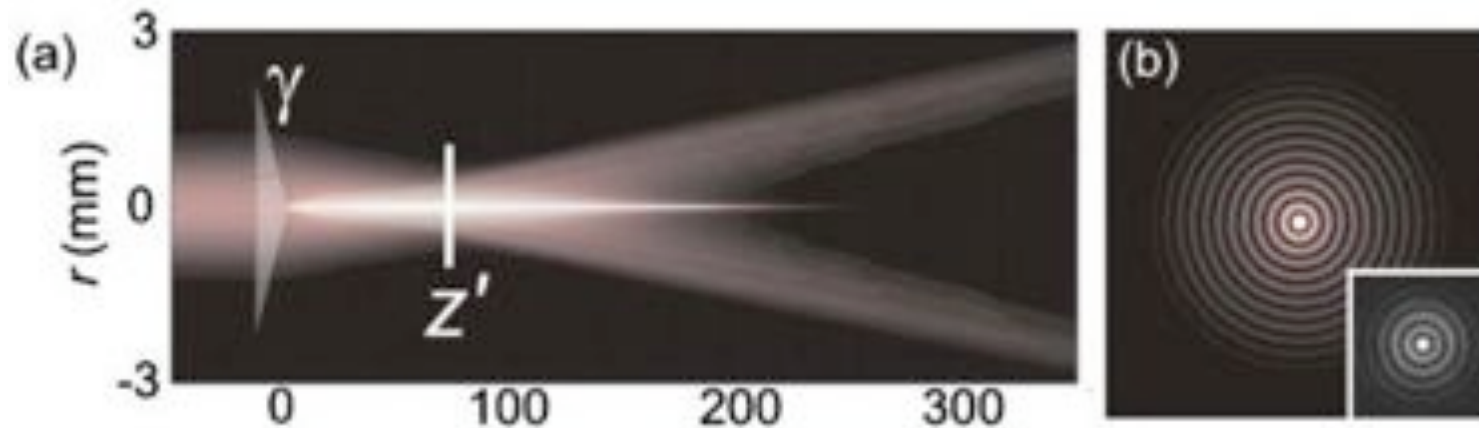


# Creating Bessel Beams



Milne (2008)

Melia Bonomo, Dickinson College

Marissa Romano, Stony Brook University

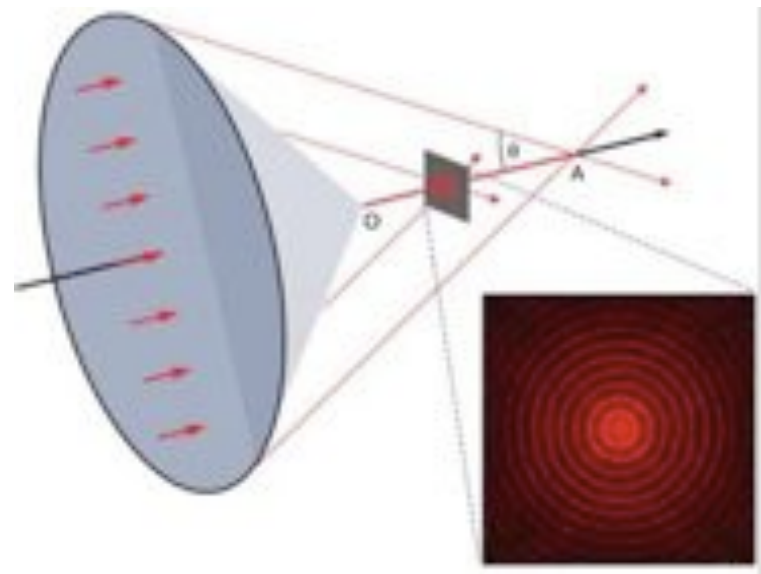


Laser Teaching Center  
Summer 2012  
Stony Brook University



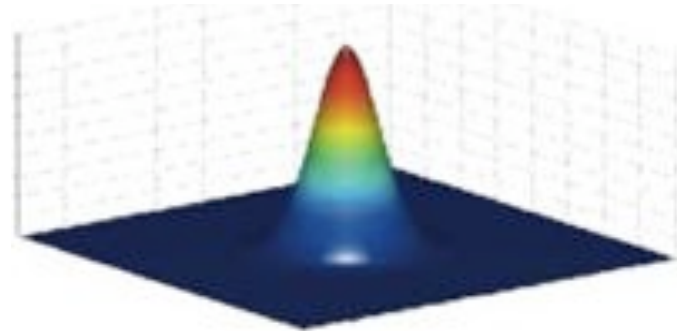
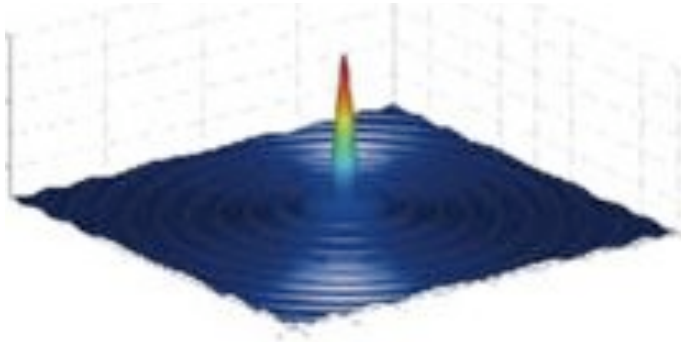
# Overview

- Bessel beam properties
- Generating Bessel beams
  - Annular Aperture
  - 4-f Spatial Filtering
  - Axicon
  - TAG Lens

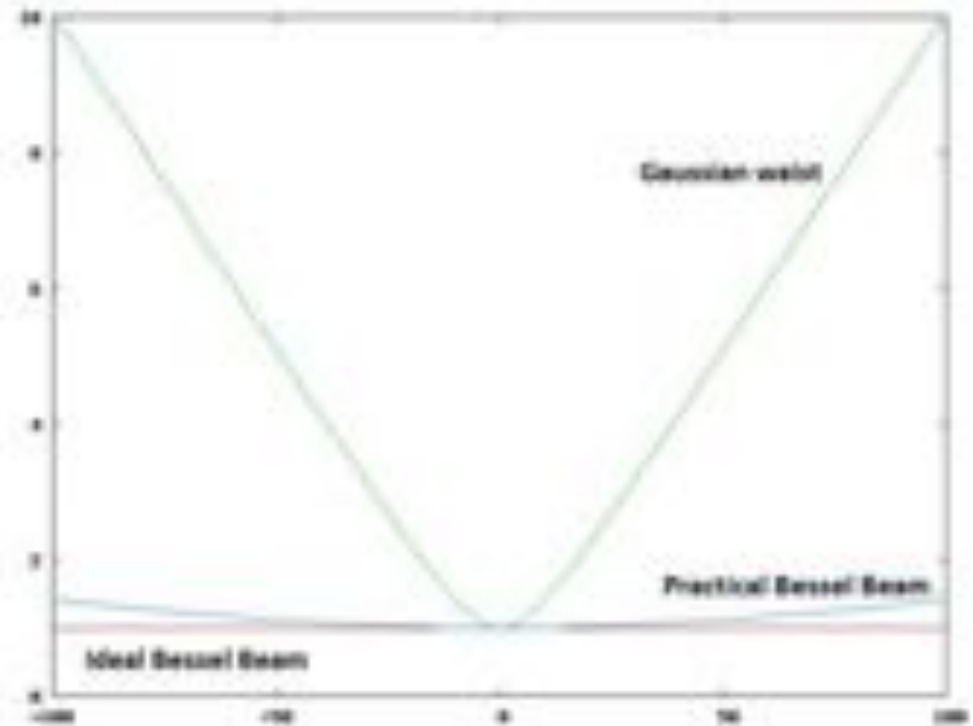


Jaroszewicz (2005)

# Bessel vs. Gaussian Beams

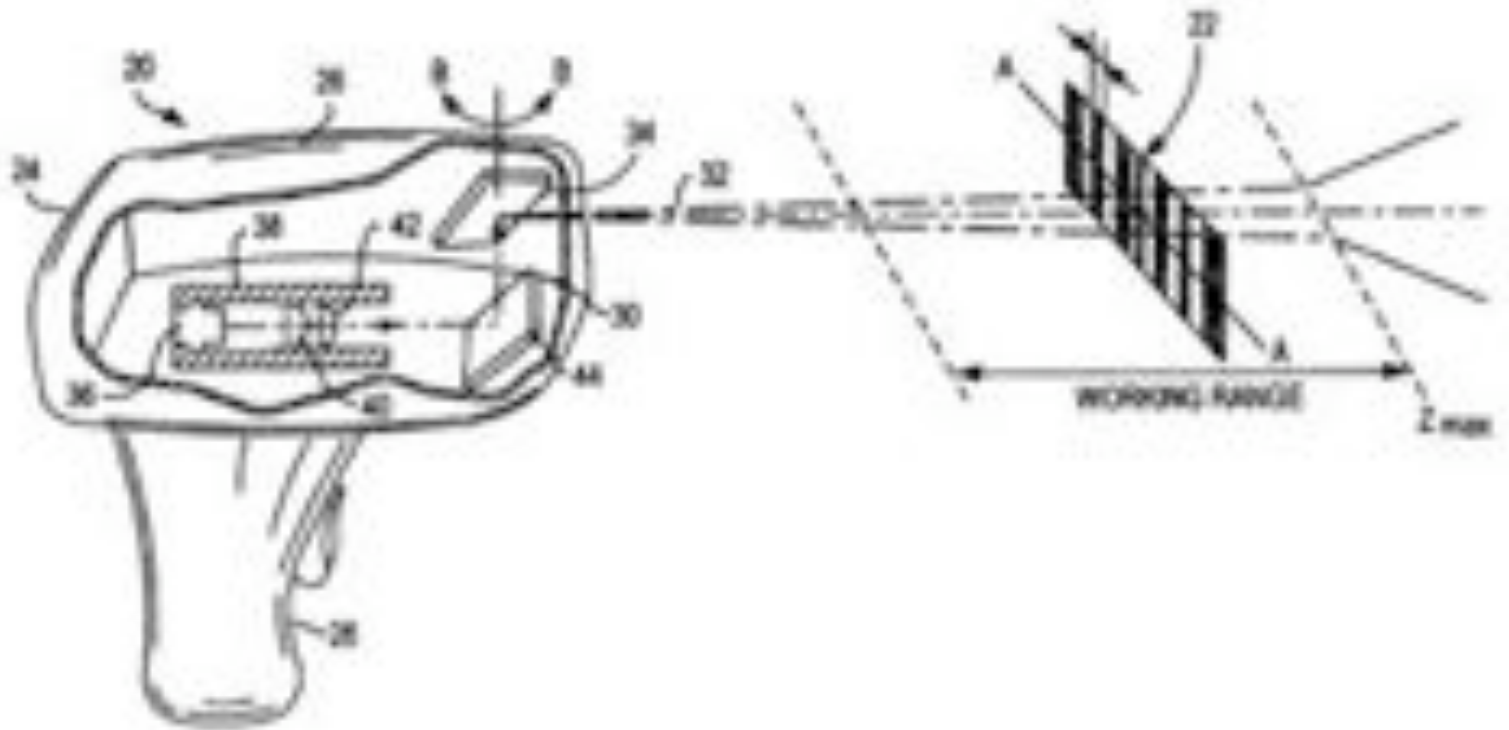


- Non-diffracting
- Thin, infinitely long core with concentric rings
- Ideal Bessel beams cannot be experimentally created
- Close approximations are possible



