity, and some showed Maltese crosses. Cellophane pieces were cut and experimented to observe the effects of layers or curve in the color patterns.

6 Future Work

Although we created various models to enhance our understanding of the color effects produced by Polage, and of intriguing birefringent properties of cellophane, we have not fully studied the complex artwork. Thus, the next step is to use the created models to analyze the artwork and to conduct further investigations.

We have demonstrated a method for measuring retardance, and would like to apply this method to various layers of cellophane, since Polage contains multiple layers of cellophane. Conducting the experiment with different colors of laser, such as violet, or using a spectrophotometer to measure both the wavelength and intensity will give us a better understanding of relationship between transmitted color and intensity. The Jones Calculus behind polarized light, (the eigenvectors being one specific example) should be studied in more detail to master the theoretical understanding of polarized light. These new additions will increase the thrills of the project.

There are other related topics in the realm of polarized light that can be explored. Some examples are photoelasticity and creation of 3D displays. Photoelasticity uses birefringence to model the distribution of stress and strain and is useful for engineering purposes. Understanding the properties of cellophane and behavior of polarized light will also allow the creation of 3D display, using a liquid crystal display (LCD) screen, cellophane, and polarizers [11].

Mentioned above are few possible examples of continuation or expansion of our research on polarized light. But as the opportunities are not limited, we will continue to research and explore in the fields of physics and mathematics to fulfill our desire for knowledge. We will learn the inner workings of nature, through which we hope to learn to appreciate it. Most significantly, we will research for our own satisfaction, because researching simply delivers us excitement and joy.

7 Conclusions

The cellophane sample we studied most extensively was found to have a retardance of 145°, or about 80% of the 180° value that corresponds to a half-wave plate. We now understand that in order to produce the most colors, or most intense colors, we need a retarder that is thicker than a half-wave plate but not too thick. Too many retarder (cellophane) layers will produce no noticeable color effects by eye. It was concluded that the retarder angle of 45° produced most colors. We confirmed that perpendicular and parallel linear polarizers with cellophane in the middle produces complementary colors.

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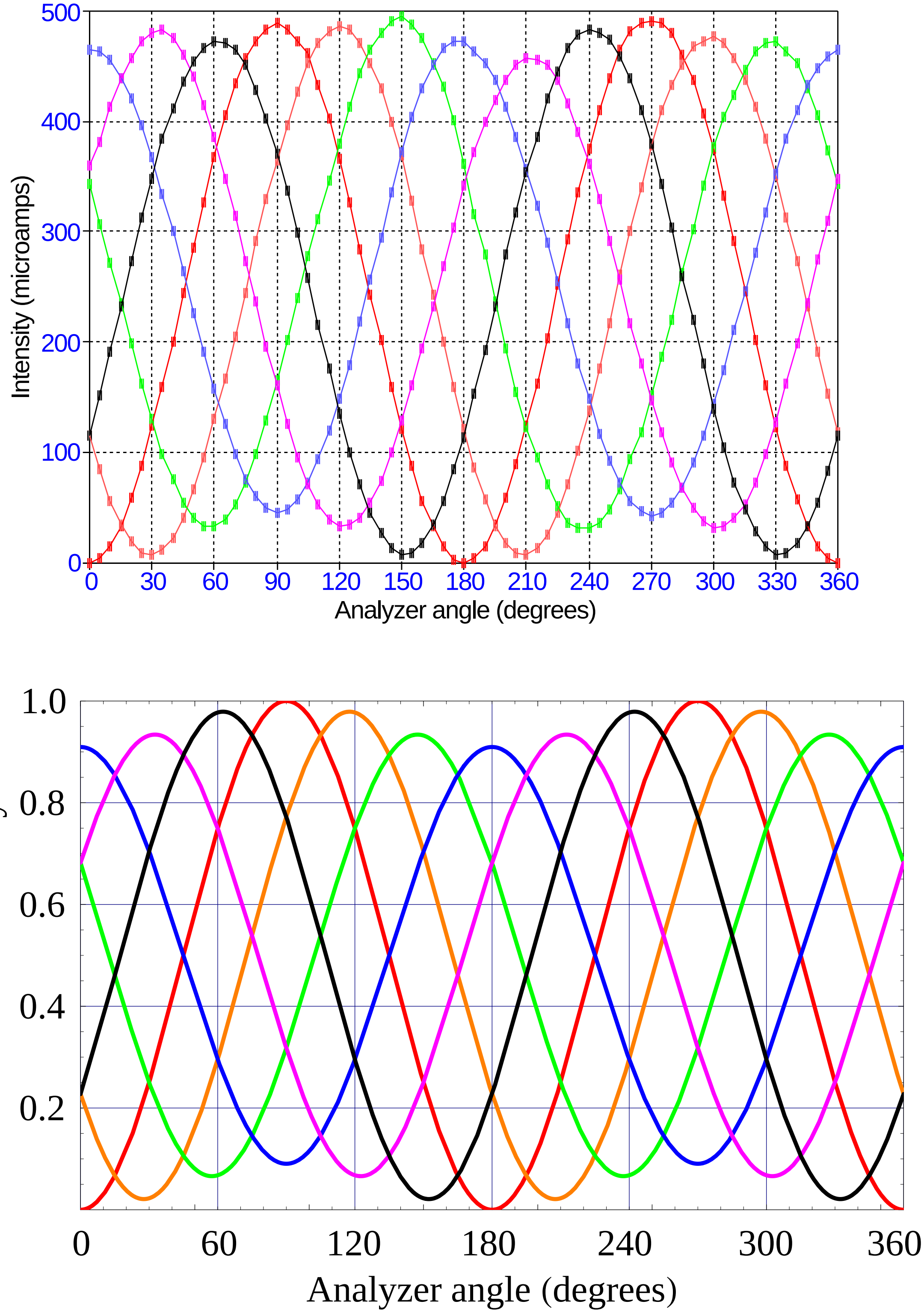

