

A Simple Technique for Studying Near-Field Diffraction

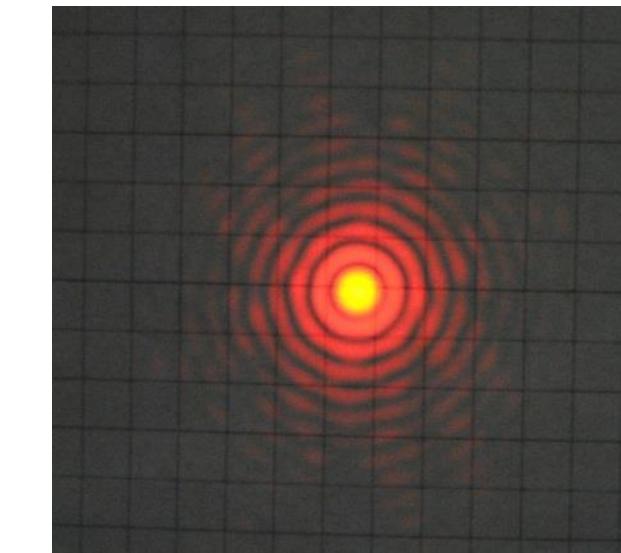
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Optics of Diffraction

Diffraction is the phenomenon in which waves encounter an obstruction such as an aperture and are thereby redirected in new directions according to a principle first described by Huygens over 300 years ago.

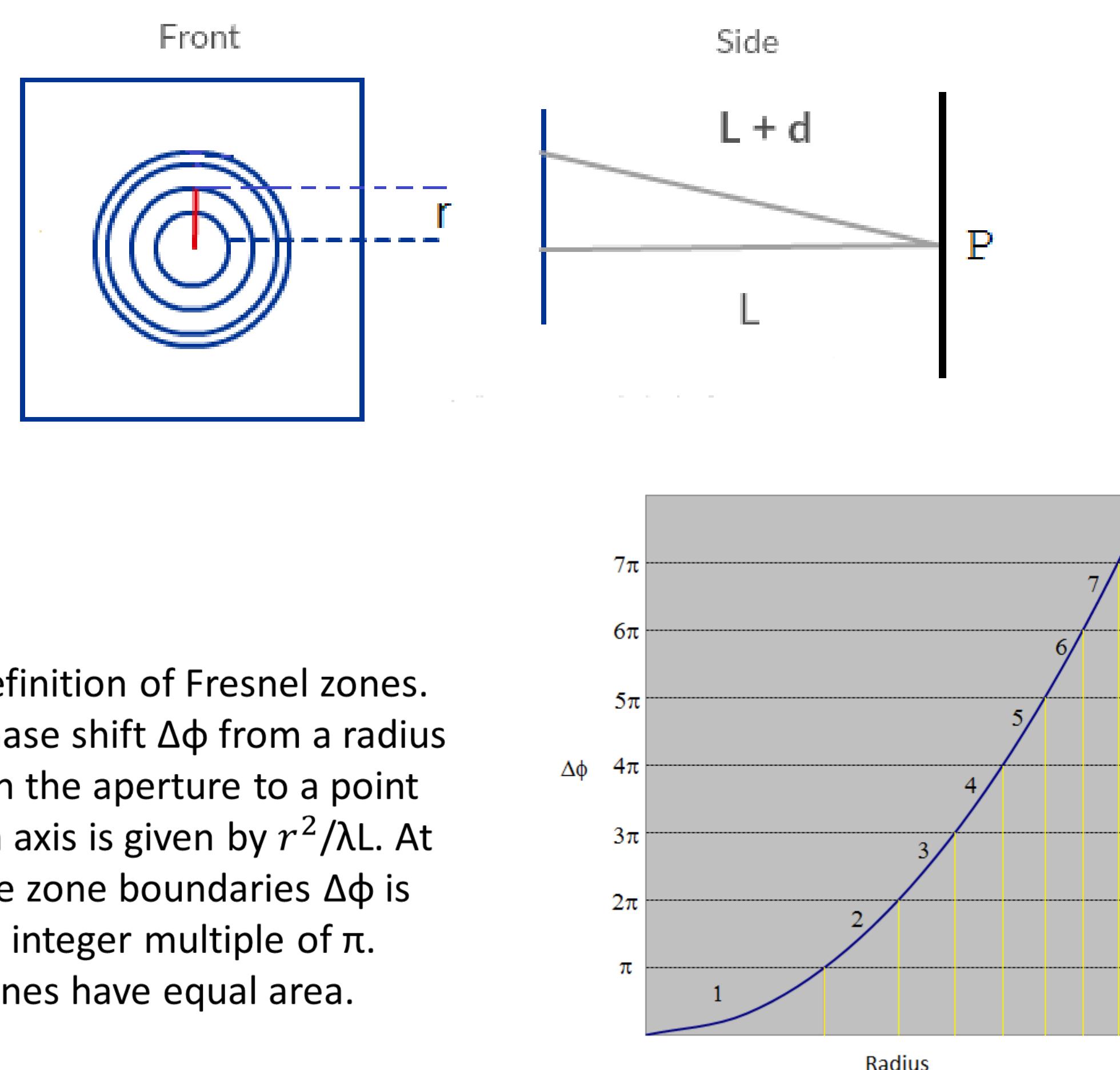
At a relatively large distance the diffraction pattern doesn't change with distance other than through an overall scale factor proportional to distance. Such far-field or Fraunhofer patterns can be readily calculated by a Fourier transform technique. For a circular aperture the pattern is described by a Bessel function and is called an Airy pattern.