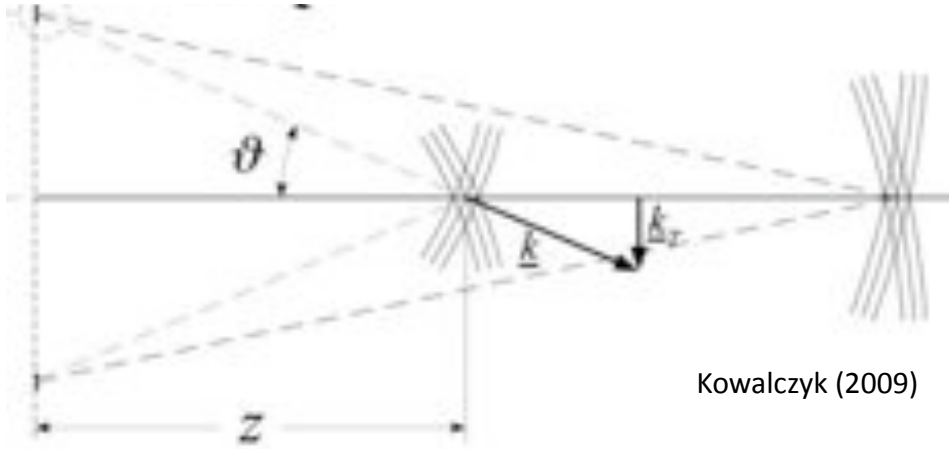
# Most Widespread Application

- Employing a Bessel beam greatly increases the working range of a barcode scanner



Gurevich 2003

# Geometry and Math

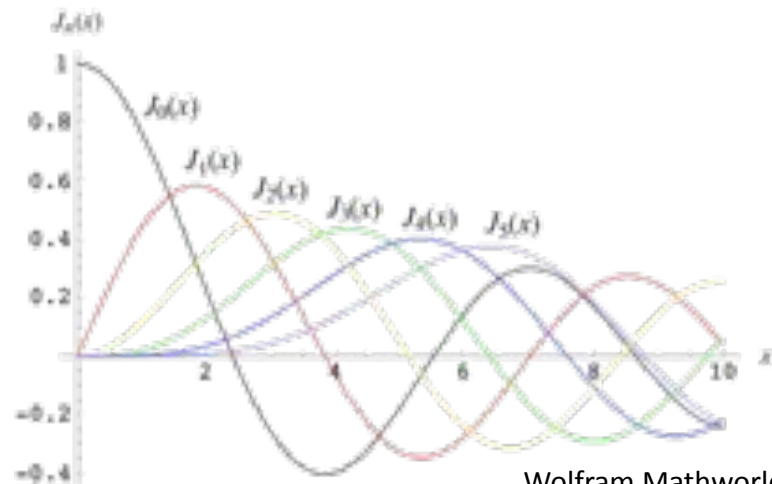


Kowalczyk (2009)

- Superposition of plane waves with k-vectors lying on surface of cone
  - Creates central max and plurality of side lobes

$$E_l(r, \phi, z) = A e^{i k_z z} J_l(k_r r) e^{i \phi l}$$

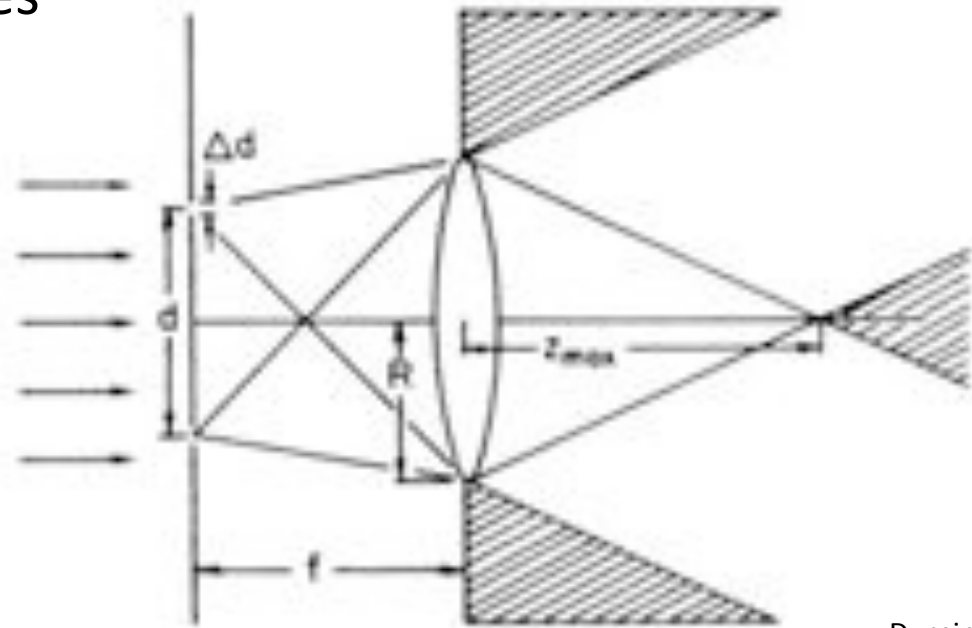
- Electric field amplitude proportional to Bessel function



Wolfram Mathworld, 2012

# Creating Bessel beams with an Annular Aperture

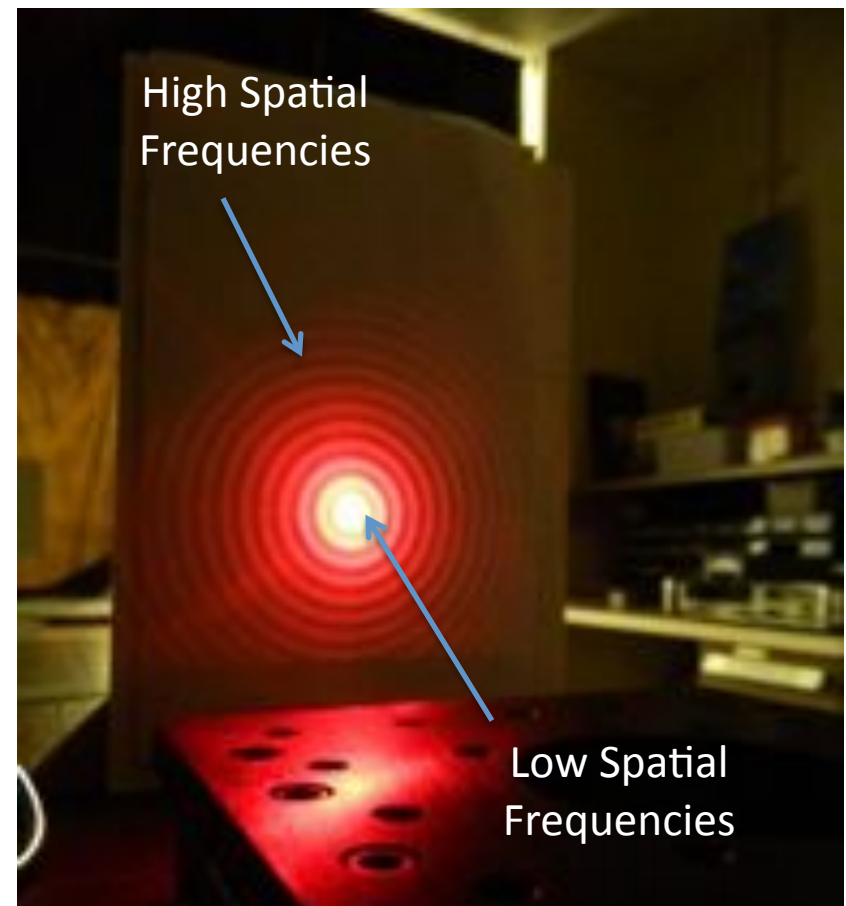
- Fresnel diffraction of thin ring light source
- Durnin and Eberly 1987
  - 2.5 mm diameter, 10 micron wide
- Lens collimates ring of light to create conical superposition of plane waves



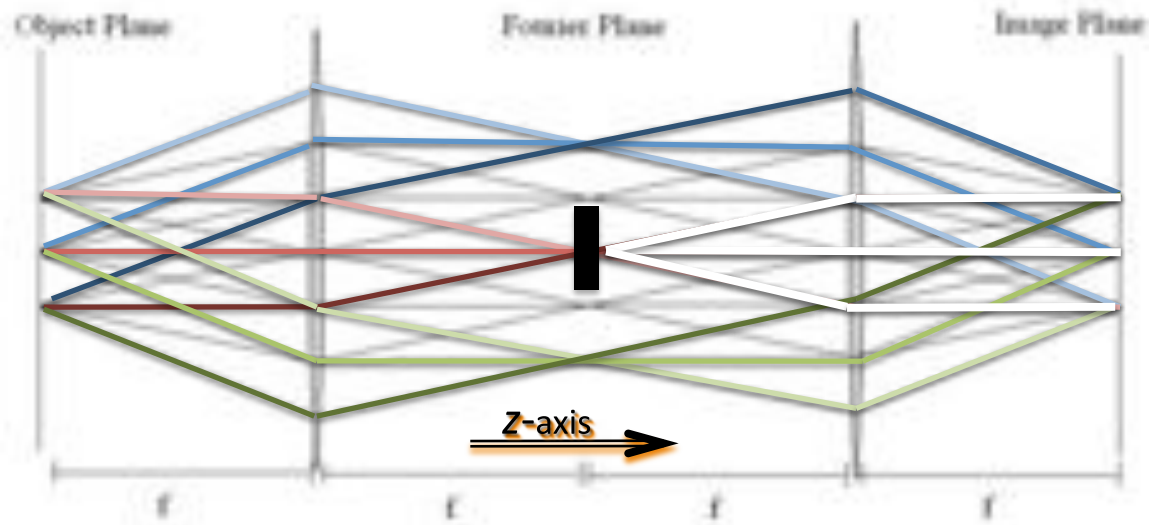
Durnin and Eberly (1987)

# Creating a ring of light through spatial filtering a circular aperture

- Pinhole diffraction pattern in Fourier plane
  - High spatial frequencies: object edges
  - Low spatial frequencies: overall quality
- Edge-enhanced image of the circular aperture



