# Creating an Exotic New Form of Light with Simple Optical Elements

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Intel Science Talent Search
November 2014

# Abstract

Airy beams are non-diffracting optical beams first predicted by Berry and Balazs in 1979 and first demonstrated experimentally by Siviloglou, Broky, Dogariu, and Christodoulides in 2007. What makes an Airy beam unique is that it can be created in one dimension and that its lobe of highest intensity propagates along a parabolic path. Airy beams can be used for a variety of purposes including particle guidance, remote sensing and plasma channel generation. Airy beams are typically generated using specialized optical devices such as spatial light modulators (SLMs) or cubic phase masks. Besides cost, both of these specialized methods have limitations: an SLM cannot be used with a high-power laser, and a cubic phase mask is not tunable. An alternative method which is both simple and inexpensive and also not subject to these limitations is to exploit aberrations found in ordinary lenses in order to produce the required cubic phase modulation of the wavefront (Papazoglou, Suntsov, Abdollahpour, and Tzortzakisin, Physical Review A, 2010).

In our research, we used the method of Papazoglou *et al.* to create a high-quality Airy beam. The setup included a 635 nm fiber-coupled diode laser with a collimating lens and negative and positive 80 mm focal length cylindrical lenses. Tilting and displacing the negative lens creates a coma aberration (cubic phase modulation); the subsequent positive lens removes aberrations other than the coma and roughly collimates the beam. The resulting wavefronts were imaged with a 200 mm focal length cylindrical Fourier transform lens into a Thorlabs DCC1545M CMOS camera and analyzed with ImageJ software. The transverse deflection of the highest-intensity lobe of the beam was observed by moving the camera along a linear rail to positions within a few cm of the focal plane. Were able to clearly demonstrate the nondiffracting and parabolic propagation of the beam. We found remarkable agreement with theoretical predictions - the observed parabolic acceleration was perfectly predicted by the observed minimum size of the beam. We hope to continue our work on Airy beams by using the current setup to demonstrate the self-healing properties of the beam, using a similar setup with mirrors instead of lenses, thereby allowing a much broader range of electromagnetic radiation to be used to create Airy beams, and to model the wavefront of the generation an Airy beam.

# 1 Introduction

Airy beams are a recently discovered form of light with several remarkable properties, including the ability to follow a curved path in free space. Airy beams are not compact light beams in the usual sense: they have one primary lobe of peak intensity and an arbitrarily large number of secondary side lobes of gradually decreasing intensity. The ability of Airy beams to follow a curved path is called *acceleration*, since the curve followed is a parabola. The second unusual property is called *non-diffraction* - the width of the primary lobe stays constant for a much greater distance than that of a conventional laser beam of the same minimum size. Finally, if the primary lobe is blocked it reforms as the beam propagates, a property known as *self-healing*. Airy beams can be created in both one- and two-dimensional forms. They are not just novel and fascinating but also have many useful applications that are now being actively explored by many researchers [1] [2] [3].

Berry and Balazs first predicted the existence of Airy beams over 30 years ago in the context of wave packets in quantum mechanics [4]. However it was not until 2007 that their findings were realized experimentally by Siviloglou *et al.* [5], who found that the Airy wave packet could be created as an optical beam by imposing a cubic phase modulation on the usual Gaussian laser beam and transforming the resulting light wave field with a Fourier transform lens. They used a sophisticated computer-controlled liquid crystal device, a spatial light modulator (SLM), to create the phase variation. Three years later Papazoglou *et al.* [6] proposed that the cubic wavefront variation could be created much more simply by passing light through a tilted lens to create a normally undesirable coma aberration. Their method is not only relatively simple and inexpensive, but also allows for the use of high-powered lasers that might otherwise damage an SLM.

