Conversions of Transverse Gaussian Laser Modes

Jay Rutledge, Max Stanley, Marcus Lo

Laser Teaching Center
Physics and Astronomy, Stony Brook University

Overview

- Production of high-order Hermite-Gaussian (HG) Modes
- Conversion of HG modes into the helical Laguerre-Gaussian (LG) base (and vice versa)
- Combination of helical LG modes to produce sinusoidal LG Modes
- Phase Analysis of achieved conversions

What's a laser mode?

- Stable pattern of oscillation within an optical cavity
- Intensity profile remains the same throughout propagation

Recognize this spot?

Only the lowest-order mode of operation!

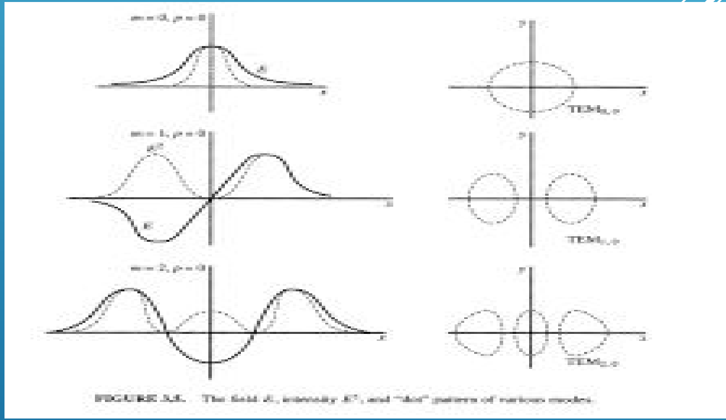
There are many higher-order modes and even several spaces of states.

Hermite-Gaussian Modes

- HG modes are discrete solutions of the paraxial wave equation in Cartesian coordinates
  - Rectangular symmetry
  - Characterized by the x & y indices TEM_{nm}.
  - Gaussian functions multiplied the Hermite polynomials.

n	
0	1
1	$2x$
2	$4x^2 - 2$
3	$8x^3 - 12x$

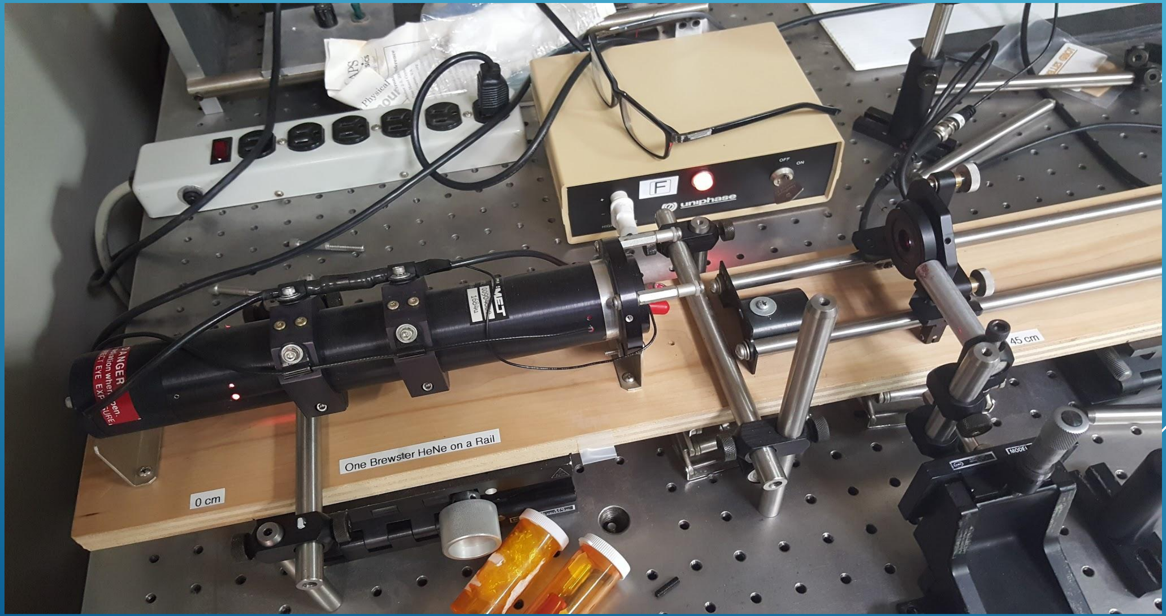
$$E_{nm}(x,y,z) = E_0 \frac{w_0}{w(z)} \cdot H_n\left(\sqrt{2}\frac{x}{w(z)}\right)\exp\left(-\frac{x^2}{w(z)^2}\right) \cdot H_m\left(\sqrt{2}\frac{y}{w(z)}\right)\exp\left(-\frac{y^2}{w(z)^2}\right) \cdot \exp\left(-i\left[kz - (1+n+m)\arctan\frac{z}{z_R} + \frac{k\left(x^2+y^2\right)}{2R(z)}\right]\right)$$



[1] Images: RP-Photonics

Open-Cavity Laser

- A Single-Brewster He-Ne laser with a rail-mounted output coupling mirror (OC).
- A variable aperture within the cavity and the tuning of the OC allows for the selection of HG modes.
    - A 10 micron wire on a translation stage enables selection of higher orders.



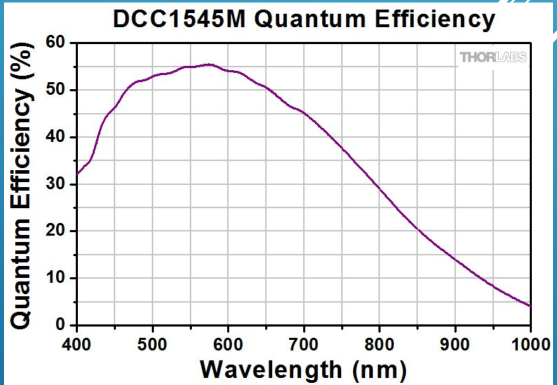
Imaging

- A Thor Labs CCD camera (and software provides images and intensity profiles.

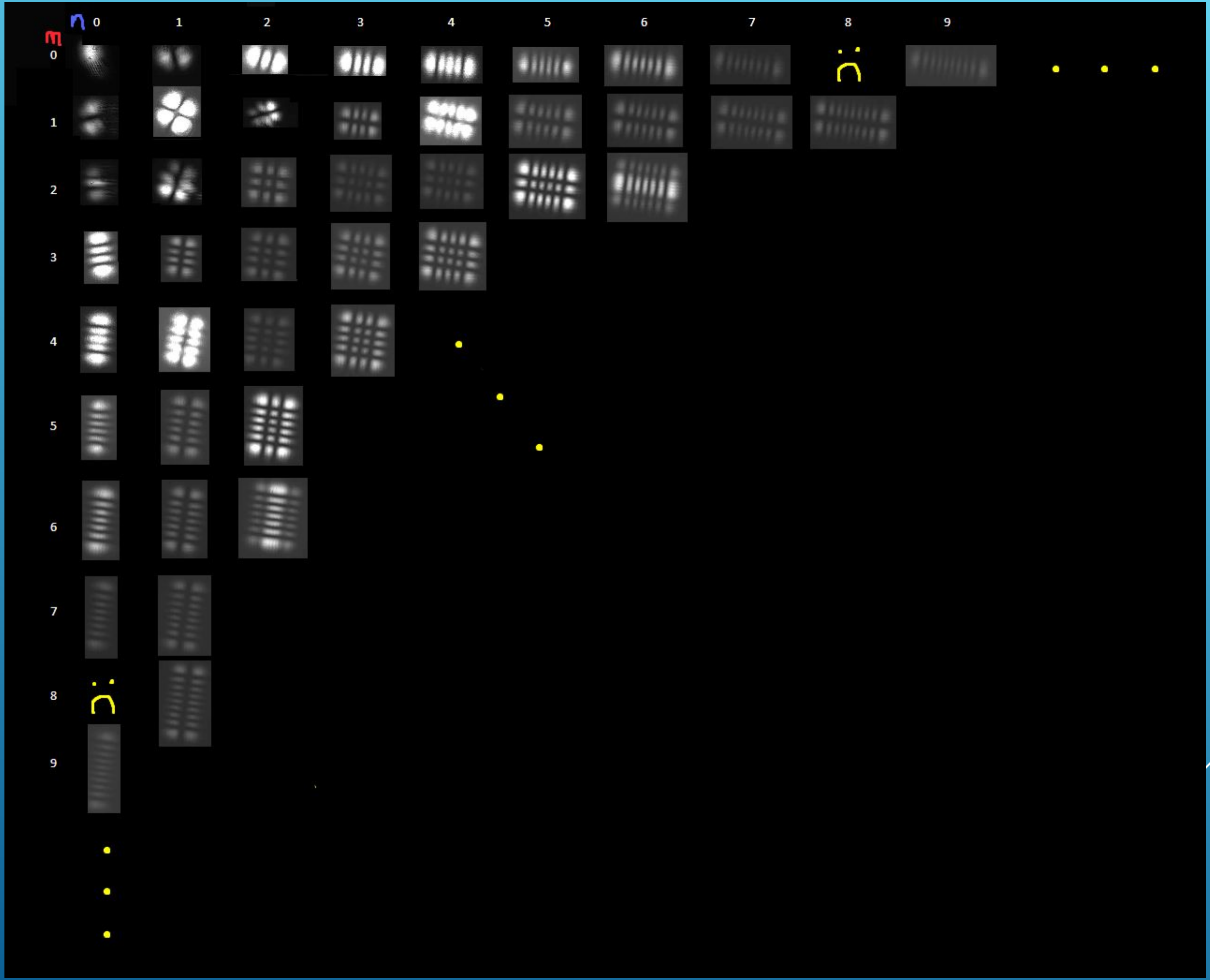
  A single linear polarizer is used for attenuation