Observed Airy pattern at 10 meters distance.

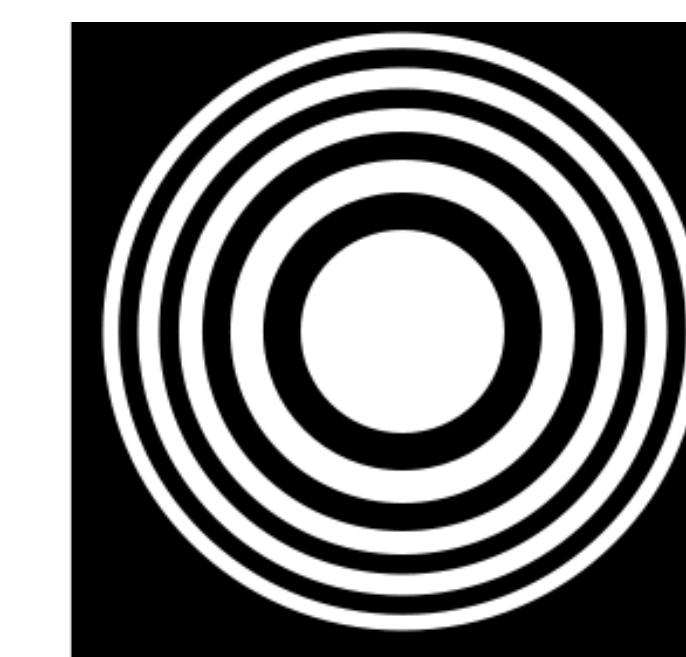
On the other hand, relatively close to an aperture or obstruction patterns are more complex and vary rapidly with changing distance. Such near-field or Fresnel patterns for a circular aperture are the subject of this project. We demonstrate through a simple experiment that the on-axis intensity varies from bright to dark and back again in a simple readily-predicted manner.

The transition from far-field to near-field can be quantified by the concept of Fresnel zones and the Fresnel number. Consider a circular aperture illuminated by a plane wave, as shown in the figure below. While the phase of the light leaving the aperture is uniform, a phase shift occurs as the light propagates a distance $L+d$ to some point P on the axis of the aperture.



Definition of Fresnel zones. Phase shift $\Delta\phi$ from a radius r in the aperture to a point on axis is given by $r^2/\lambda L$. At the zone boundaries $\Delta\phi$ is an integer multiple of π . Zones have equal area.

In a Fresnel zone plate alternate zones are obstructed. Zone plates can be used to focus x-rays. (Figure from Wikipedia.)



The Fresnel number F is given by $a^2/\lambda L$, where a is the aperture radius. Far field corresponds to $F \ll 1$ while near field has $F \geq 1$.