In this research, we created very high quality Airy beams by this simple method [6] and studied their properties with an inexpensive CMOS-camera. We observed both parabolic acceleration and the nondiffracting property and are currently studying the self-healing property. We found remarkable agreement with theoretical predictions - the observed parabolic acceleration was perfectly predicted by the observed minimum size of the beam. Our one-dimensional Airy beams were created with a matched pair of inexpensive cylinder lenses. We are now investigating the possibility of creating Airy beams by introducing aberrations via mis-aligned mirrors, and we have created high-quality mirrors by coating cylinder lenses with thin gold films. Utilizing mirrors would allow the same setup to work over a wide range of wavelengths. Our research has shown that the formation and properties of Airy beams is an excellent topic for student experiments in an upper-level undergraduate laboratory.

# 2 Background

## 2.1 The Airy Wave Packet

The existence of Airy beams was first predicted by Michael Berry and Nandor Balazs in 1979 [4]. Berry and Balazs described a wave packet (a localized "burst") that does not spread and curves as it propogates from its origin. The Airy packet has a probability distribution described by an Airy function at the origin ($t = 0$), and evolves according to Schrödinger's equation. The wave packet at any point in time is described by [4]:

$$\psi(x,t) = Ai[\frac{B}{h^{2/3}}(x - \frac{B^3t^2}{4m^2})]e^{(\frac{iB^3t}{2m})[x-(\frac{B^3t^2}{6m^2})]} \tag{1}$$

where $B$ is a constant, $m$ is particle mass, $h$ is Planck's constant and $Ai(x)$, Airy function, is given by [7]:

$$Ai(z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i(zt+\frac{t^3}{3})} \tag{2}$$

Berry and Balazs gave a quantum mechanical description of this wave packet using semi-classical orbits [4]. Another explanation was provided in a comment on the paper using the equivalence principle. Greenberger showed that in a free-fall reference frame, the Airy wave packet is a stationary state, and so does not spread [8]. Though it may seem like a wave packet should not be able to propagate along a curved path, it is not in violation of Eherenfest's theorem [3], which states that in quantum mechanics, as in Newtonian mechanics, the center of mass of a force-free system cannot accelerate. The Airy wave packet is not square integrable, so it does not have a defined center of mass.

## 2.2 Bessel Beam

Discovered [9] and experimentally demonstrated [10] in 1987, Bessel beams are the first type of nondiffracting beam ever created. The Bessel beam is described by [10]:

$$U(x,y,z;k) = exp(i\beta z)J_0[\alpha(x^2+y^2)] \tag{3}$$

where $J_0$ is the zero-order Bessel function, and $\beta^2 + \alpha^2 = k^2$. A Bessel beam appears as an inner lobe surrounded by an arbitrarily large number of weaker rings. It can be thought of as a conical superposition of plane waves that constantly interfere to create the unique profile of the Bessel beam. Like the Airy wave packet, the ideal Bessel beam is nondiffracting and self healing, and is not square integrable, requiring an infinite amount of energy to produce it. In Fig. 1, the intensity profiles of the two beams are compared. A key difference between the two beams is that the Bessel beam exhibits radial symmetry while the Airy beam does not.

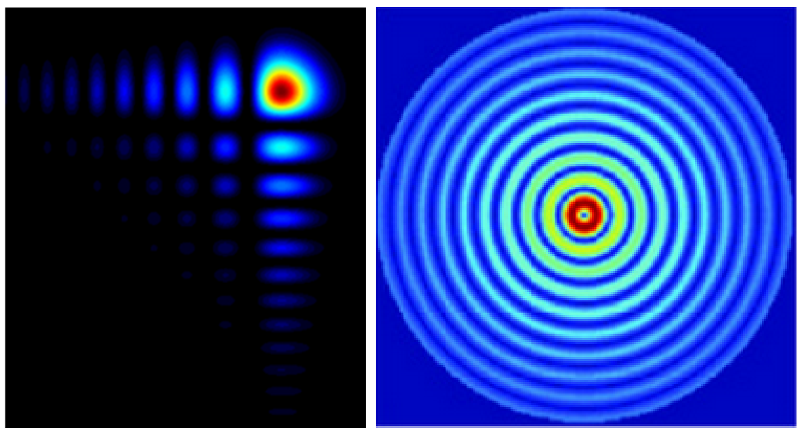

Figure 1: A false-colored, 2D Airy beam (left) [6] and a false colored Bessel beam [11]

As it turns out, the Airy function is a Bessel function of fractional order [12]. This justifies several of the similarities between the two beams, and allows for comparisons to be made between them.