Figure 3: Results of measurements (above) and Mathematica simulation (below). The lines in the upper graph simply connect the points.

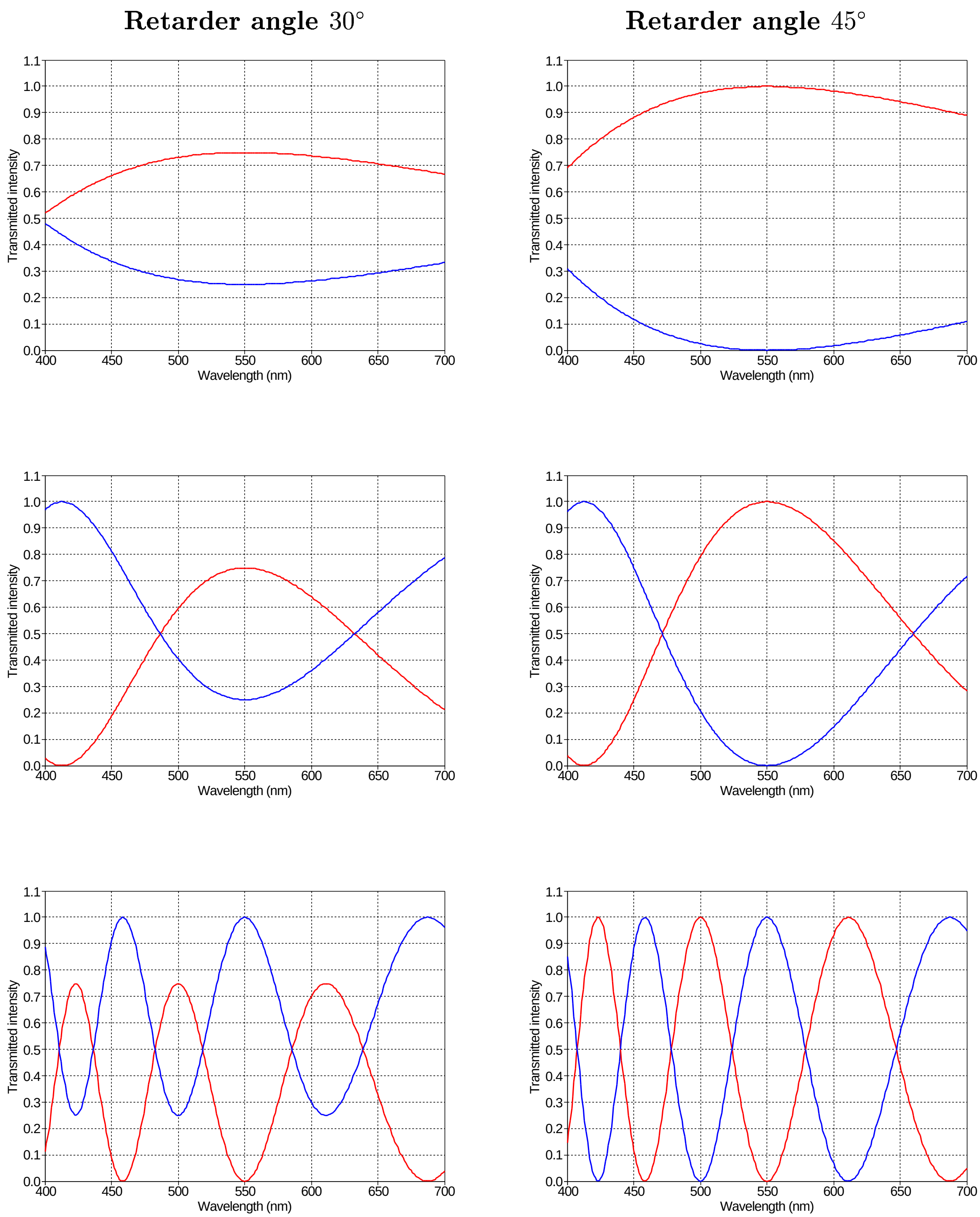