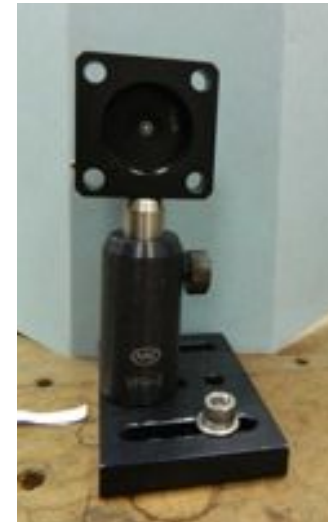
# 4-f Spatial Filtering Method



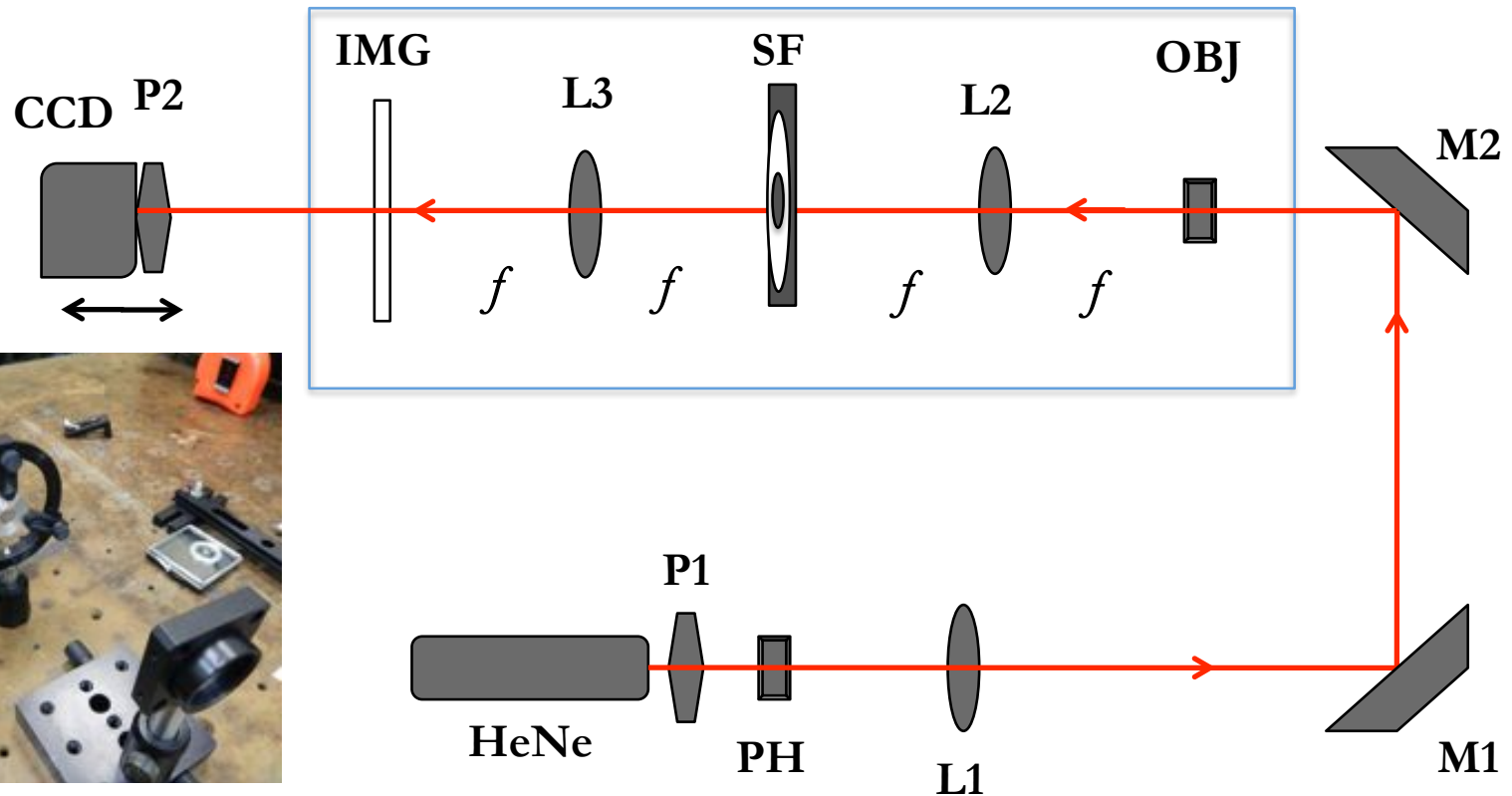
- Lens organizes spatial frequencies of object
  - Rays bent at same angle are part of the same diffraction order of the pattern in the Fourier plane
- The diffraction pattern is the Fourier transform of the object
- Inverse image created in image plane



# Setup Design

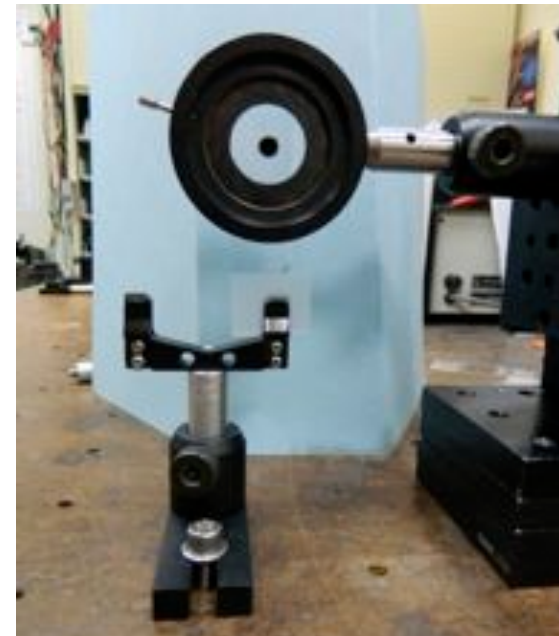
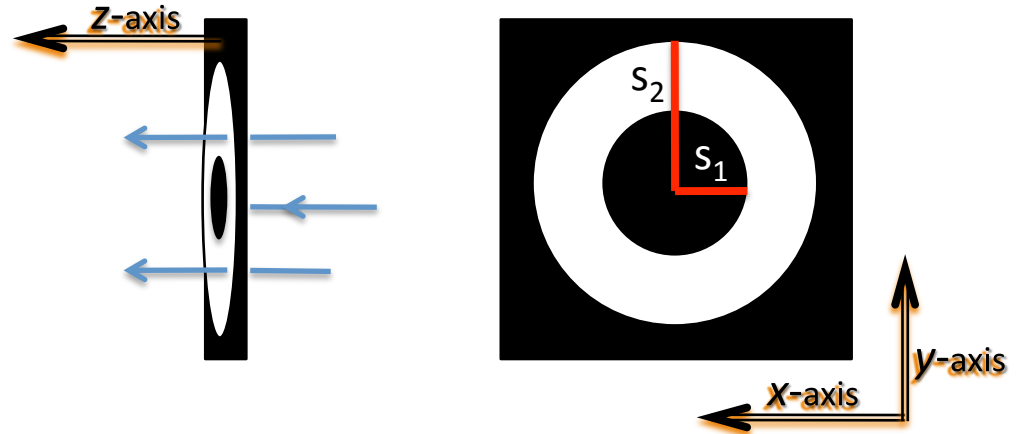


← Z-axis



# The filter dimensions

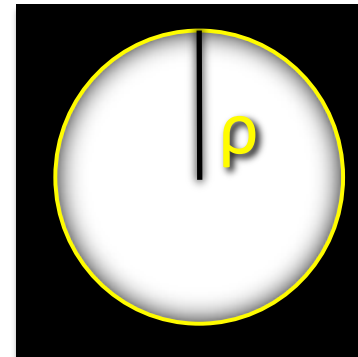
- Inner radius 3 mm
  - To block the low frequencies
- Outer iris diaphragm radius 10 mm
  - Limits the high frequencies coming through too
- Why is there a limit on the outer dimension?



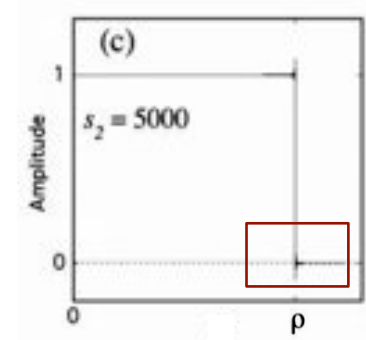
# Gibb's phenomenon

- Always a zero in the middle of the diffracted field at  $r = \rho$
- Causes double-lobed amplitude at edge of aperture
- This is the difference between our ring and that which would have come from a uniformly illuminated annular slit

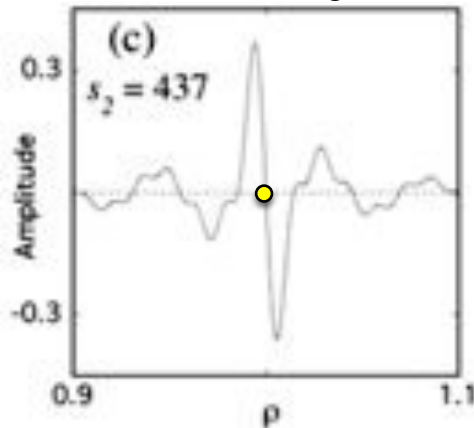
circular aperture



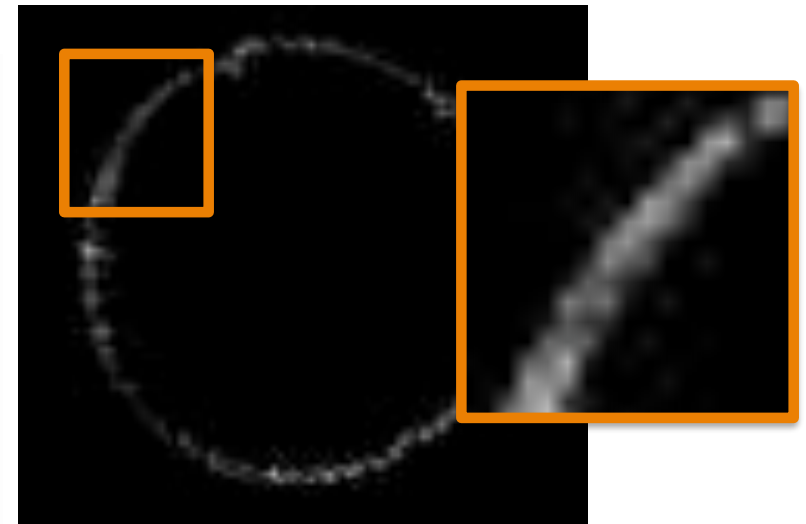
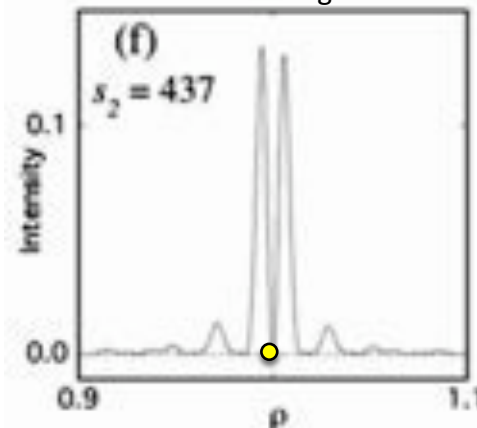
amplitude at edge of aperture



amplitude at edge of filtered image

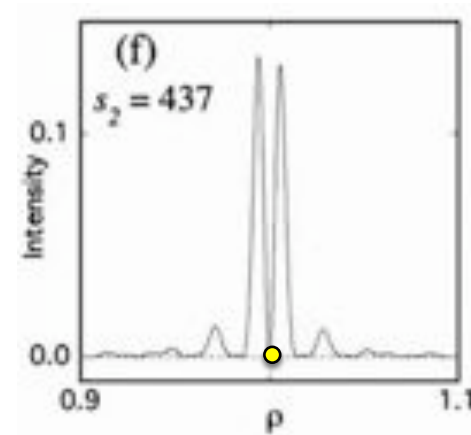


intensity at edge of filtered image

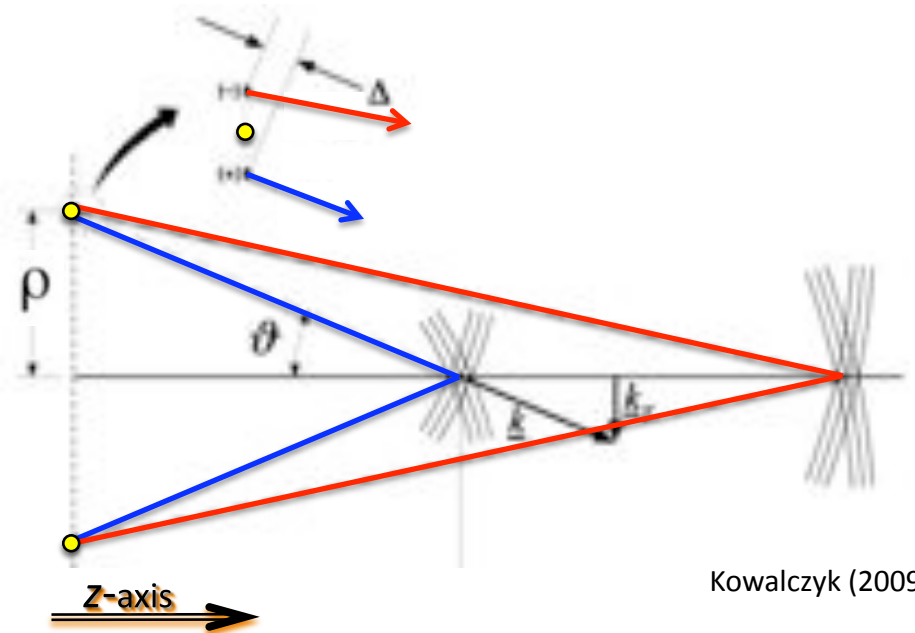
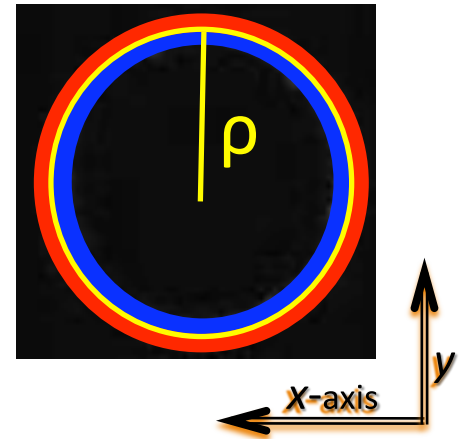