## 2.3 Truncated Airy Beams

An ideal Airy beam, like an ideal Bessel beam, requires infinite energy in order remain completely diffraction-free and to reconstruct after being partially blocked, as well as to follow a curved path. It is the infinite number of auxiliary lobes that enable it to have these three properties. Therefore, in order to realize the beam experimentally, a finite version of the Airy beam has to be implemented. The Airy function can be exponentially truncated. At $z = 0$, the finite optical beam is then described by [13]:

$$U(x/x_0, z = 0) = Ai(x/x_0)exp(ax/x_0) \tag{4}$$

where $a > 0$ and $x_0$ is a transverse scale.

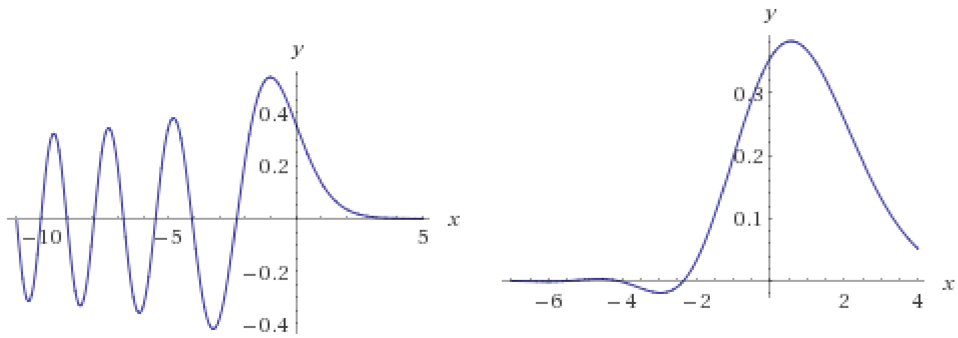


Figure 2: Ai(x) (left) and Ai(x)exp(ax), a=1 (right); generated using Wolfram Alpha

It is evident from Fig. 2 that, in contrast to the unconfined Airy function (left), the truncated function (right) converges on both sides. Therefore, a beam described by such a function can be generated. Of consequence is this new function's Fourier transform:

$$U'(k) = exp(-ak^2)exp[\frac{i}{3}(k^3 - 3a^2k - ia^3)] \tag{5}$$

where $k$ is the new transverse coordinate. The function in Equation 5 is Gaussian. A beam described by (5) is a Gaussian beam with cubic phase modulation.

# 3 Properties of the Airy Beam

## 3.1 Parabolic Acceleration

One of the most fascinating properties of the Airy beam is its parabolic acceleration. The intensity profile of an exponentially truncated beam is given by [14]:

$$I(x,y,z) \propto Ai^2[\frac{x}{x_0} - (\frac{\xi}{2})^2 + ia\xi] \cdot Ai^2[\frac{y}{x_0} - (\frac{\xi}{2})^2 + ia\xi] \cdot exp(2a\frac{x+y}{x_0}) \cdot exp(-2a\xi^2) \quad (6)$$

where $x_0$ is a transverse scale, $\xi = z/kx_0^2$ is a normalized propagation distance, and $k$ is the wavenumber, equal to $2\pi/\lambda$. From the above equation (6), it can be seen that the beam changes in the x and y directions as it propagates. The acceleration of the beam, the rate at which the peak lobe of the beam shifts transversely as it propagates, can be calculated by [14]

$$A = \frac{\sqrt{2}}{16\pi^2} \cdot \frac{\lambda^2}{x_0^3} \approx 0.037 \cdot \frac{\lambda^2}{W_A^3} \quad (7)$$

The curving nature of the beam is useful in optical micromanipulation as well as curved plasma channel generation.