$$F = \frac{a^2}{\lambda L}$$

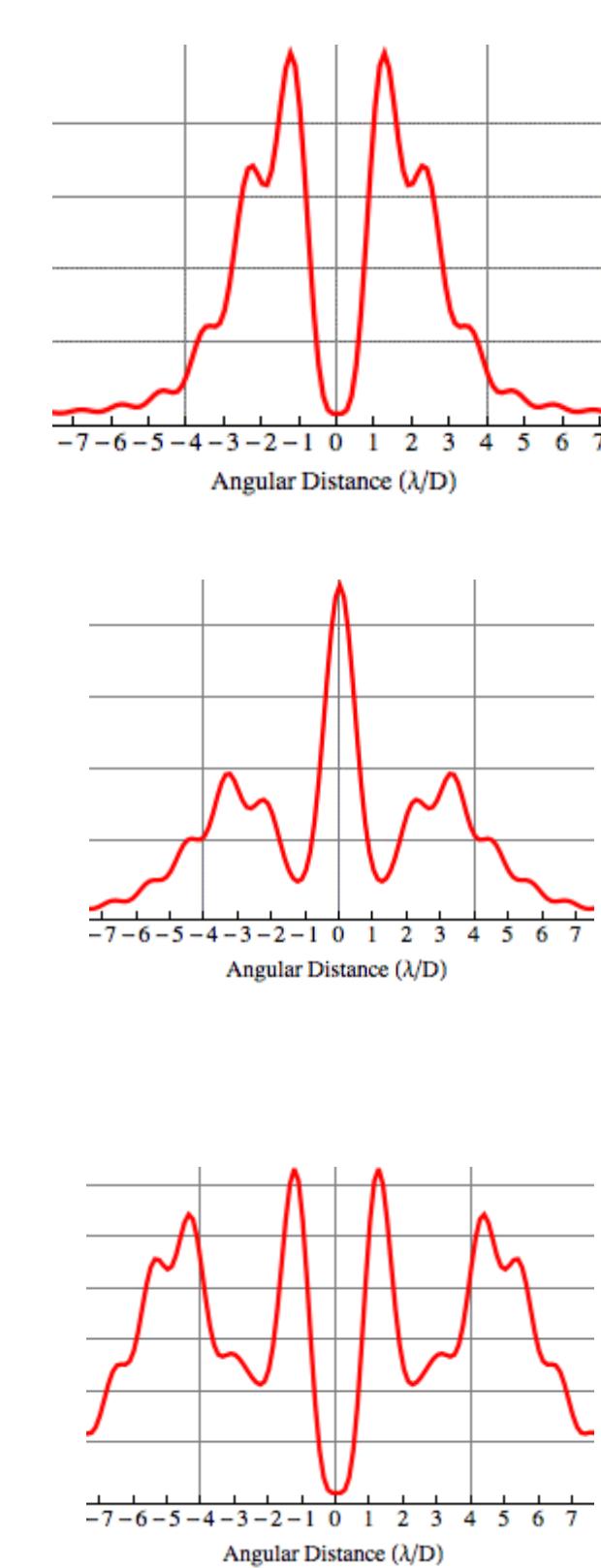
Progression of the Near-Field

Fresnel Number