DCC1545M Camera:

- Monochrome
- 1280x1024 Resolution



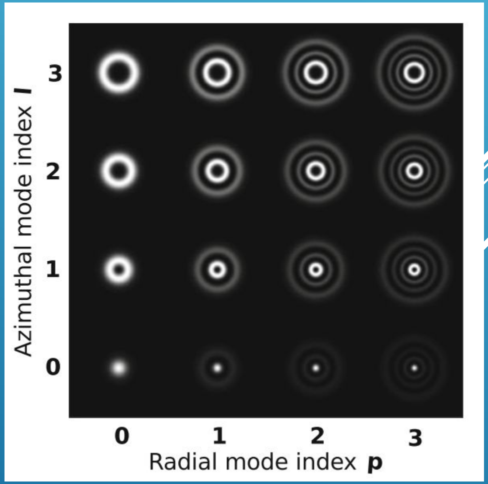
Achieved HG Modes



Conversion to Laguerre-Gaussian Modes

- Another independent set of solutions in cylindrical coordinates
  - Characterized by the radial and azimuthal indices p and .ℓ
  - Gaussian functions multiplied by the Laguerre polynomials.
- Carry orbital angular momentum independent of polarization (spin angular momentum) in integer quantities $\hbar$ per photon.
- ℓ
- Indices convert as $HG_n^m \rightarrow LG_p^\ell = LG_{\min(n,m)}^{|n - m|}$

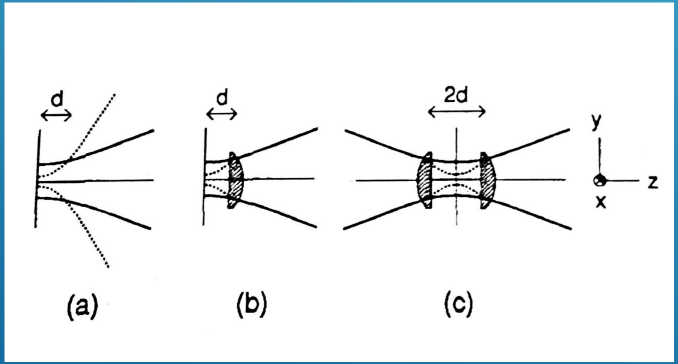
$$u^{\mathrm{hel}}_{p,l}(r,\phi,z) = \frac{1}{w(z)}\sqrt{\frac{2p!}{\pi(|l|+p)!}}\; e^{\mathrm{i}\,(2p+|l|+1)\Psi(z)} \times \left(\frac{\sqrt{2}r}{w(z)}\right)^{|l|} L_p^{(|l|)}\left(\frac{2r^2}{w(z)^2}\right) e^{-\mathrm{i}k\frac{r^2}{2q(z)}+\mathrm{i}l\phi}$$



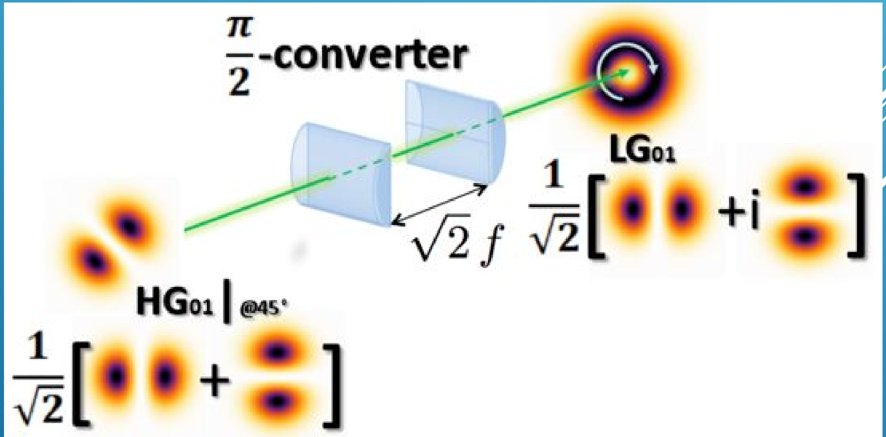
[2] Image: 2Physics

Astigmatic Mode Converter

- A mode-matching lens, and pair of cylindrical lenses separated precisely by f√2.
- Exploits the Gouy phase:
  - First lens introduces an astigmatism and definite phase difference (π/2) between HG components oriented along axes of astigmatism.
  - Second lens removes the astigmatism, leaving a modal beam.



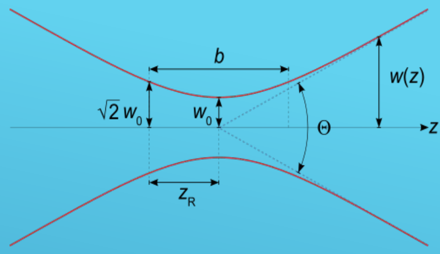
[3] Beijersbergen, 1992



[4] Image: Wikipedia: Angular Momentum of Light

Beam Profiling and Mode-Matching

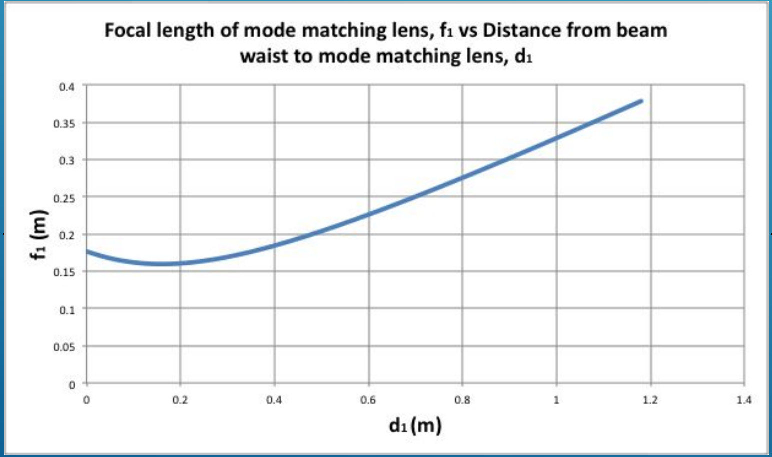
- Matching a beam to a mode for coupling into an optical instrument requires Gaussian beam optics
- We must know:
  1) The beam waist size, w_0
  2) The Rayleigh range, z_R