## 3.2 Nondiffracting

Another useful feature of the Airy beam is that, like the Bessel beam, it is nondiffracting. The Airy wave packet [4]:

$$\psi(x,t) = Ai[\frac{B}{h^{2/3}}(x - \frac{B^3t^2}{4m^2})]e^{(\frac{iB^3t}{2m})[x-(\frac{B^3t^2}{6m^2})]} \quad (8)$$

clearly does not spread as it propagates. As shown in Fig. 3, its intensity profile remains the same during propagation, though it does shift transversely.

A finite optical Airy beam is not completely diffraction-free. However, it does maintain its minimum peak lobe width (FWHM) over far greater propagation distances than does a Gaussian beam of the same minimum width. Notice that in the finite beam (Fig. 4), while the peak lobe follows a parabolic path and does not spread rapidly, the auxiliary lobes spread in the opposite direction so that the center of mass of the beam does not accelerate.

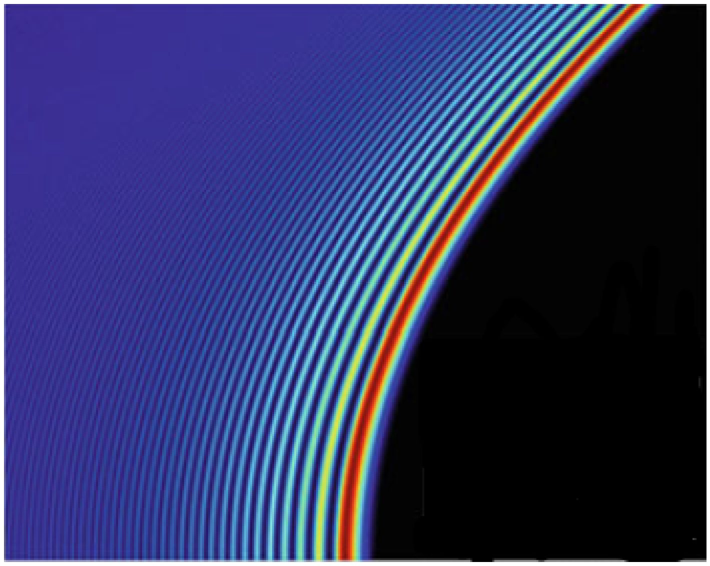

Figure 3: Propagation of an infinite Airy wave packet [3]

Figure 4: Propagation of a finite optical Airy beam [3]

## 3.3 Self-Healing

Also like the Bessel beam, a partially blocked Airy beam can reconstruct. Ideal nondiffracting beams are solutions to the Helmholtz equation and can be described by [9]:

$$U(x, y, z) = u(x, y)exp(i\beta z) \tag{9}$$

An opaque object causing diffraction is said to be the complement of an aperture of the same size and shape. Babinet's principle states that the disturbances caused by two complements are of equal amplitude and opposite phase.[15]. Babinet's principle can be applied to find the far-field result of partially blocking a beam. To find $U_f$, the resulting amplitude of a beam disturbed by an opaque object, we use the equation [16]:

$$U_f = U_i - Uc \tag{10}$$

where $U_i$ is the incident light and $U_c$ is the complex amplitude of the resulting diffraction pattern from a complementary object (an aperture of the same size and shape). The far-field intensity of the beam is then given by[16]:

$$\lim_{z\to\infty} |U_f|^2 = \lim_{z\to\infty} (|U_i|^2 + |U_c|^2 - U_i U_c^* - U_i^* U_c) \tag{11}$$

The field diffracted by the complementary aperture, $U_c$ decreases by a factor of $1/z$ and is equal to 0 at $z \to \infty$. Far-field intensity of the partially blocked beam can then be written $\lim_{z\to\infty} |U_f|^2 = |U_i|^2$[16]. The ideal nondiffracting beam is completely reconstructed in the far field.

## 3.4 One-Dimensional

Another remarkable feature of the Airy beam is that it can exist in one dimension. Besides a plane wave, the Airy beam is the only non-trivial, propagation invariant solution to the paraxial wave equation that can exist in one dimension [12]. Its propagation takes place in $(1 + 1)$ dimensions (one transverse dimension + propagation or time dimension), whereas beams such as the Bessel beam, which is radially symmetric, must exist in $(2+1)$ dimensions. While Airy beams do exist in $(1+1)$ dimensions, 2-dimensional Airy beams can be generated as well.