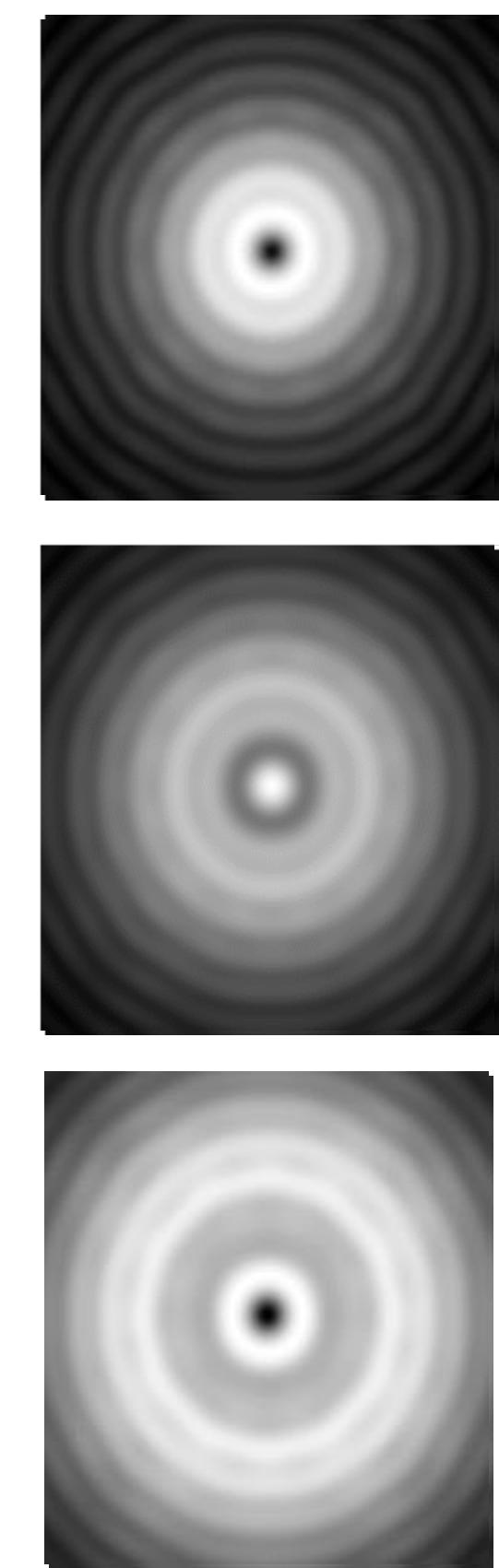
2



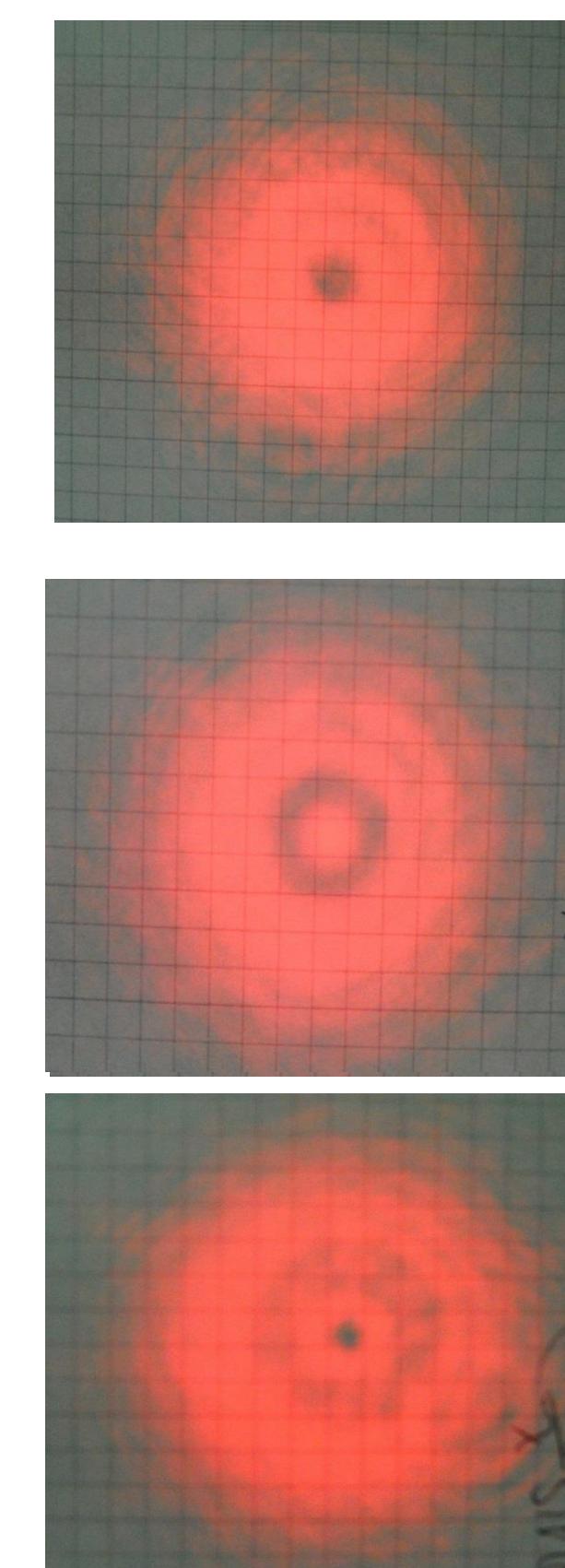
Calculated Profile



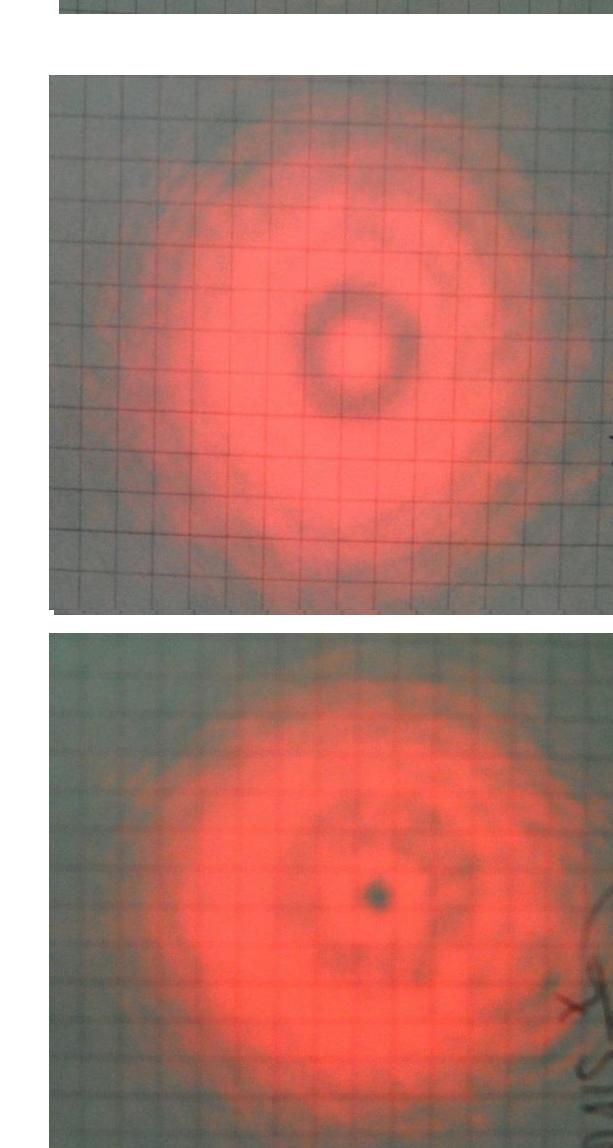