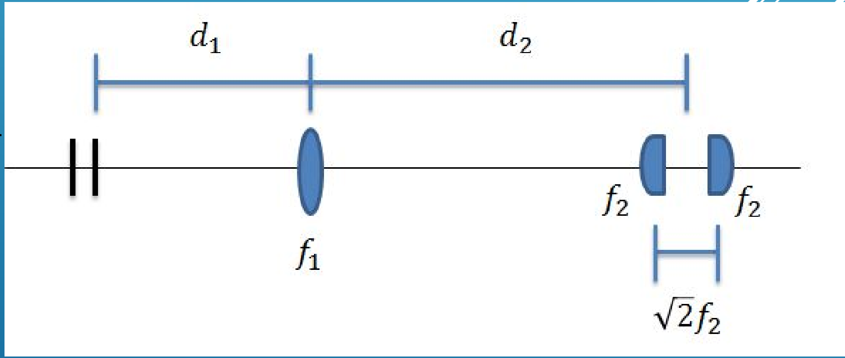


$$w(z) = w_0\sqrt{1 + \left(\frac{\lambda(z - z_{waist})}{w_0^2\pi}\right)}$$

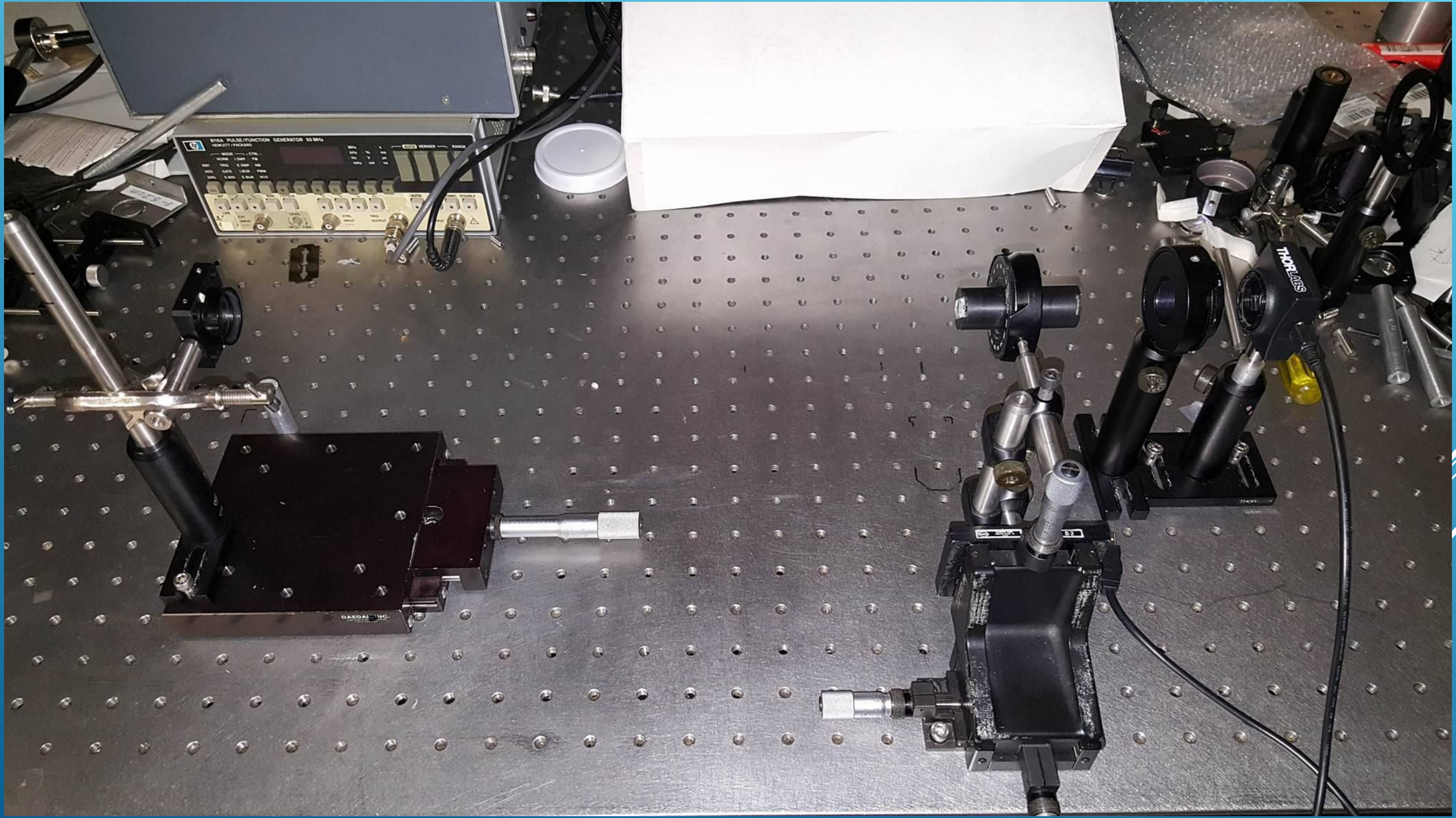
$$\Theta \approx \frac{2\lambda}{\pi w_0}$$

$$z_R = \frac{\pi w_0^2}{\lambda}$$

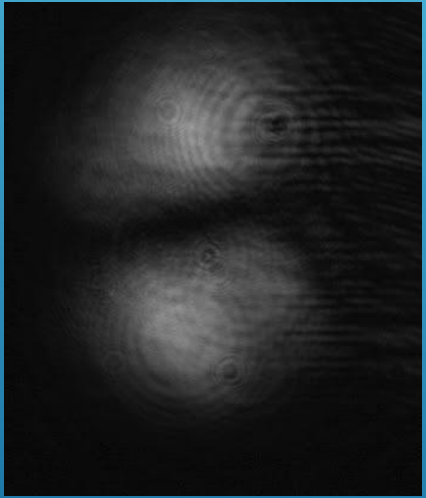




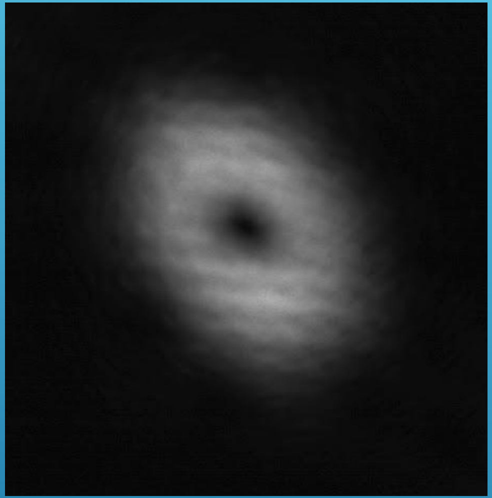
Astigmatic Mode Converter



Achieved Conversions

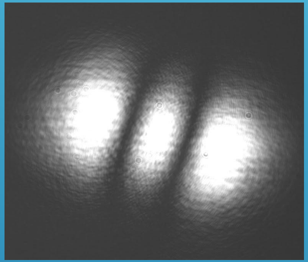


HG_1^0

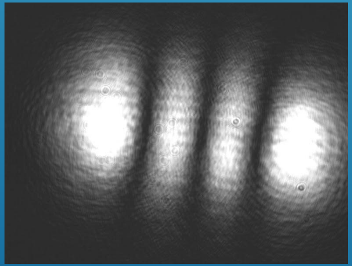


LG_1^0

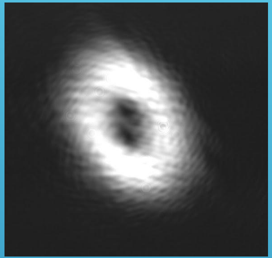
Achieved Conversions



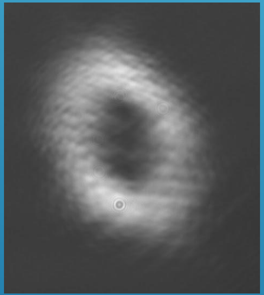
HG_0^2



HG_0^3



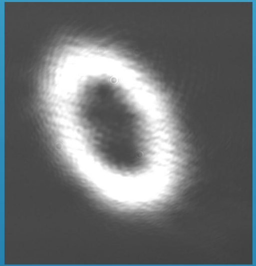
LG_3^0



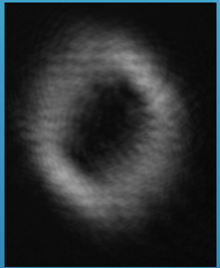
LG_3^0

Conversions Cont'd

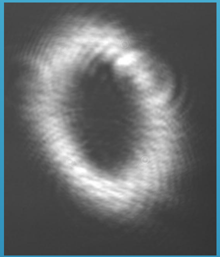
- All HG's with either n or m = 0 convert to a simple ring.