# 4 Generating Airy Beams

Airy beams are generated by imposing a cubic phase modulation on a Gaussian beam and then Fourier transforming the result. A laser typically produces a Gaussian beam. There have been a number of methods proposed and used in order to produce the cubic phase modulation. Using a liquid crystal device called a spatial light modulator (SLM) [5], using specially made phase masks [1], and using lenses [6]. Finally, a lens is used to Fourier transform the beam. We are most concerned with the method invloving an SLM (the first method ever used) and the method involving only lenses (used in this project), which evolved from SLM method.

## 4.1 Using a Spatial Light Modulator

One can generate finite optical Airy beams by exploiting the fact that the Fourier transform of the truncated Airy beam is a Gaussian beam with a cubic phase modulation. A laser provides the Gaussian beam. A spatial light modulator (SLM), a reflective or transmissive liquid crystal device, can be used to impose a cubic phase modulation. A converging cylindrical lens is then used to optically Fourier transform the cubically modulated Gaussian beam.

## 4.2 Using Lenses

Any lens will have aberrations. One such aberration is called the coma.

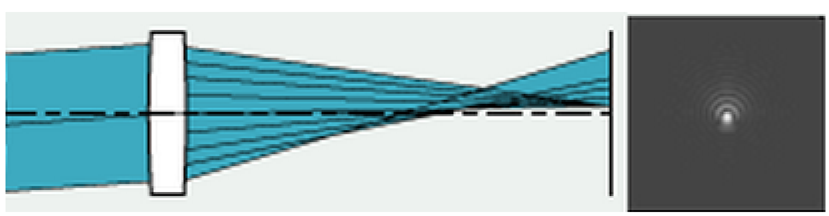

Figure 5: a diagram illustrating the coma aberration [17]

A coma occurs when off-axis light enters a lens. The resulting image appears to have a tail not unlike that of a comet. A coma is one of the Seidel aberrations and, in a spherical lens, is described by the term [18]:

$$F x_0 \rho^3 cos\theta \tag{12}$$

where $F$ is the coma aberration coefficient, $x_0$ is the height of the object, $\rho$ is the radial coordinate (the distance from the center of the lens), and $\theta$ is the angular coordinate. In a cylindrical lens, the coma term changes to:

$$Fx_0x^3 \tag{13}$$

where $x$ is the distance from the center of the lens. The total wavefront aberration in the cylindrical lens is given by [6]:

$$W = -\frac{1}{4}Bx^4 - \frac{1}{2}(2C + D)x_0^2x^2 + Ex_0^3x + Fx_0x^3 \tag{14}$$

In order to compensate for aberrations in a lens, a second lens of opposite focal length can be used. In the manner show in Fig 5., a tilted lens can be used to produce the coma aberration and a second lens perpendicular to the beam can be used to compensate for non-cubic aberrations.

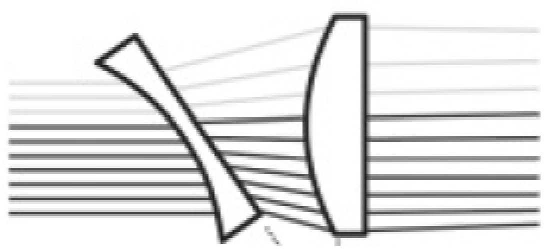

Figure 6: A picture raytrace of of light passing through the matched focal length lenses [6]

Light hitting the upper half of the lens (shown by the light gray rays) is dominated by spherical aberration and is therefore undesirable for the purpose of creating an Airy beam.