# Forming the Bessel beam

- Propagation delay  $\Delta$  between double-lobed intensity
- As you go farther on z-axis the lobes spread apart
  - constructive interference
  - start of Bessel beam
- Until a certain point where the lobes are too spread
  - destructive interference
  - end of the Bessel beam




edge-enhanced image



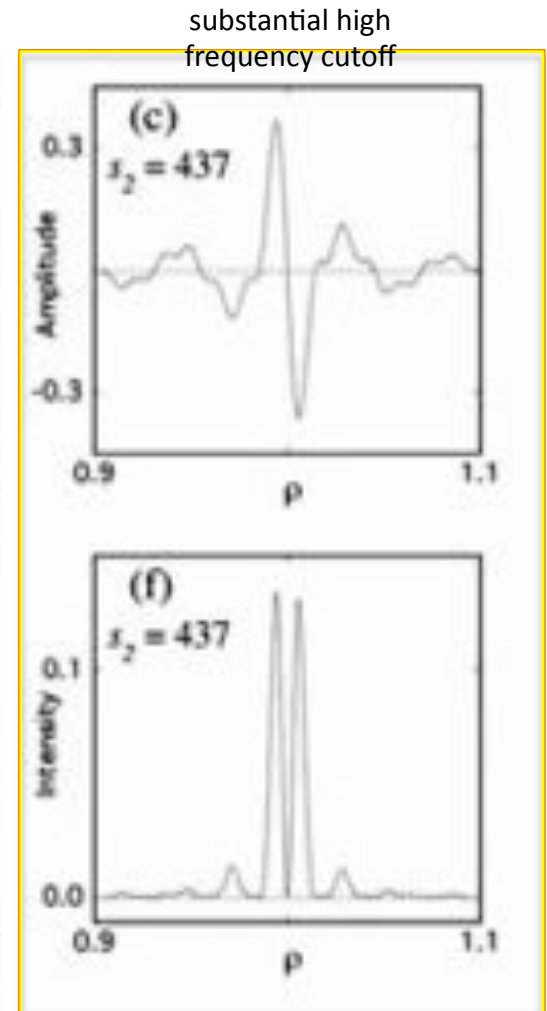
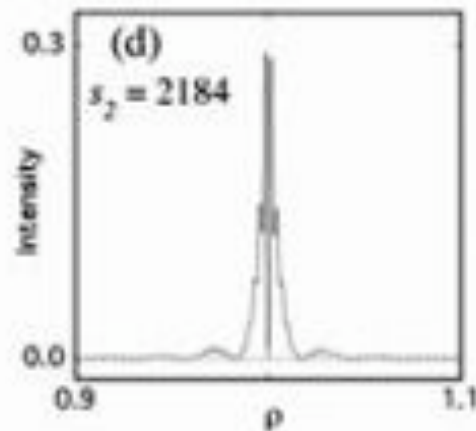
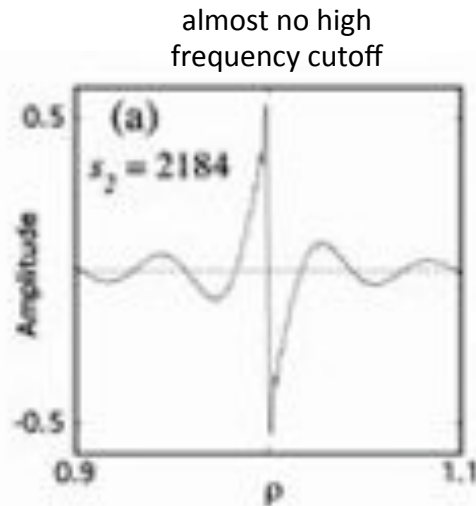


## Technically speaking

- “Bessel beam” normally used to describe non-diverging beams
  - Here, we’re making use of a diverging pattern to create the Bessel-like beam
  - Still has a Bessel function radial profile and exhibits the characteristic properties
- 

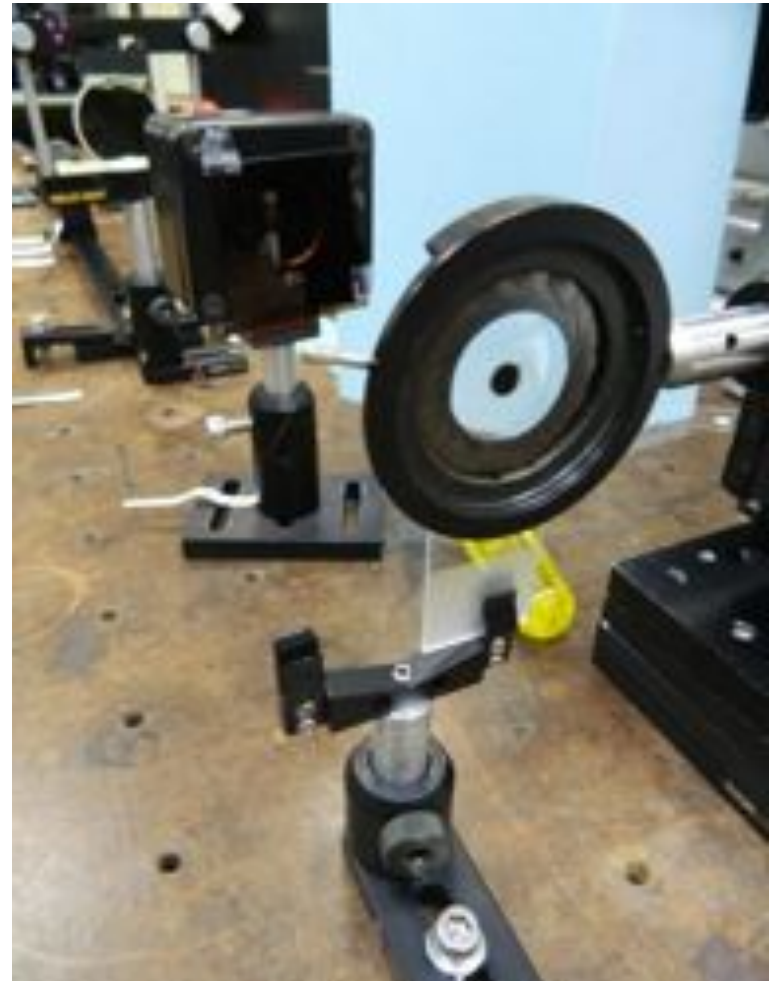
# Effect of outer frequency cutoff on separation between intensity lobes

- Cutting off high frequencies:
  - less of an overshoot
  - lobes are more distinguishable from each other
  - larger propagation delay between lobes
  - larger region of constructive interference on axis
  - longer Bessel beam