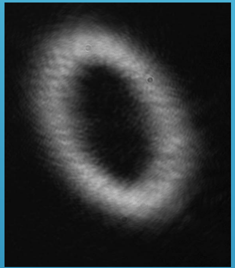
LG_5^0



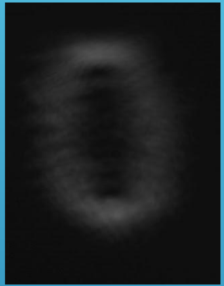
LG_6^0



LG_7^0

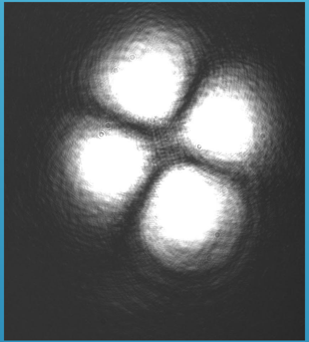


LG_8^0

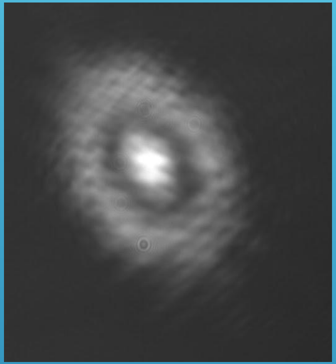


LG_9^0

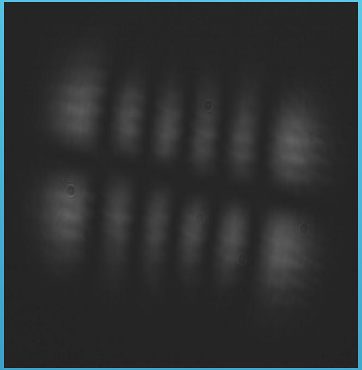
Conversions Cont'd



HG_1^1



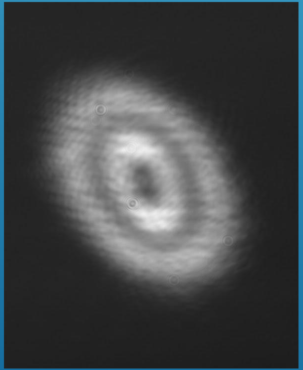
LG_0^1



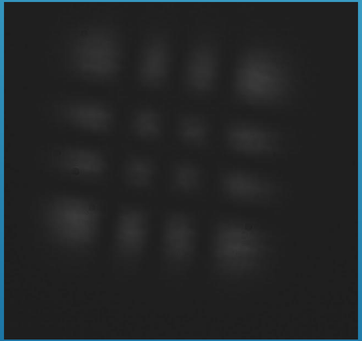
HG_1^5

LG_1^4

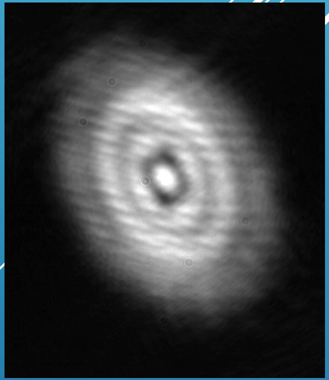
HG_1^3



LG_1^2



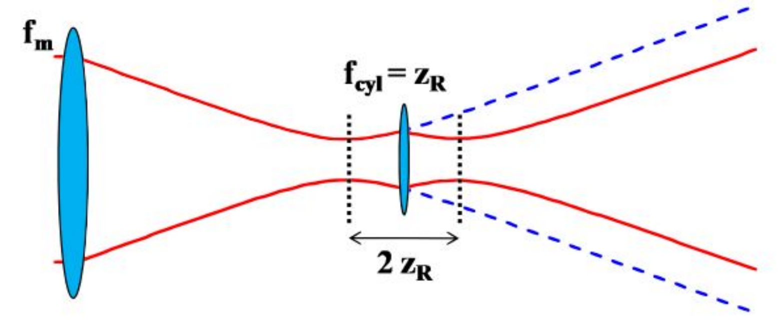
HG_3^3



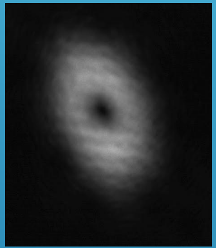
LG_0^3

Mode Reversion (Just for Kicks)

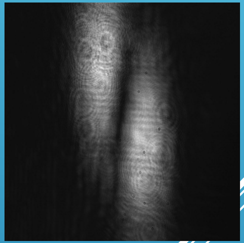
- Each set of solutions is an independent basis set, so in principle any set may be converted to another, i.e. LG modes can be reverted back to the HG base.
  - A mode-matching lens and single cylindrical lens can perform this.



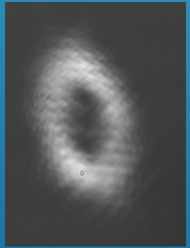
[5] Image: Hamsa Sridhar, Laser Teaching Center



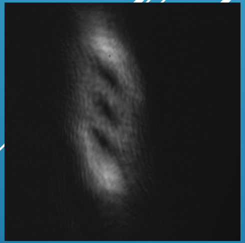
LG 10



HG 10



LG 03

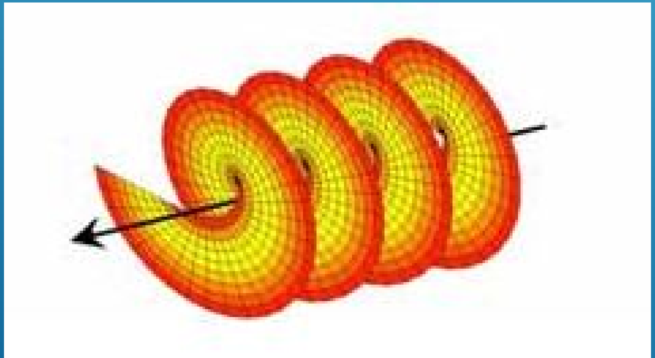


HG 03

Helical LG Modes and Optical Vortices

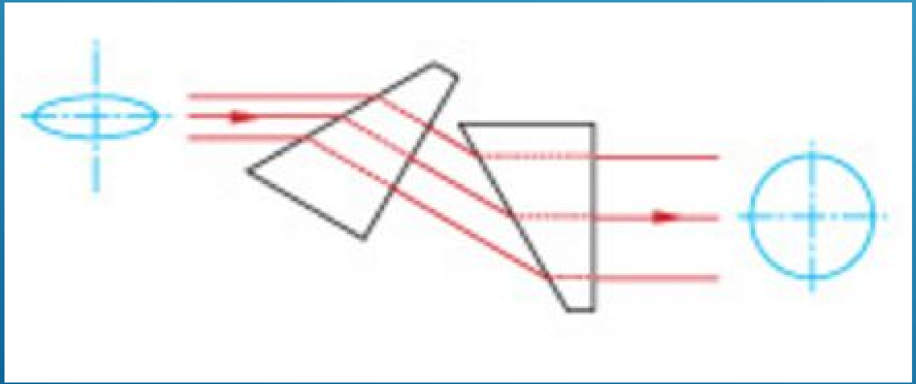
- So far, all converted modes have been “Helical” LG modes.
  - The orbital angular momentum of the LG mode is a result of a helical wavefront.