# 5 Setup

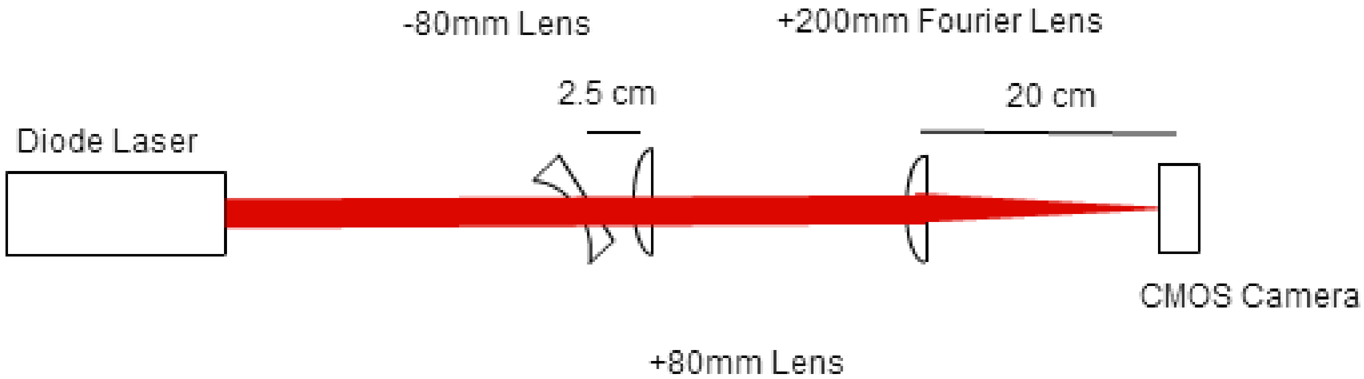


Figure 7: A diagram of the setup. A Gaussian laser beam passes through the first two. The recollimated beam the passes through the third, Fourier, lens and into the camera.

Our setup was assembled on an optical table using standard components, with one exception. In the final setup, cylindrical lenses of -80 mm, +80 mm and +200 mm focal length were used. The laser was a Cambridge Collimators LM635 fiber-coupled diode laser with a collimating lens. We used a Thorlabs DCC1545M CMOS camera. Lenses as well as the CMOS camera were mounted on a linear rail.

## 5.1 Imposing a Cubic Phase Modulation

Initially, we used lenses with mismatched focal lengths. We found that while Airy beams can be generated in this manner, they are of very poor quality due to undesired aberrations. A reason for starting with lenses of different focal lengths is the spacing between two lenses required in order to collimate the beam. This distance can be calculated using Gullstrand's equation [19]:

$$f = \frac{f_1 f_2}{f_1 + f_2 - d} \tag{15}$$

In order to recollimate the beam, we want $f \to \infty$:

$$f_1 + f_2 - d = 0 \tag{16}$$

By Equation 16, one can determine that two lenses of opposite focal length would have to be placed in contact with each other in order to recollimate a beam. This would make it impossible to tilt one of the two lenses. Fortunately, tilting a lens changes the lens' effective power. The power of a tilted lens is found by [20]:

$$P = P_0(1 + \frac{\sin^2\theta}{2n}) \tag{17}$$

$$f = \frac{f_0}{(1 + \frac{\sin^2\theta}{2n})} \tag{18}$$

Despite the increased distance due to the change in focal length, it was still impossible to collimate the beam using "off-the-shelf" lens mounts. This problem was solved by carefully securing one lens directly to a post using double-sided tape. The beam was aimed at one half of the lenses as shown in Fig. 6.