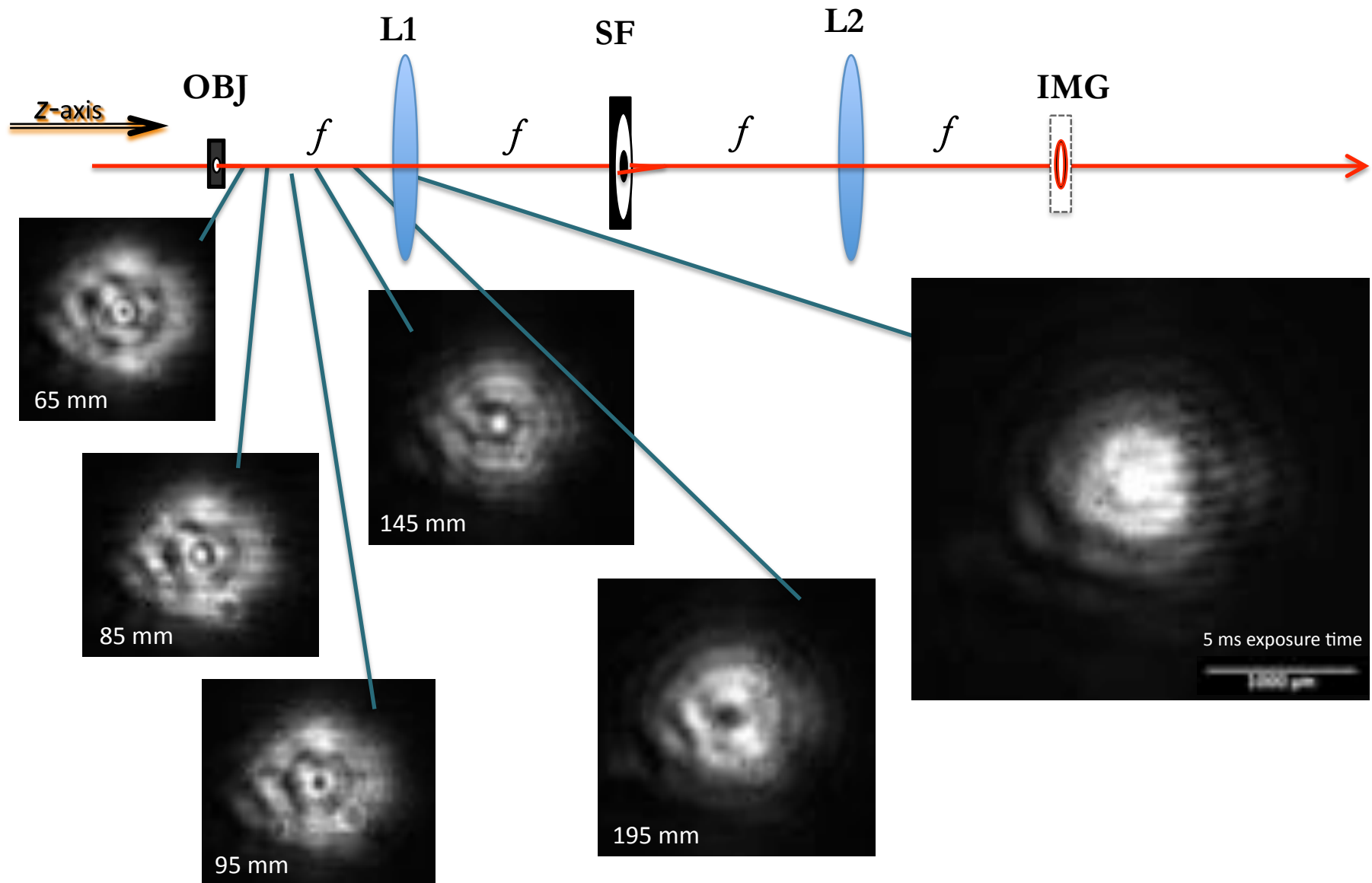


# Electrim EDC 1000N images

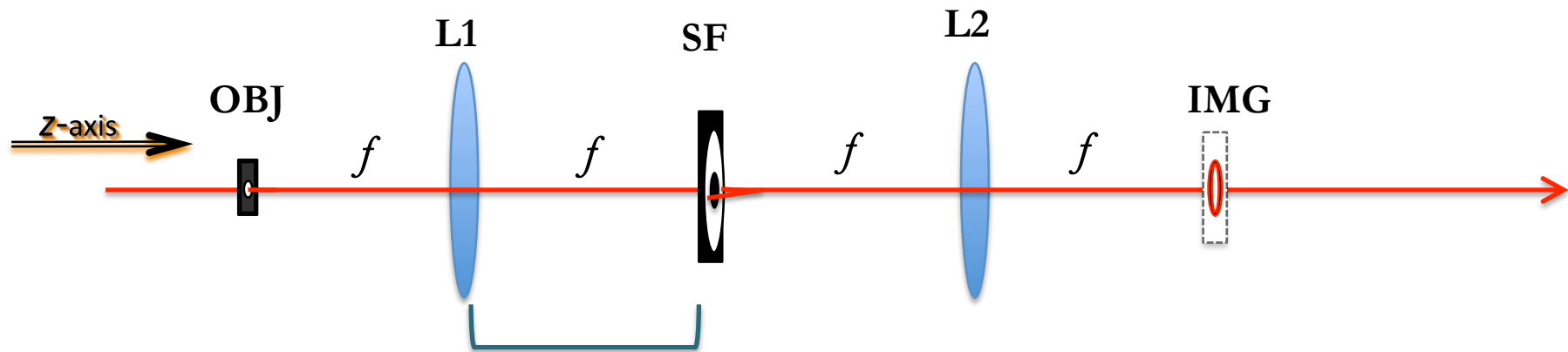
- 7.4 micron square pixels
- Polarizers to prevent saturation
- Recorded the evolution of the light field from the initial aperture to the final Bessel beam
- Images are transverse taken in transverse plane



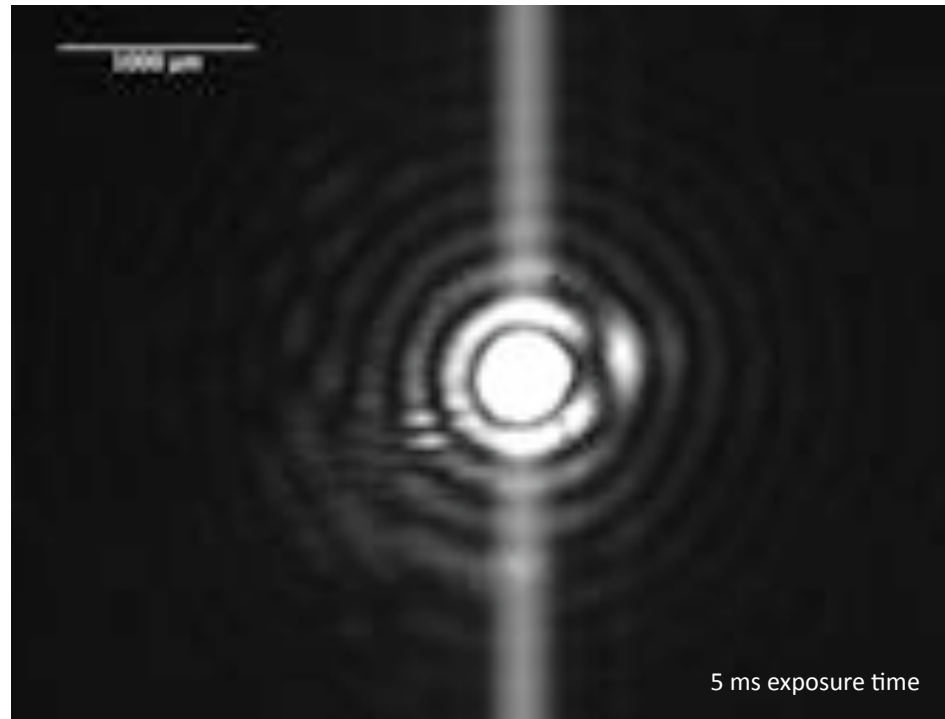
# Object to Lens 1



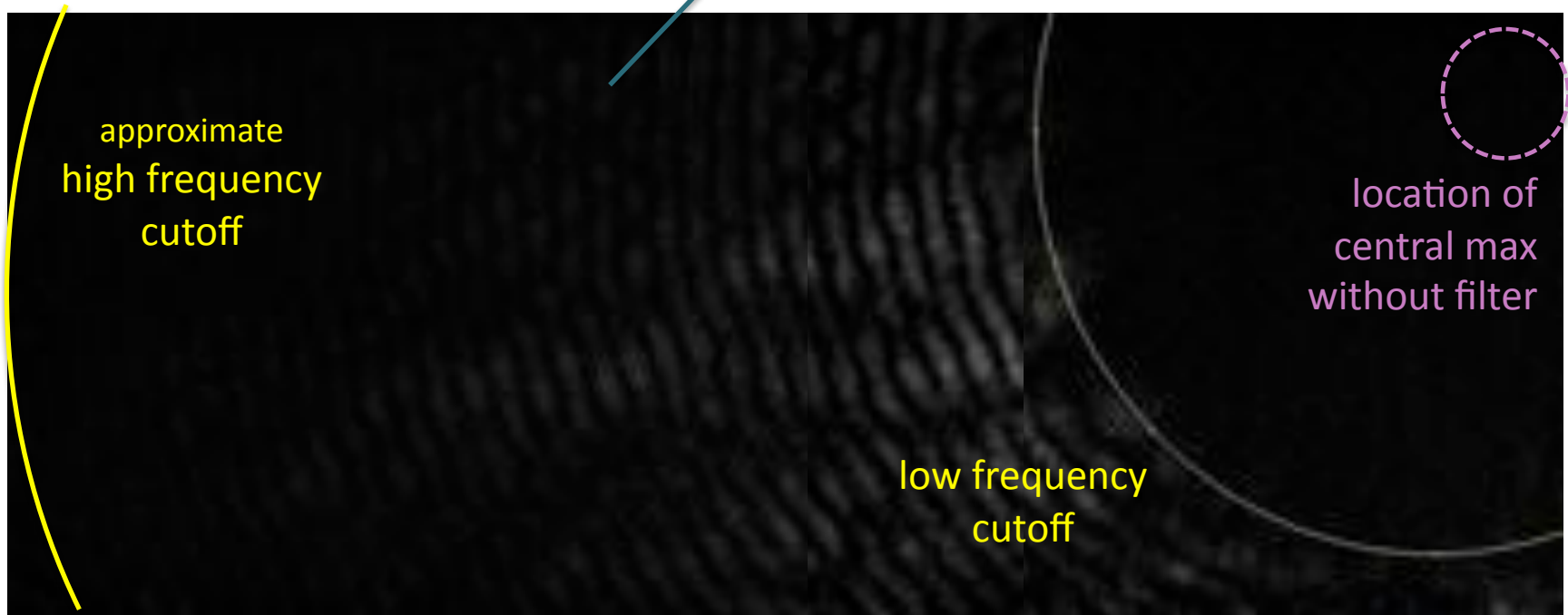
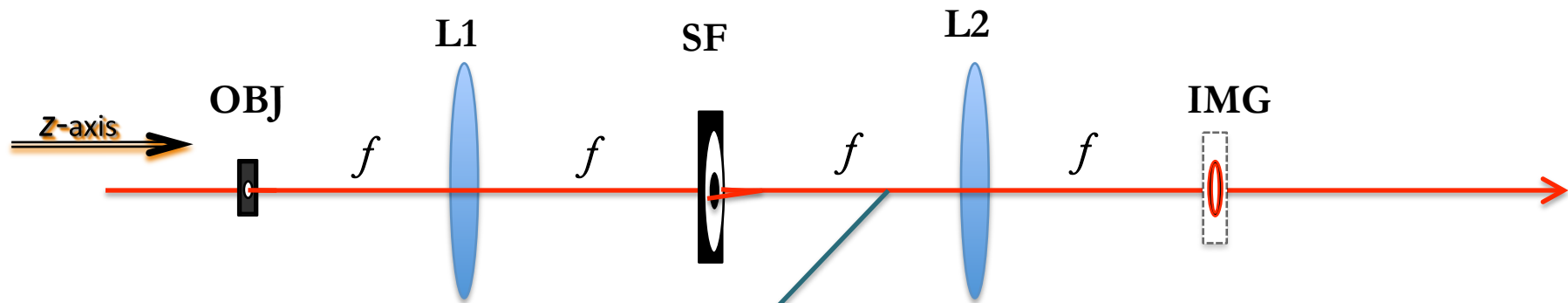
# Lens 1 to Spatial Filter