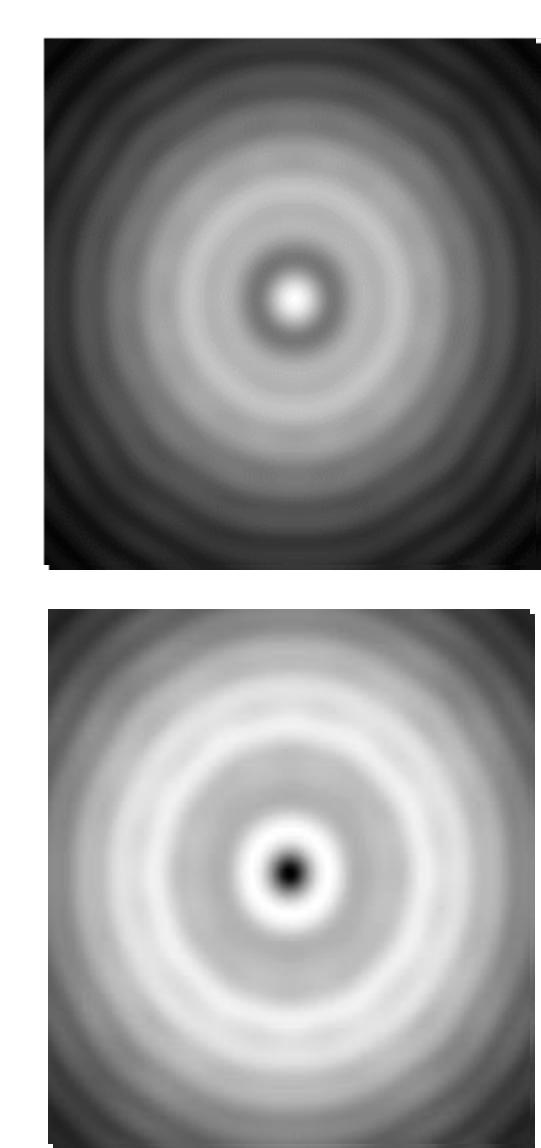
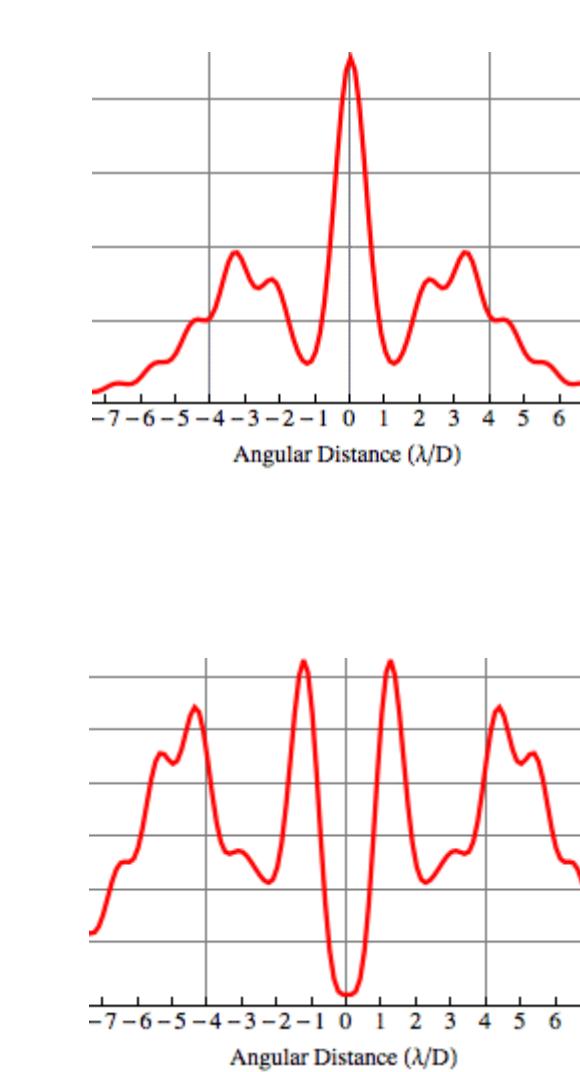
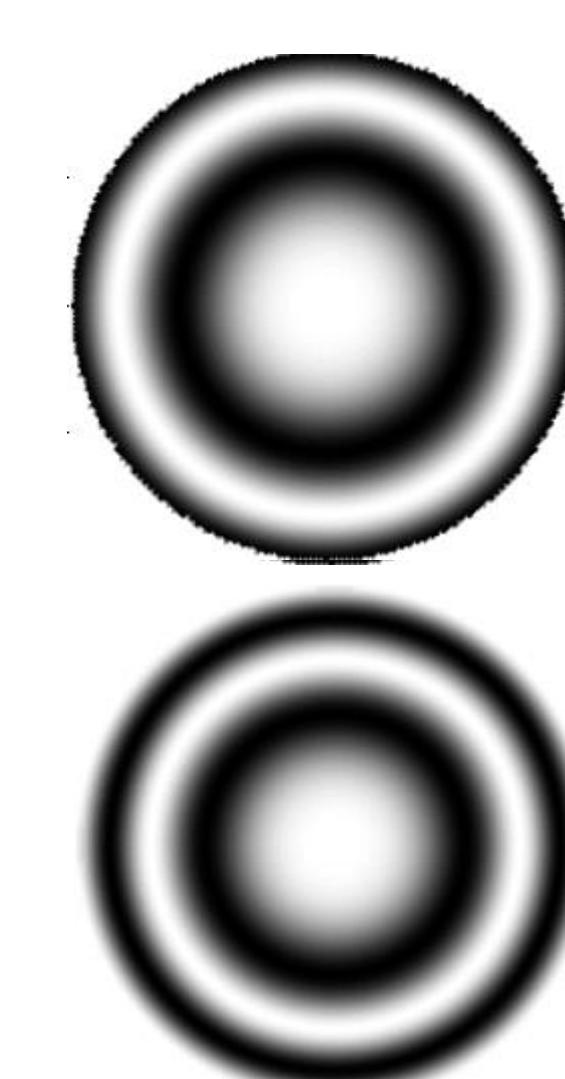
Calculated Pattern