Figure 4: Transmitted intensity versus wavelength graph with retarder at a fixed angle.

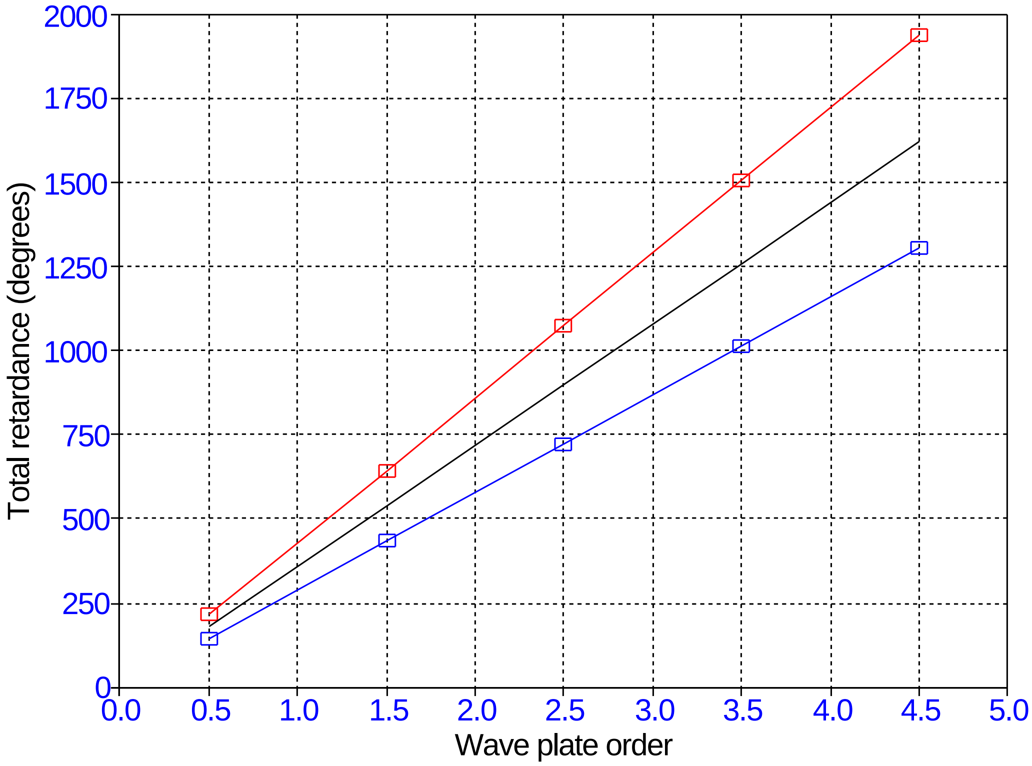

Figure 5: Wave plate order and resulting retardance.