classic Airy  
pattern

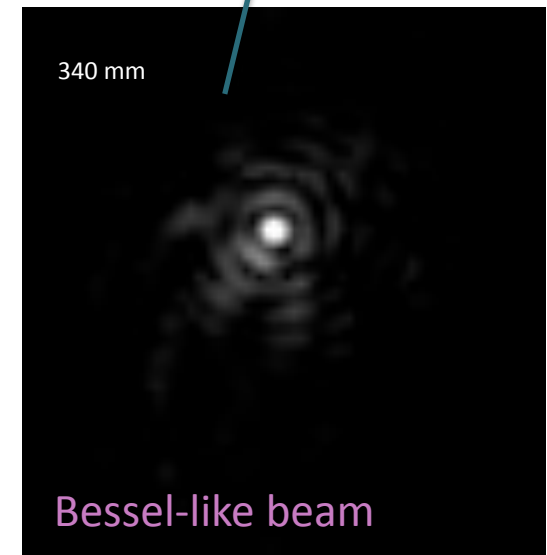
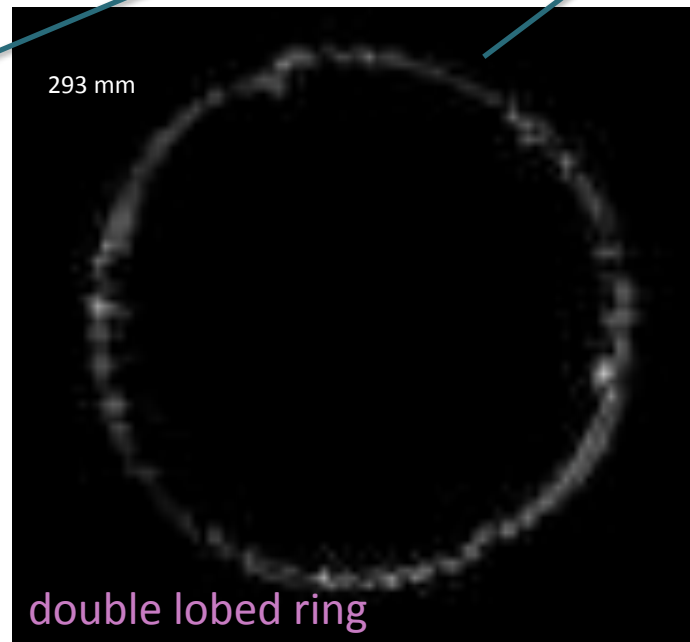
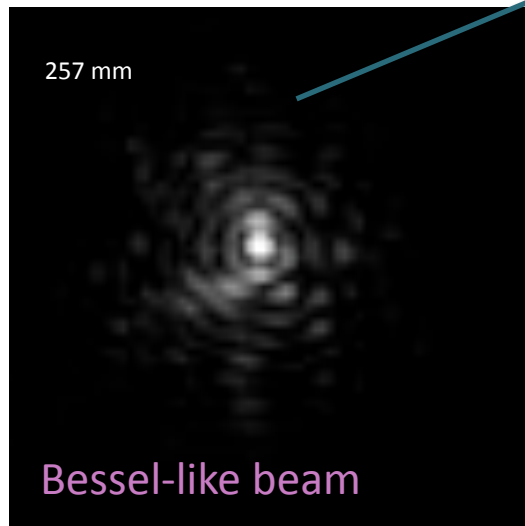
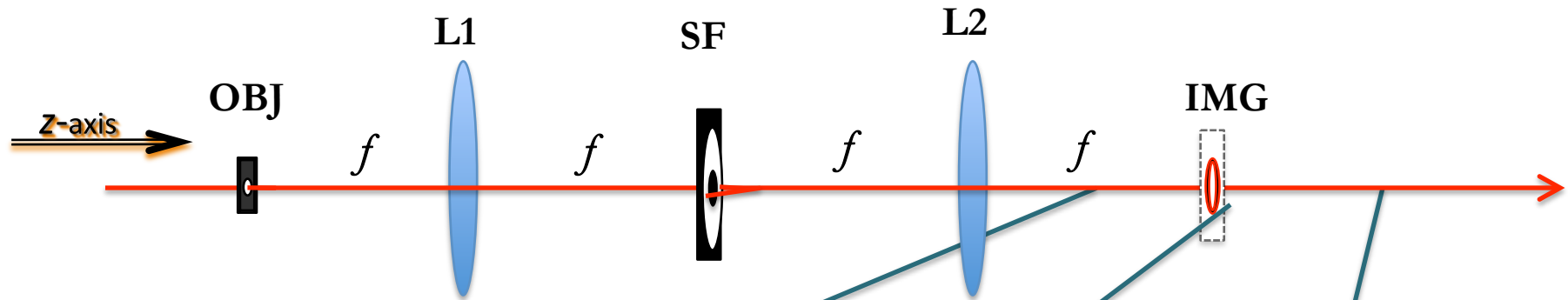


# Spatial Filter to Lens 2



190 mm from Fourier plane

# Lens 2 to Image Plane and Beyond



# The formation of the Bessel beam

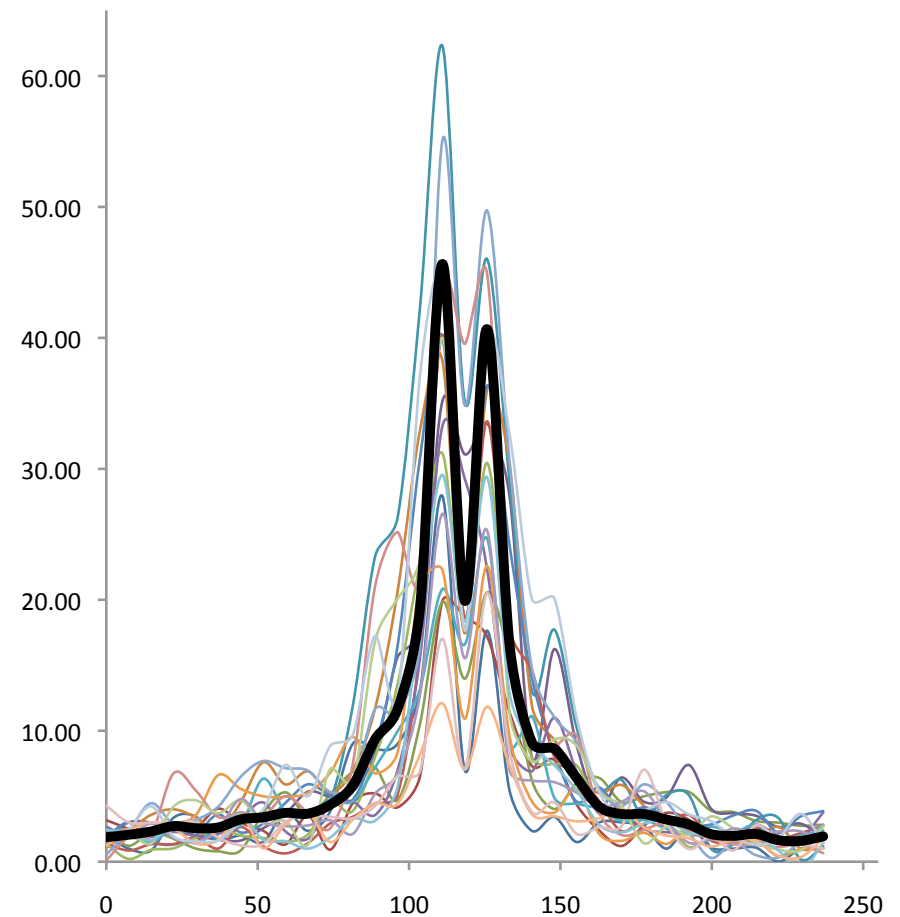
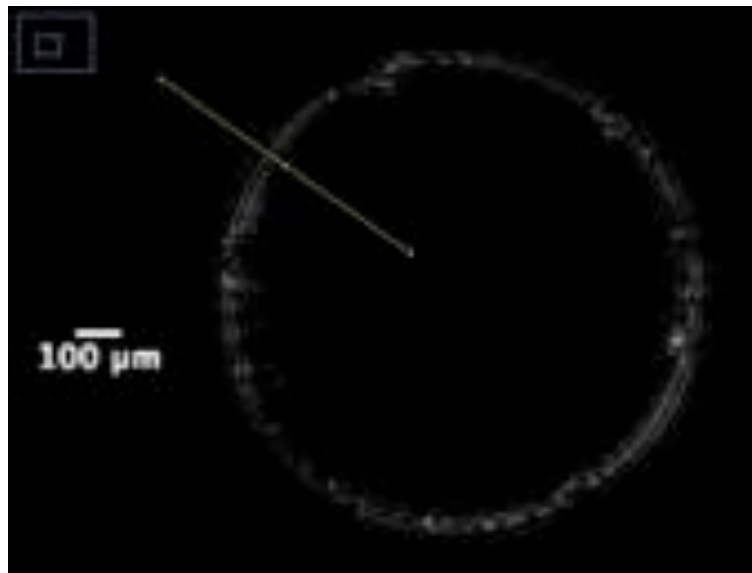


- First starts at about 220 mm from the final lens
- Propagates 47 mm
- Disappears 267 mm
- Ring of light comes to focus at 293 mm
- Bessel beam appears again at 320 mm
- 47 mm propagation
- 376 mm it disappears

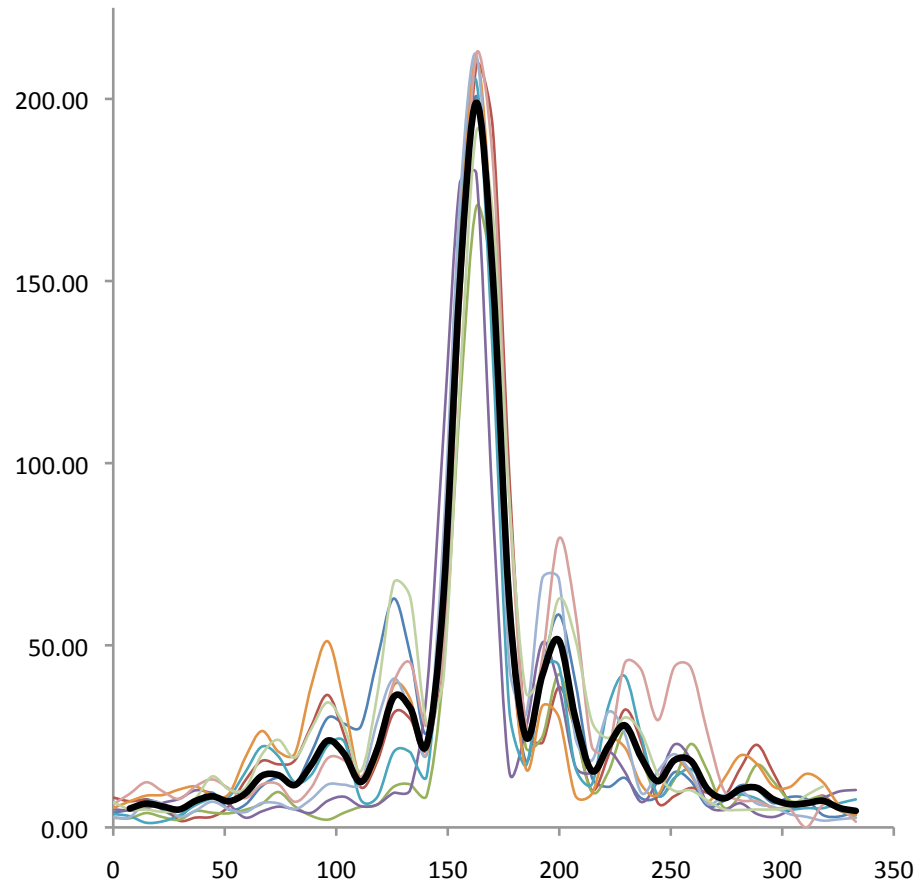
# ImageJ Analysis: thin ring of light



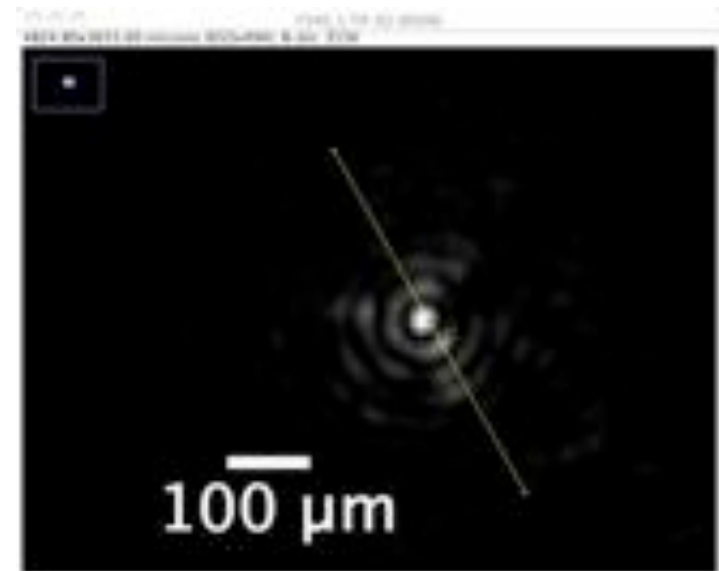
- Average radial intensity profile of double-lobed ring source at focal point
- Spacing between lobes about  $14.8\ \mu\text{m}$