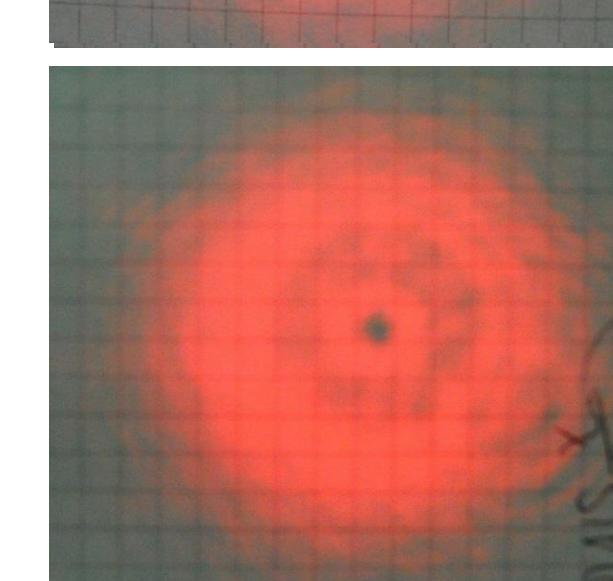
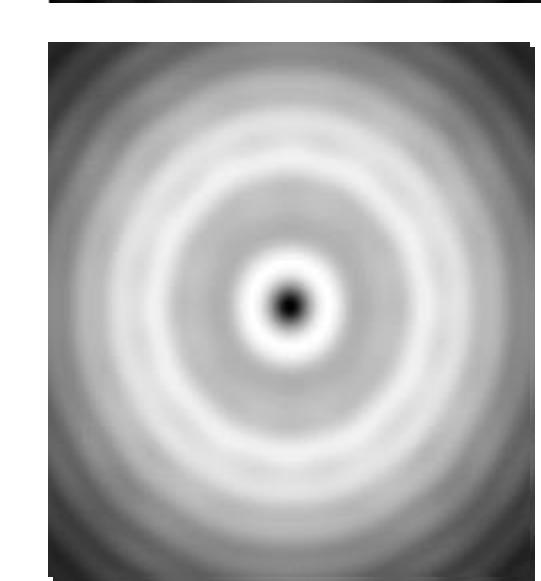
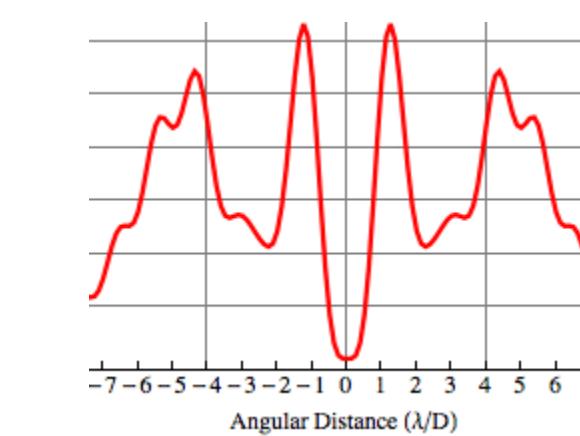
Observed Pattern