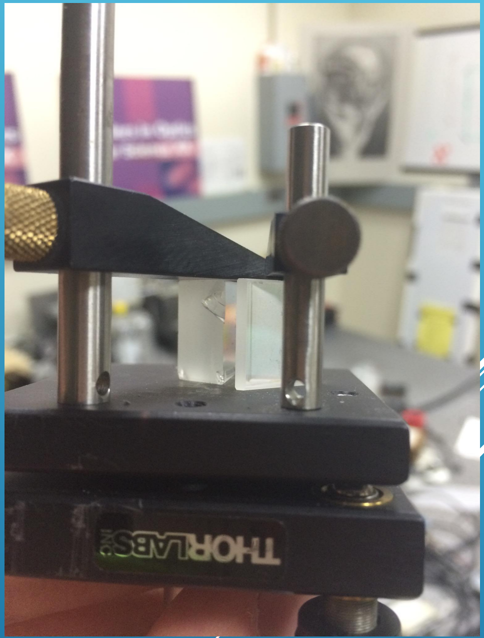
- Those with dark centers are in the class of beams “Optical Vortices” (though not all vortices are necessarily modes!).
  - A phase singularity exists at the center--a region of undefined phase and consequently vanishing intensity.
  - Applications in trapping and exerting torques on small particles.



Ellipticity

- Troubleshooting:
  - Spherical Lens
  - Anamorphic Prisms
  - Just realign!



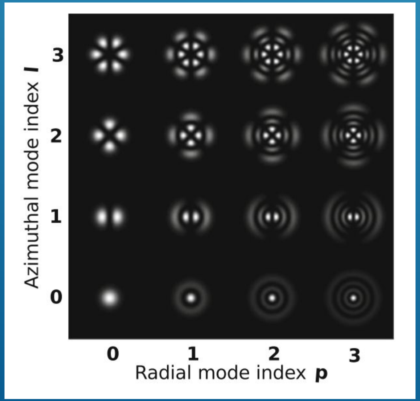


Generation of Sinusoidal Laguerre-Gaussian Modes

- This is where the distinction of a "helical" LG mode becomes important:
  - The superposition of two oppositely-handed helical LG modes results in a sinusoidal LG mode.
    - Cancellation of orbital angular momentum (no more helical wavefront!)
      - Interests in LIGO (Laser Interferometer Gravitational Wave Observatory)
    - Sinusoidal variation in azimuthal intensity.

$$u^{\mathrm{hel}}_{p,l}(r,\phi,z) = \frac{1}{w(z)}\sqrt{\frac{2p!}{\pi(|l|+p)!}}\; e^{\mathrm{i}\,(2p+|l|+1)\Psi(z)} \times \left(\frac{\sqrt{2}r}{w(z)}\right)^{|l|} L_p^{(|l|)}\left(\frac{2r^2}{w(z)^2}\right) e^{-\mathrm{i}k\frac{r^2}{2q(z)}+\mathrm{i}l\phi}$$

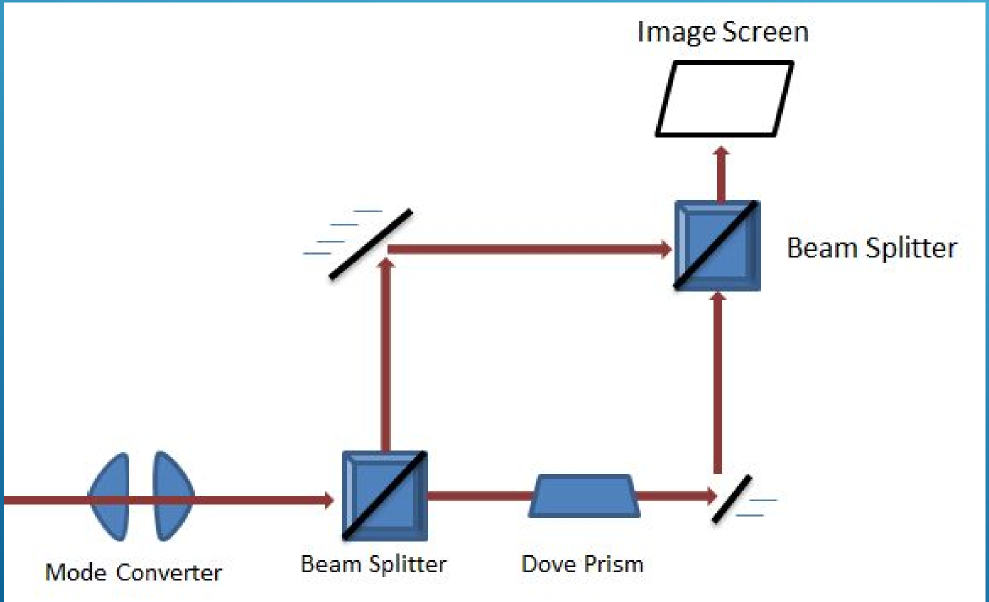
$$u^{\mathrm{cosine}}_{p,l}(r,\phi,z) = \frac{2}{w(z)}\sqrt{\frac{2p!}{\pi(|l|+p)!}}\exp\left(\mathrm{i}\,(2p+|l|+1)\,\Psi(z)\right) \times \left(\frac{\sqrt{2}r}{w(z)}\right)^{|l|} L_p^{l}\left(\frac{2r^2}{w(z)^2}\right)\exp\left(-\mathrm{i}k\frac{r^2}{2q(z)}\right)\cos(l\phi)$$

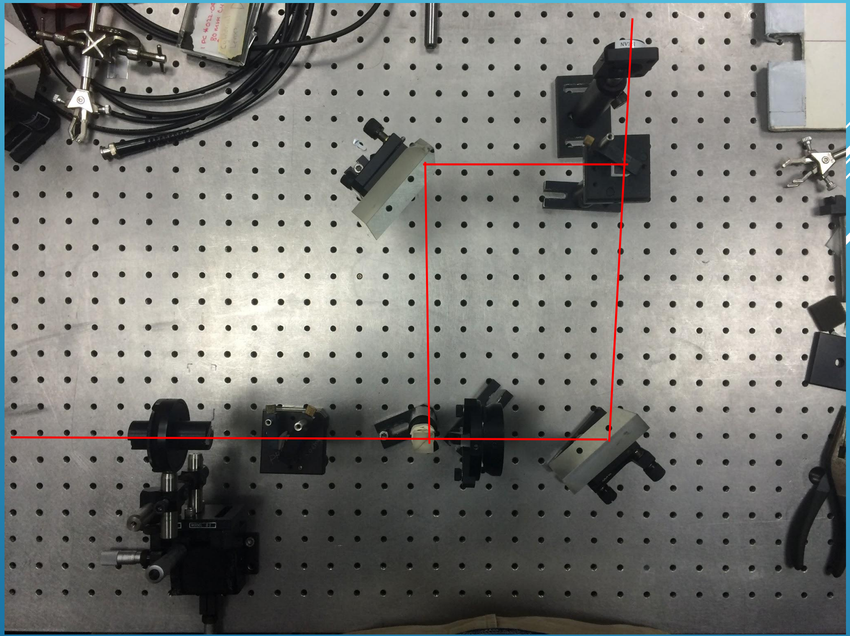


$$\cos x = \mathrm{Re}\{e^{ix}\} = \frac{e^{ix}+e^{-ix}}{2}$$

Mach-Zehnder Interferometer

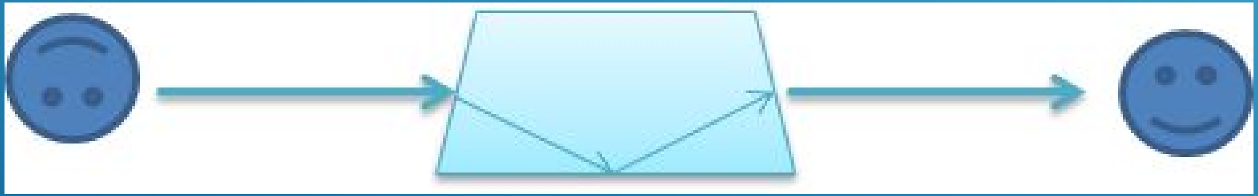
- Output helical LG is split and recombined to produce a sinusoidal mode.