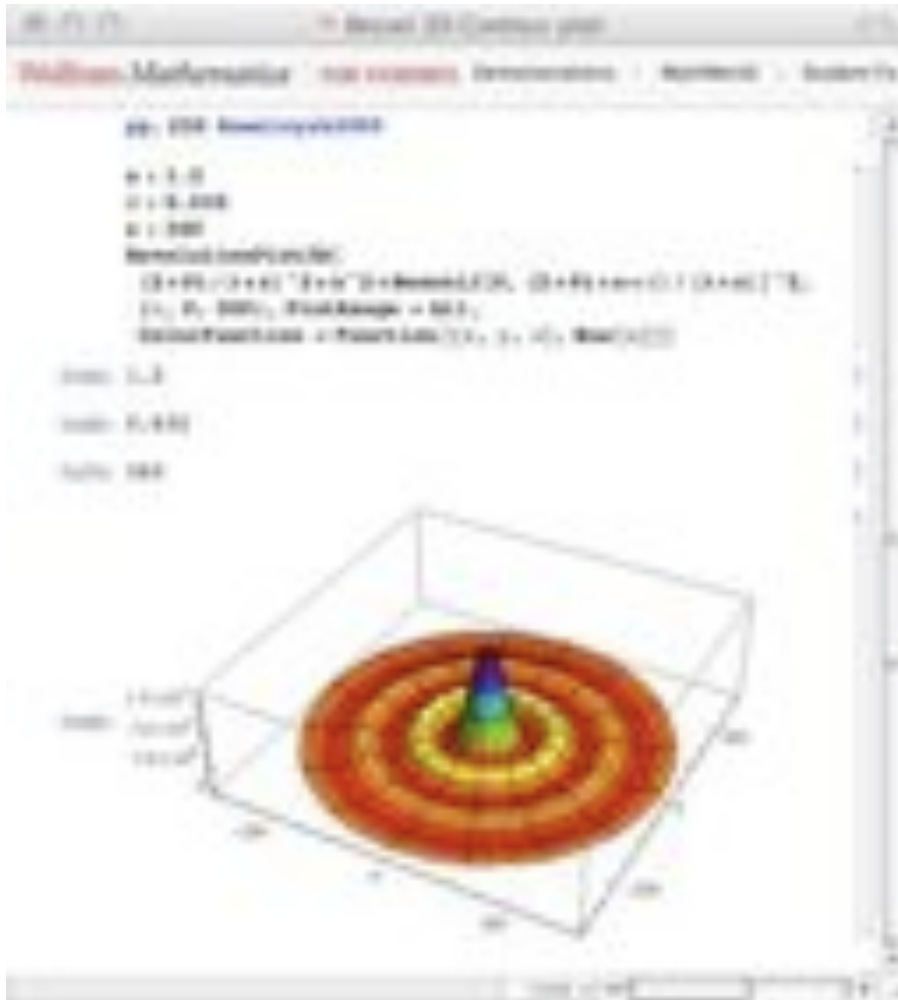
# ImageJ Analysis: Bessel beam



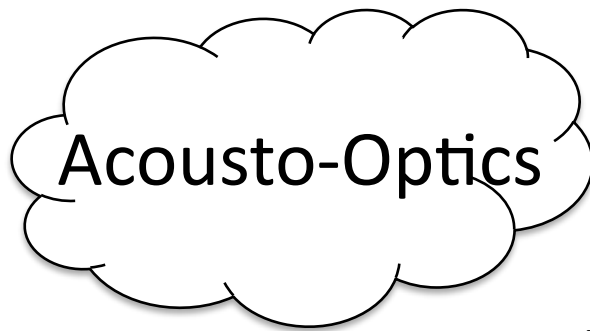
- Azimuthally averaged radial intensity profile of Bessel beam at a distance  $Z = 340$  mm behind second Fourier lens
- Central spot size about  $44.4 \mu\text{m}$



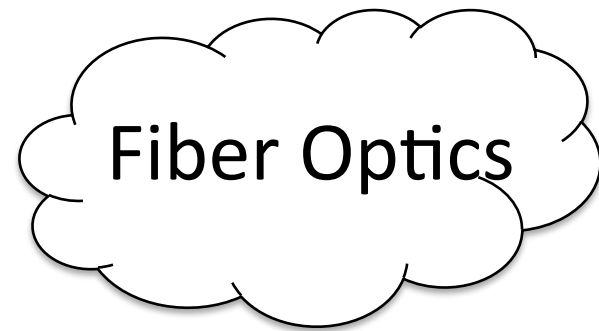
# Modeling in Mathematica



- Currently creating theoretical model of the beam's intensity profile
  - Fit this model to the average intensity of the beam at a certain  $z$  distance
- Also graphing the  $z$  dependence of the theoretical and experimentally determined central spot size



Acousto-Optics



Fiber Optics

Hundreds of papers later...

Acoustic Vortices

**Dashti 2006**

TAG Lens

**McLeod & Arnold 2008**

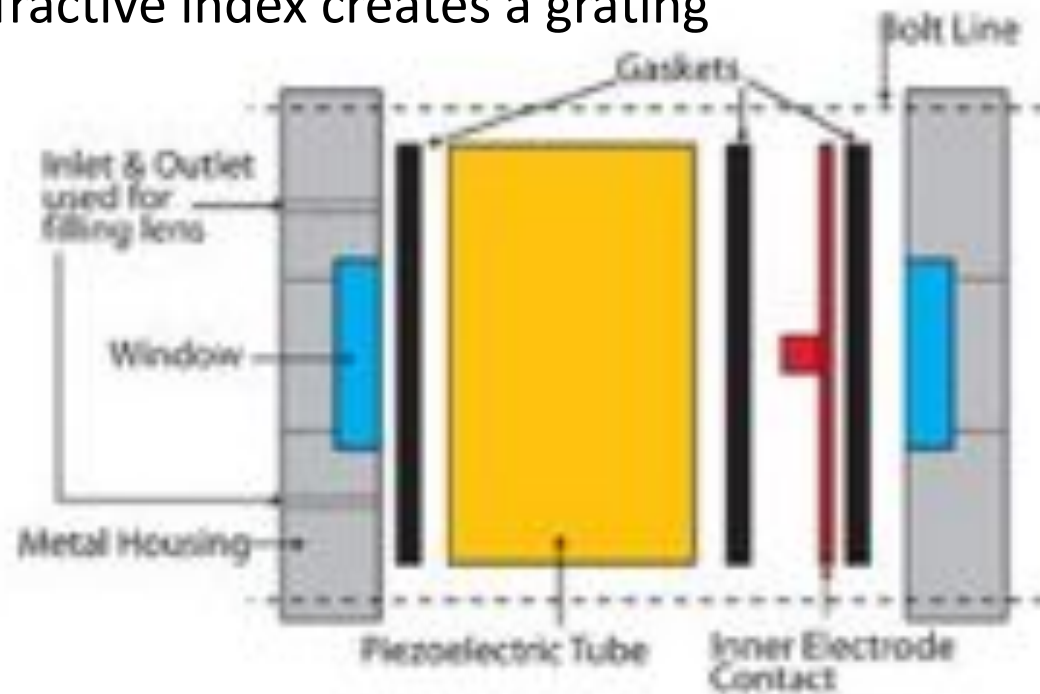


Bessel Beams

Melia

## Tunable Acoustic Gradient Index of Refraction Lens (TAG lens)

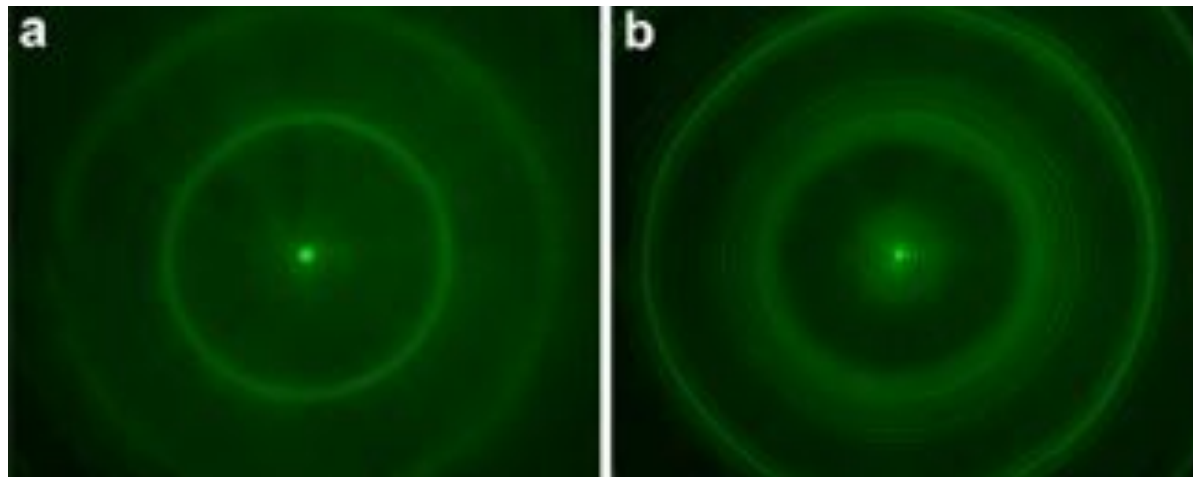
- A cylindrical cavity filled with a viscous fluid (silicone oil)
- A piezoelectric ring is driven to provide an radially symmetric acoustic wave
- Frequency range of 250-500 kHz at amplitudes from 0-100 V
- Change in refractive index creates a grating
- Tunability!



E. McLeod and C.B. Arnold (2008).

# TAG Lens

- Invented by Craig Arnold, a professor at Princeton (2008 paper)
- Co-founded TAG Optics Inc. to commercialize
- Receive a device in several weeks
- Does not make higher-order Bessel beams