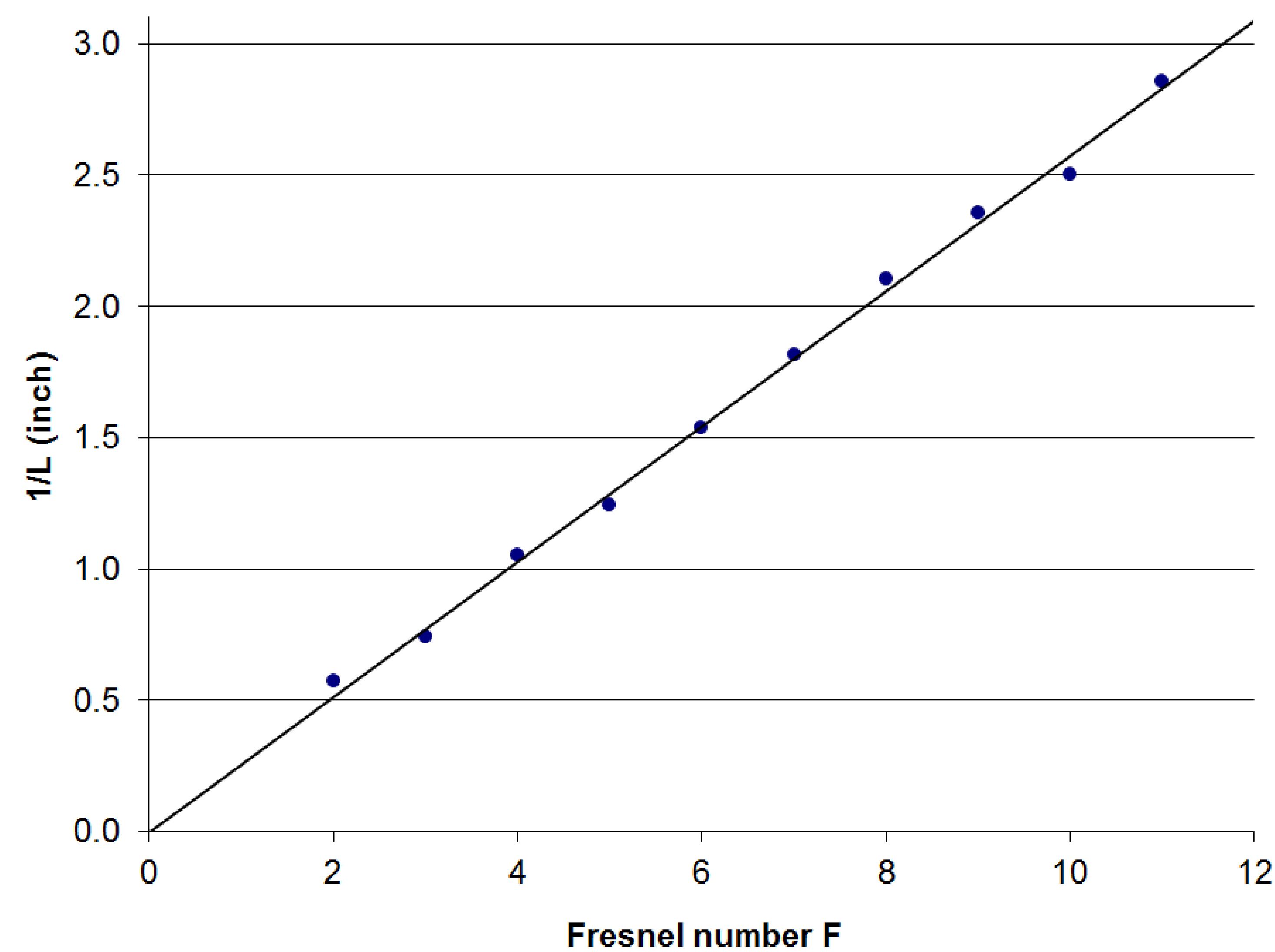
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4