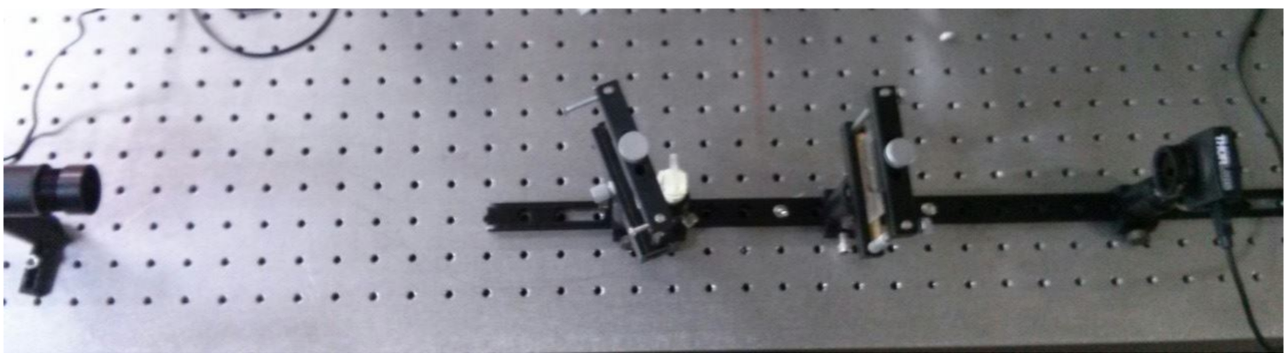

Figure 8: A photograph of the final setup

In order to produce the desired coma aberration, the negative lens was tilted 18 °. The positive lens of equal focal length was placed 2.5 centimeters away from the negative lens in order to collimate the light and to cancel out aberrations other than the coma.

## 5.2 Fourier Transforming Lens

The Fourier transform of a wave field is found in the diffracting wave field's Fraunhofer region [21]. The Fraunhofer region, or far-field, is a great distance from an aperture. However, the equivalent of the far-field is found at the focal plane of a lens. A lens can therefore be used to create the spatial Fourier transform of a source of light.

In our setup, a +200 mm cylinder Fourier transform lens was placed between the coma-inducing lenses and the camera. The Fourier focal plane of the Airy beam is then 200 mm after the Fourier lens. This is the $z = 0$ of our beam, and is where the beam is at its clearest.

### 5.3 Imaging the Beam

We used a Thorlabs DCC1545M CMOS camera, not unlike a consumer webcam, in order to view the Airy beam. At first, the camera was overloaded by the beam. We worked around this by attaching an ND 3.0 filter directly to the camera. Using the filter also had the advantage of attenuating ambient room light. The camera was mounted to a linear rail, and could be moved in line with the propagation of the beam in order to image it at different distances from the focal plane.

# 6 Results

We were able to observe very high-quality Airy beams. Below is a picture of our Airy beam at the focal plane of our Fourier lens (Fig. 9). The primary lobe of the Air beam, at less than 50 microns, is narrower than a human hair.

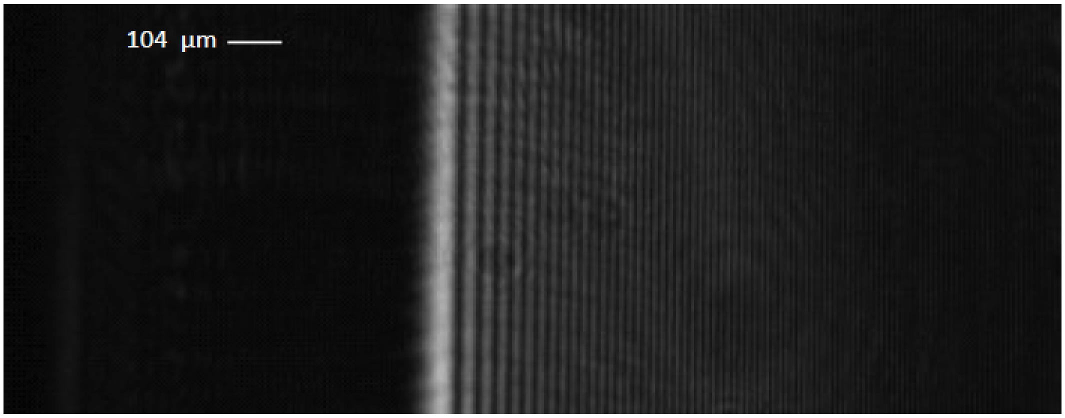


Figure 9:

## 6.1 Intensity Profile

The intensity profile of our Airy beam was found using a free software ImageJ. Below is the intensity profile of our Airy beam (Fig. 10) at focus along with a graph of a prediction based on the square of the Airy function (Fig. 11).