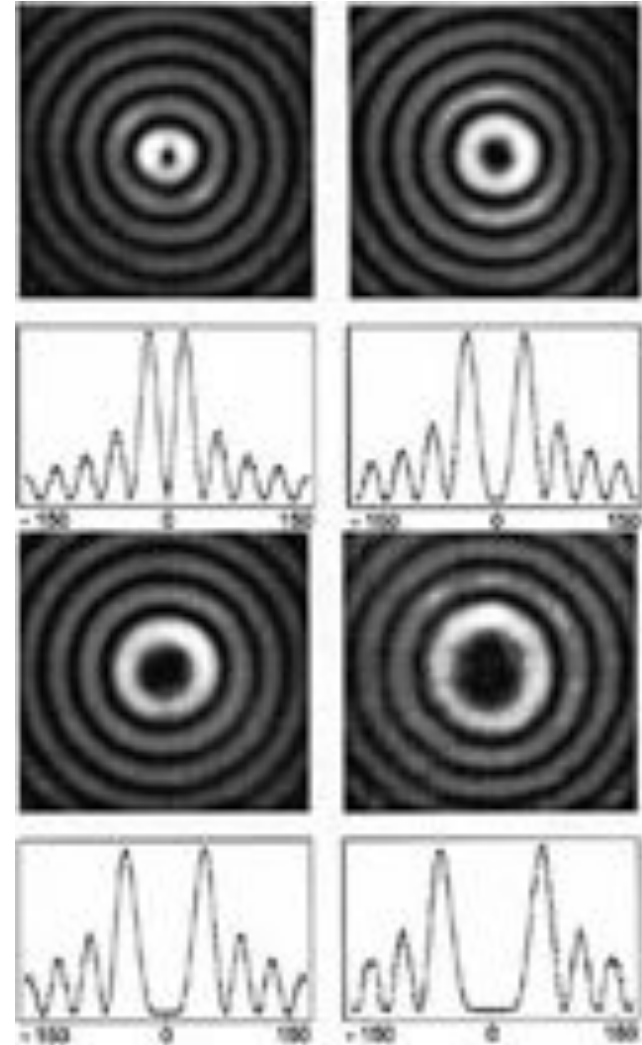


E. McLeod and C.B. Arnold (2008).

# Azimuthal Phase Variation

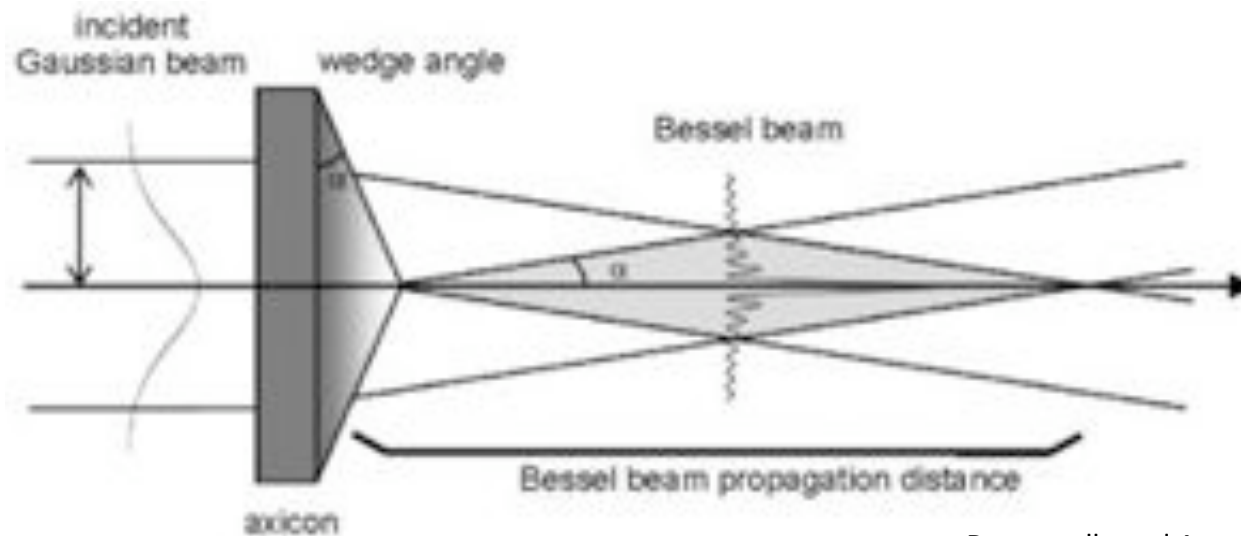
$$E_l(r, \phi, z) = A e^{ik_z z} J_l(k_r r) e^{i\phi l}$$

- Zero order ( $l = 0$ )
  - No phase variation
  - Bright central maximum
- Higher-order ( $l > 0$ )
  - Azimuthal phase variation
  - Dark central
- Conservation of topological charge
  - Laguerre-Gaussian (LG) beam with initial orbital angular momentum



Arlt and Dholakia (2000)

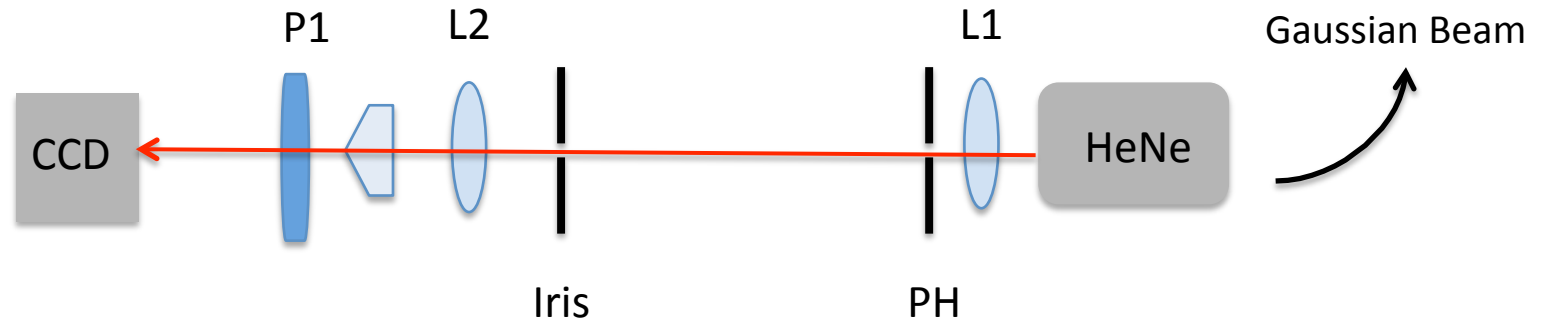
# Axicon (Conical) Lens



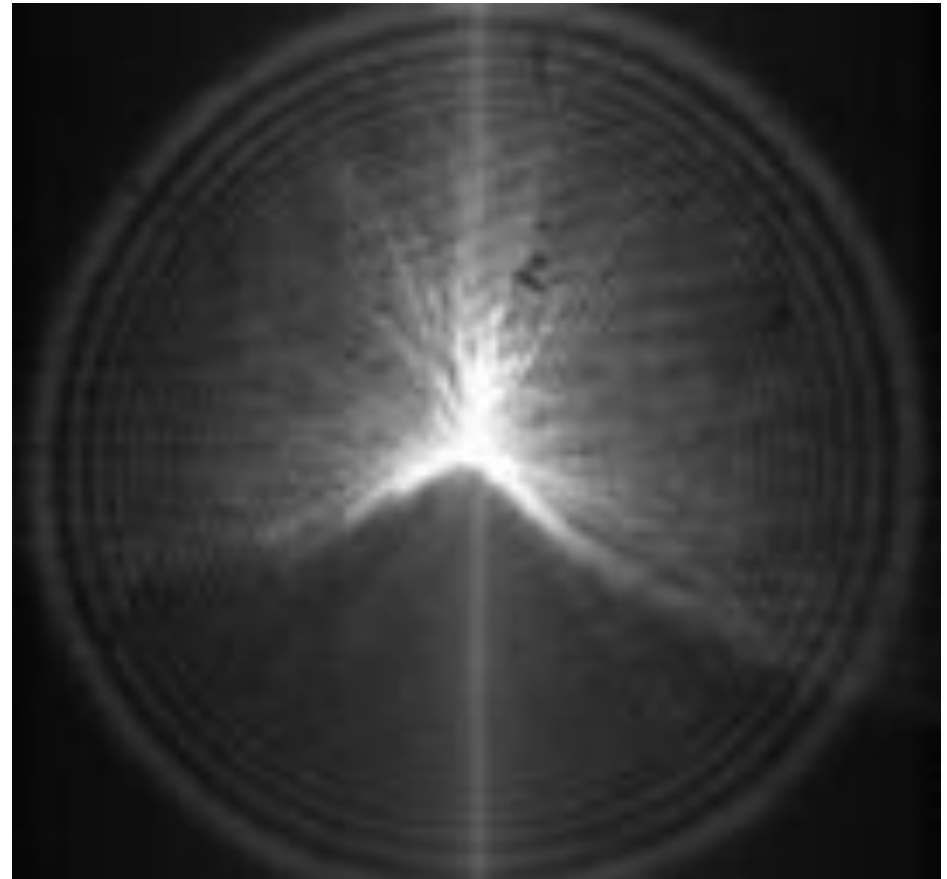
Duocastella and Arnold (2012)

- Conical lens with apex angle  $\alpha$  and index of refraction  $n$
- Bessel beam length proportional to incident beam diameter
- Most efficient method of creating Bessel beams

# Zero- Order Setup

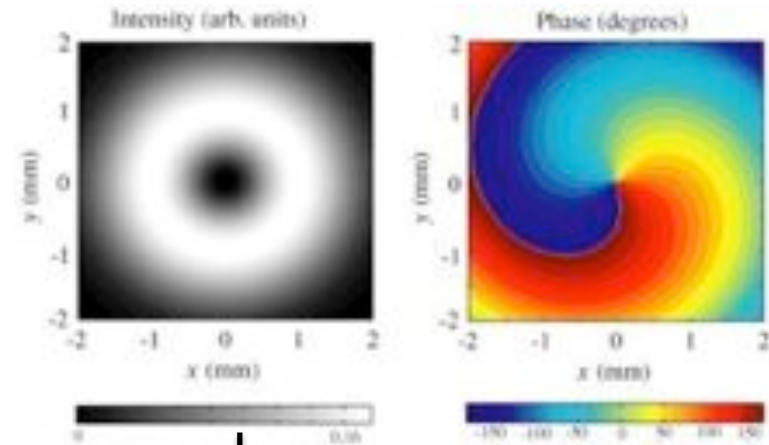


- Lens 1 and 75 micron pinhole
- Iris diaphragm and lens 2
  - Cleans up Airy pattern and collimates
- Polarizer
- Axicon (borrowed from CCNY)
  - $1^\circ$  axicon angle & 25.44 mm diameter
  - $n \approx 1.55$
- CCD for recording

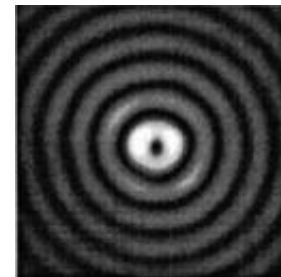


# Higher-Order Setup

- Open Cavity HeNe Laser
- Astigmatic Mode Converter  
→ convert to LG beam
- Pair of Lenses (Telescope)  
→  $f = 35 \text{ mm}$  &  $750 \text{ mm}$
- Axicon lens



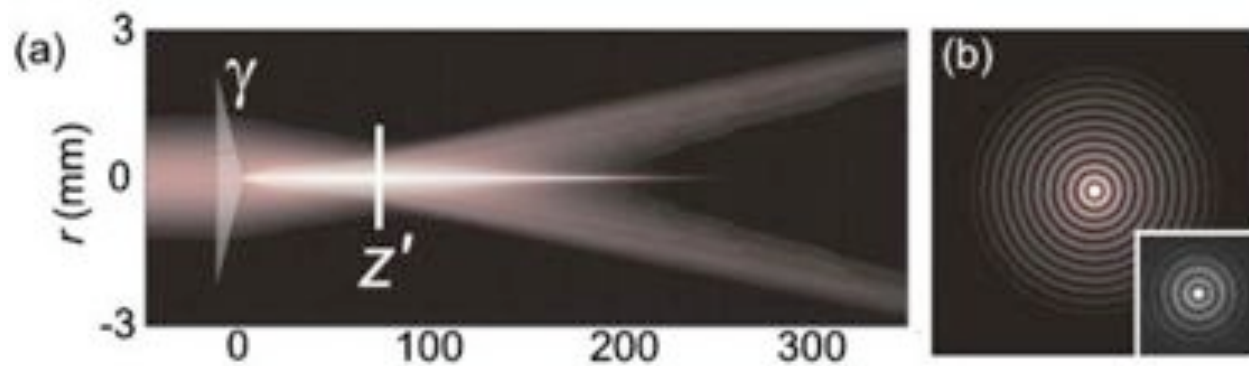
C.H Gan (2007).



Arlt and Dholakia (2000)

# Conclusion

- Bessel beams have interesting properties and important applications
- There are a variety of ways to make them
- This was a good topic for a teaching laboratory!



Milne (2008)

# Acknowledgements

Founder of the Laser Teaching Center

- Hal Metcalf



Mentors

- John Noé
- Marty Cohen



Research funded by the Physics and Astronomy REU program  
and the Laser Teaching Center at Stony Brook University

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- McLeod, J. M. (1953). "The Axicon: A New Type of Optical Element." *Optical Society of America* 44(8): 592-597.