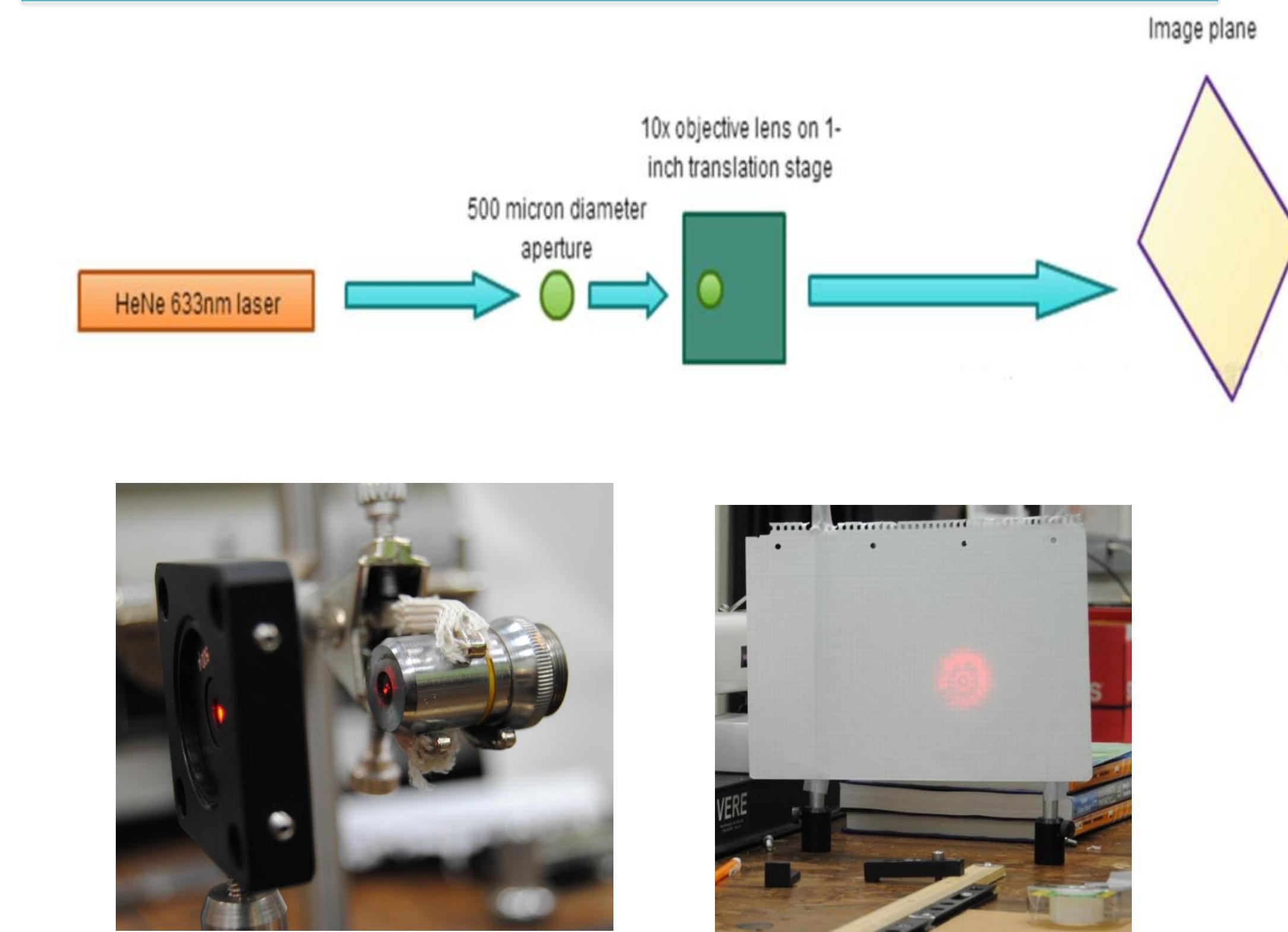
These images show the patterns we observed for Fresnel numbers 2, 3 and 4 and related calculations. Images were recorded with a standard DSLR camera from behind the paper viewing screen. Calculations are from James C Wyant's Mathematica Fresnel calculator at <http://wyant.optics.arizona.edu/math.htm>



Experimental results (points) and predicted linear relationship

$$\frac{1}{L} = \frac{\lambda}{a^2} F$$

Experimental Setup



We set our laser ~ 1 meter back from the 500 micron aperture to allow enough beam expansion to cover the hole. A magnified image of the diffracted light was projected by a 10x microscope objective lens onto a screen ~ 2 meters away. The lens was mounted on a translation stage.

Analysis and Results

Our recorded data consists of translation stage readings R_n at which we observed patterns with dark or bright centers, corresponding to even or odd integer Fresnel numbers, respectively. The separation of these R_n values is relatively accurate, but they share a (very nearly) common distance offset due to uncertainty in the position of the lens' front focal plane relative to the aperture. We accounted for this in the spreadsheet analysis by introducing a parameter A that relates the nominal and absolute lens positions. The positions of our data points in the figure are very sensitive to this parameter and for $A = 1.75 \pm 0.01$ inches we obtain excellent agreement with the Fresnel equation prediction.

Discussion

Similar to the well-known Arago spot demonstration, in which a bright spot appears in the center of the shadow of a circular obstruction, the dark central spots that appear in near-field diffraction from a circular aperture are a dramatic demonstration of Fresnel's theory of the wave nature of light. We obtained excellent quantitative agreement with the Fresnel prediction with very simple equipment and techniques.

While this project was conceived independently of any prior work qualitative demonstrations of Fresnel diffraction are well known (eg, MIT Open Course Ware). A quantitative experiment similar to ours (but with a sodium lamp) was reported in 1949 by L.A. Sanderman and R.S. Bradford (AJP 17, page 514).

Acknowledgments

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