Reversing the Handedness of an LG Beam

- A Dove Prism inverts an image through a single instance of total internal reflection:
    - Provides the necessary additional “flip” in one path for sinusoidal generation.



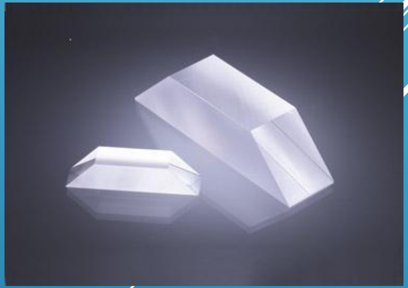
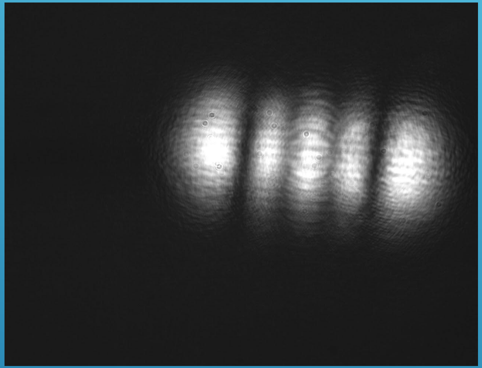
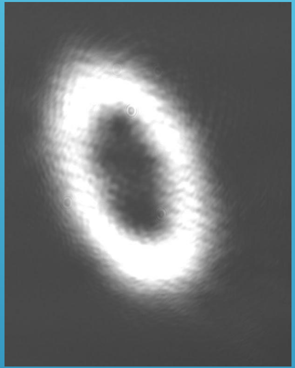


Image: Ealing Catalog

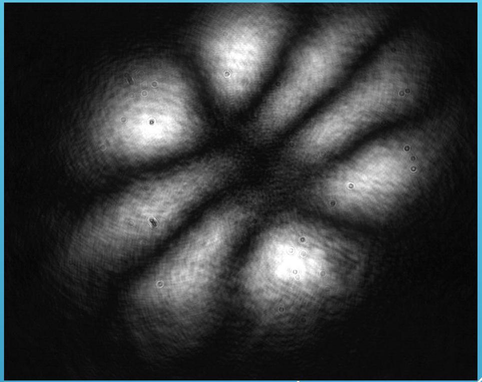
Full HG→LG_{hel}→LG_{sin} Conversions



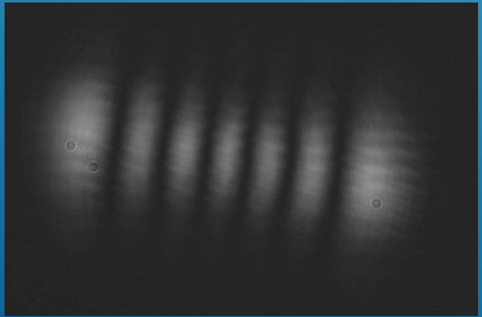
HG_0^4



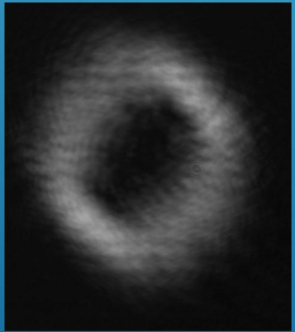
$LG_{hel\ 0}^{4}$



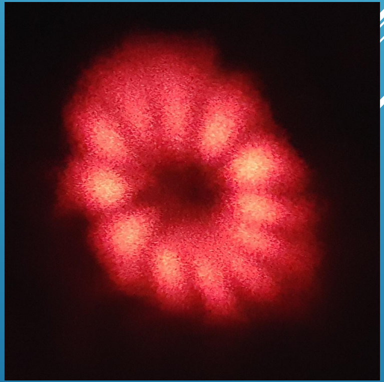
$LG_{sin\ 0}^{4}$



HG_0^6



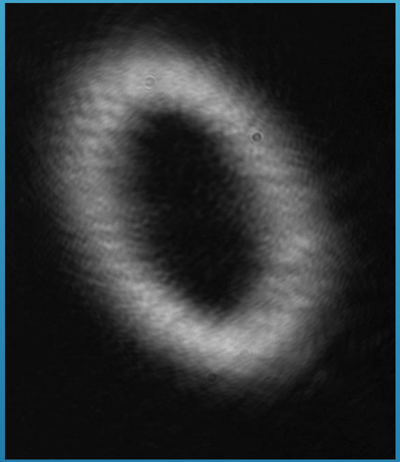
$LG_{hel\ 0}^{6}$



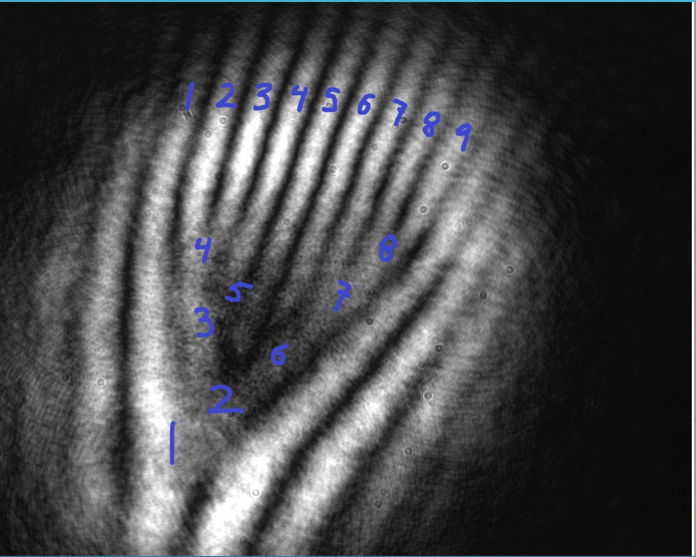
$LG_{sin\ 0}^{6}$

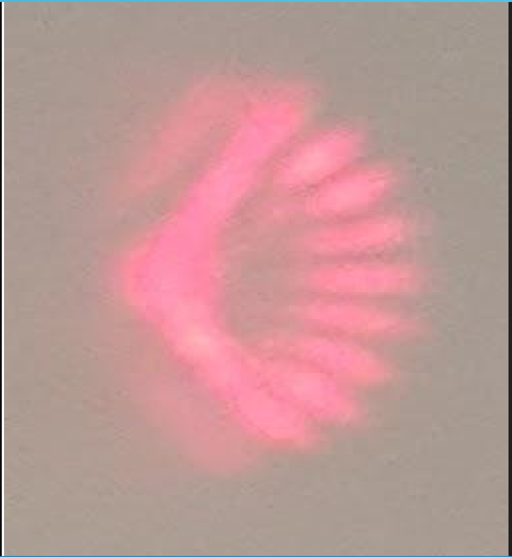
Full HG$\rightarrow$LG$_{hel}$$\rightarrow$LG$_{sin}$ Conversions

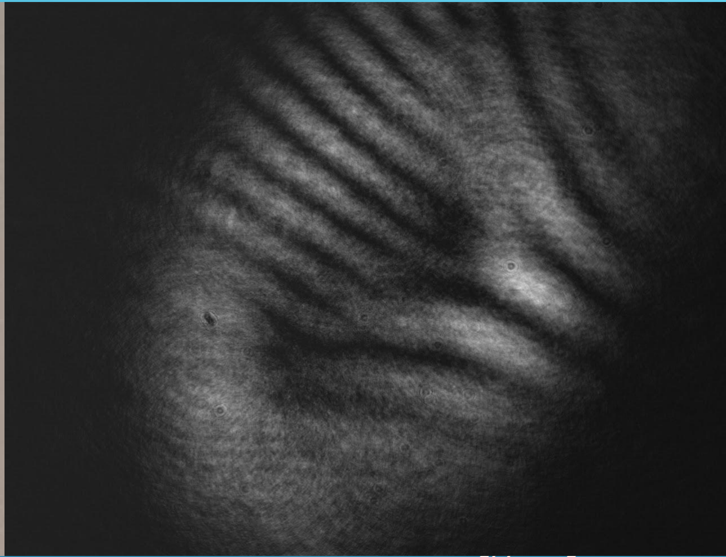
MIA



The Fork







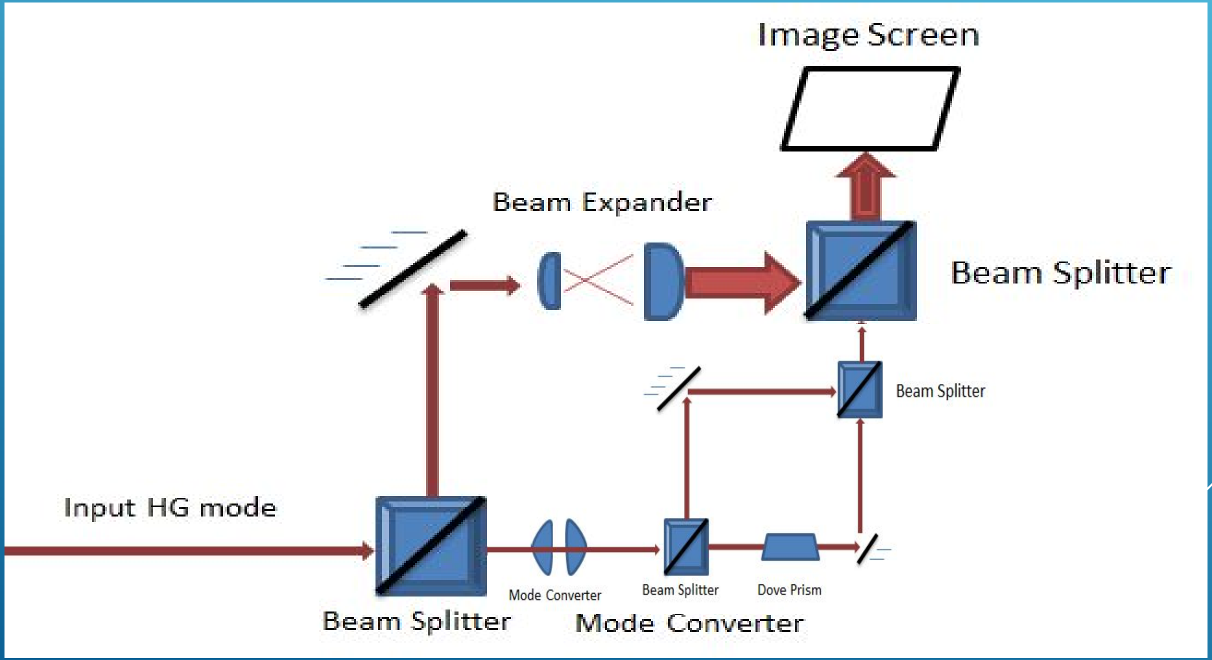
- Fork patterns arise when interfering helical LG modes with its opposite-self ($-\ell$) or a reference plane wave (Ex. HG). The fork arises from the phase discontinuity at the dark center of an optical vortex.

- Against a reference, the number of forks, or # of prongs - 1, indicates the topological charge ℓ of the vortex, or its integer multiple of $\hbar$ orbital angular momentum.

- Against its opposite-handed pair, however, we found that the # of forks is *twice* the topological charge.

Phase Analysis of Helical and Sinusoidal LG Modes

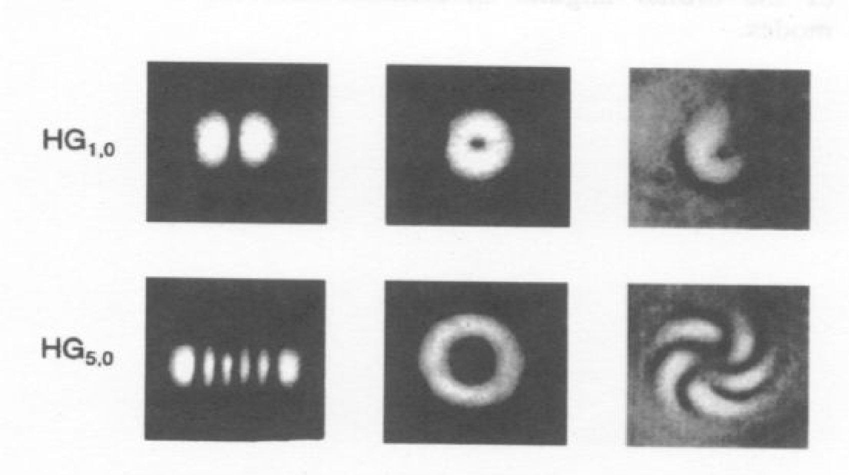
- A Mach-Zehnder Interferometer enveloped within another Mach-Zehnder ("Mach 2")
  - Mode in question interfered with an expanded HG lobe (a reference plane wave).



Mach 2

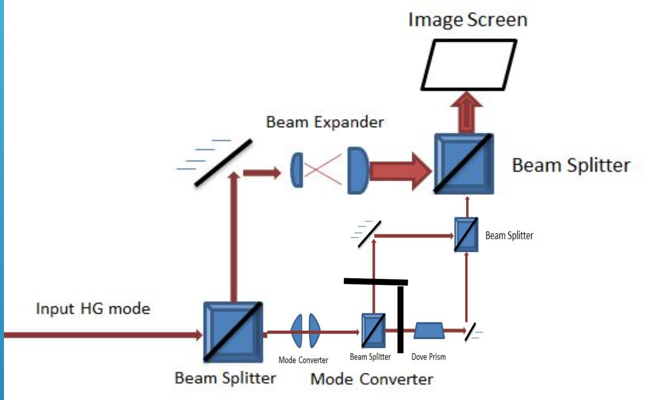
Phase Analysis of Helical

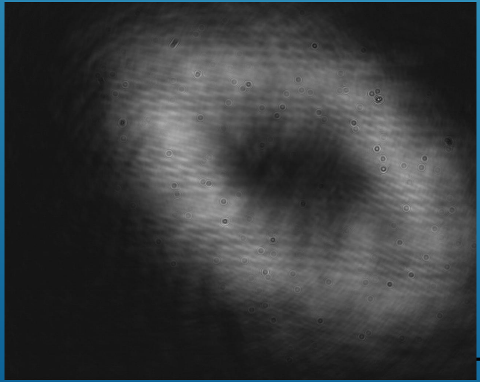
Hermite Gaussians, Laguerre Gaussians, Interferograms



American Journal of Physics, Jan. 1996, Padgett

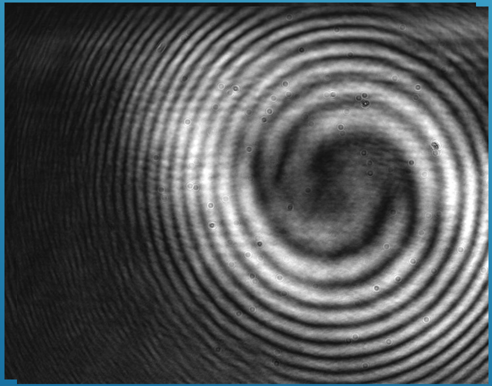
Phase Analysis of Helical LG Modes





LG_{03}

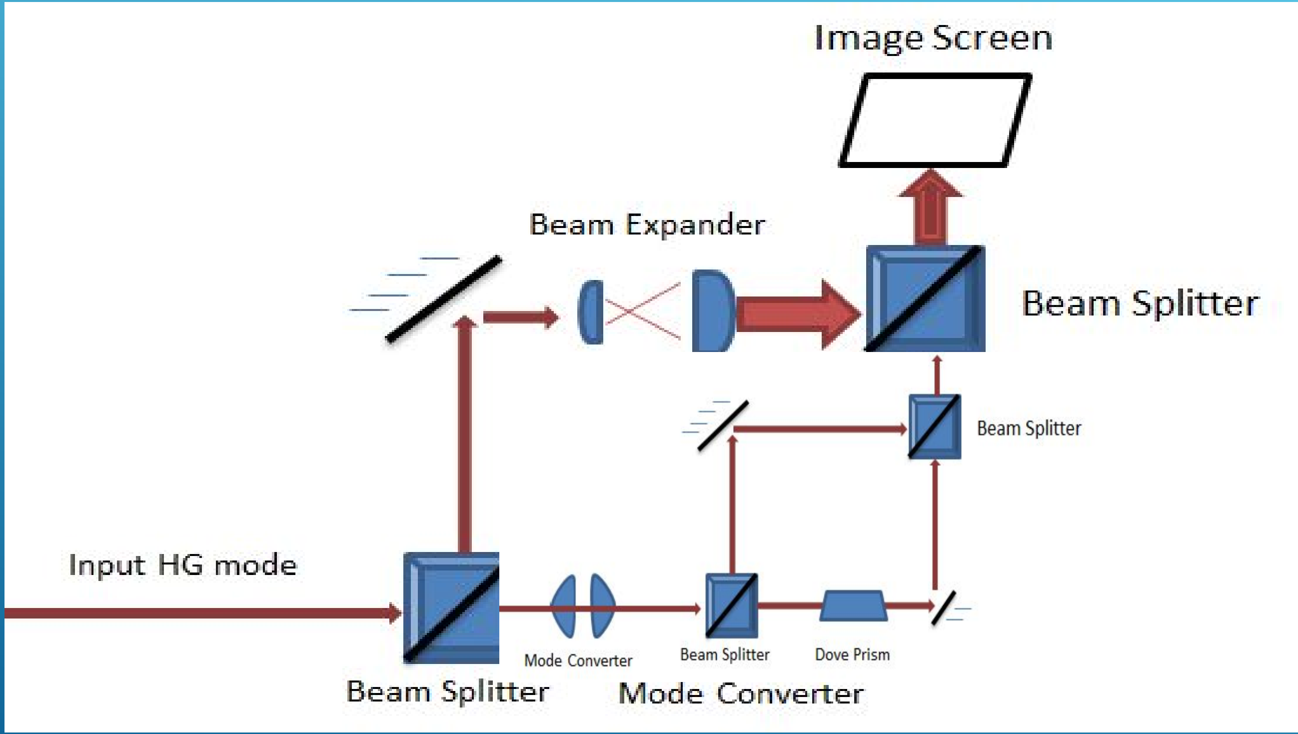
LG_{03}



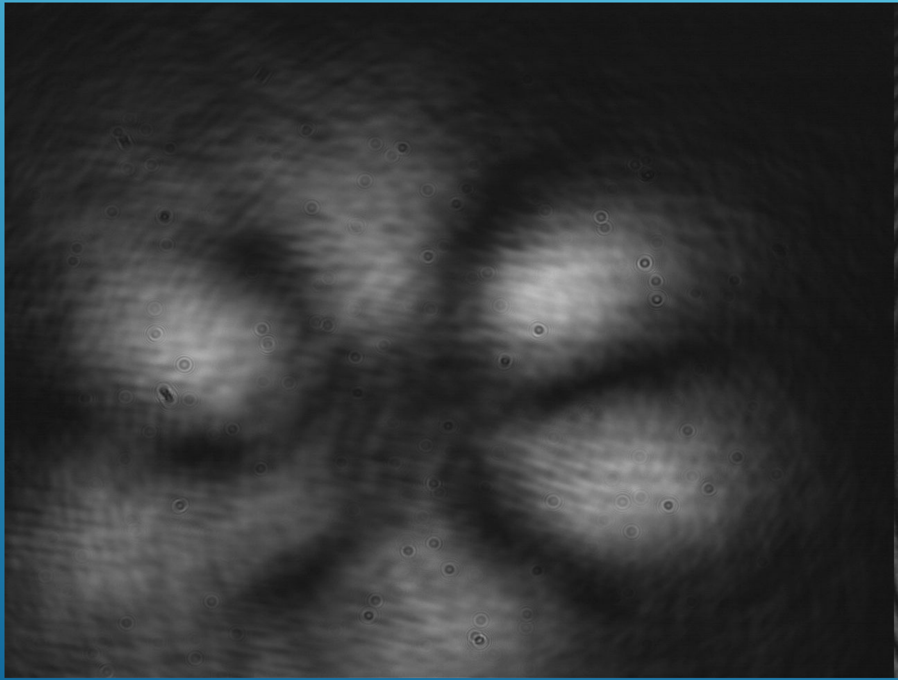
LG_{03} (passed through upper path) + plane wave reference

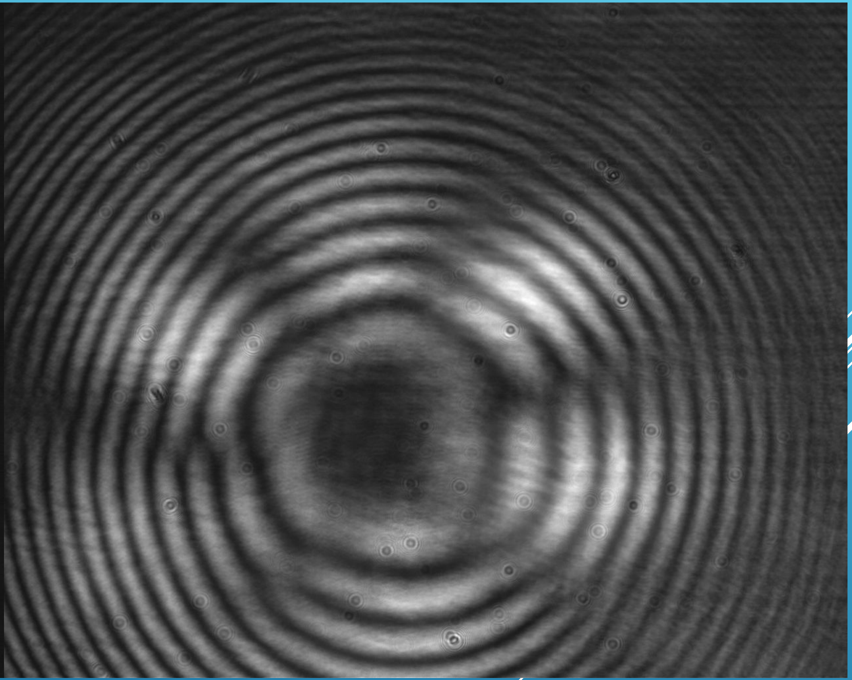
LG_{03} (passed through lower path) + plane wave reference

Phase Analysis of Sinusoidal LG Modes



Phase Analysis of Sinusoidal LG Modes





LG_{03}

LG_{03} + plane wave reference

Acknowledgments

- We thank Dr. Sean Bentley from Adelphi University for his leadership, Dr. John Noe for his presence and guidance in the Laser Teaching Center, Dr. Martin Cohen for his helpful insights, and Dr. Harold Metcalf for his coordination and supervision of this research opportunity. And, of course, our peers who have made these seven weeks informative and, above all, entertaining!

References

[1] https://www.rp-photonics.com/hermite_gaussian_modes.html
[2] http://www.2physics.com/2013/05/precision-interferometry-in-new-shape.html
[3] Astigmatic laser mode converters and transfer of orbital angular momentum; *M.W. Beigersbergen, L. Allen, H.E.L.O van der Veen and J.P. Woerdman, 1992*
[4] https://en.wikipedia.org/wiki/Angular_momentum_of_light
[5] Creating optical vortex modes with a single cylinder lens, *Hamsa Sridhar, Martin G. Cohen and John W. Noe, Laser Teaching Center, Stony Brook University*
[6] http://www.ealingcatalog.com/optics/prisms-polarizers/prisms/dove-prisms.html
An experiment to observe the intensity and phase structure of Laguerre-Gaussian laser modes **by M. Padgett, J. Arlt, and N. Simpson (*American Journal of Physics, Jan. 1996*)**