## 6.2 Nonspreading

In order to understand the significance of the nonspreading property of the Airy beam, a comparison should be made between the divergence from focus of the Gaussian and Airy beams. The divergence of a Gaussian beam is described by [22]

$$W(z) = W_0[1 + (\frac{z}{z_0})] \tag{19}$$

where $W(z)$ is the beam radius, $W_0$ is the beam waist, the radius of the narrowest point, $z$ is distance in the along propagation direction of the beam and $z_0$ is the Rayleigh range.

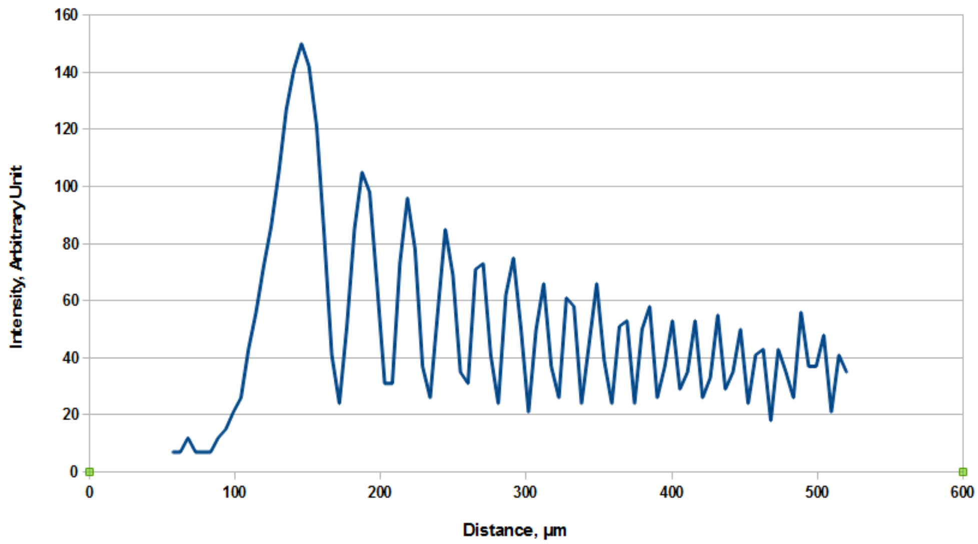


Figure 10: Our Airy beam intensity profile at focus

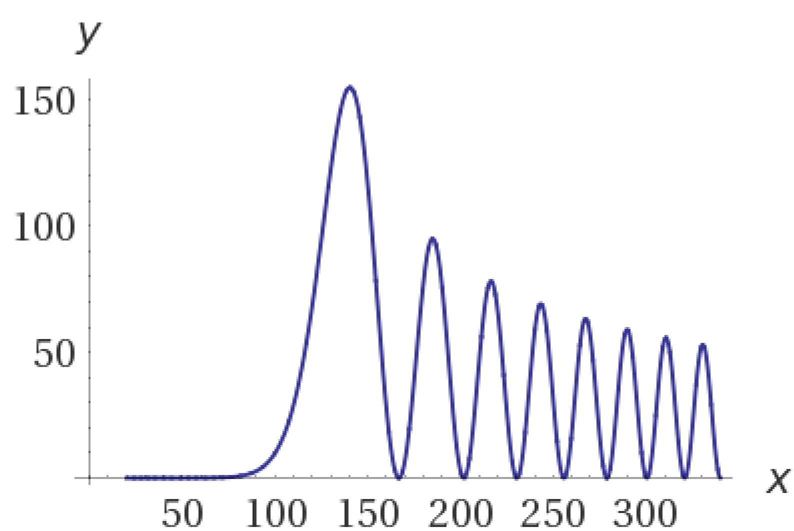


Figure 11: The theoretical intensity profile of the Airy beam at focus, generated using Wolfram Alpha.

Rayleigh range is the distance from the beam waist over which a beam's cross-sectional area doubles, given by [22]:

$$z_0 = \frac{\pi W_0^2}{\lambda} \tag{20}$$

Our the peak lobe of our Airy beam exhibited very little divergence. The divergence of our Airy beam's peak lobe is compared with the theoretical divergence of a Gaussian beam with the same beam waist (FWHM)

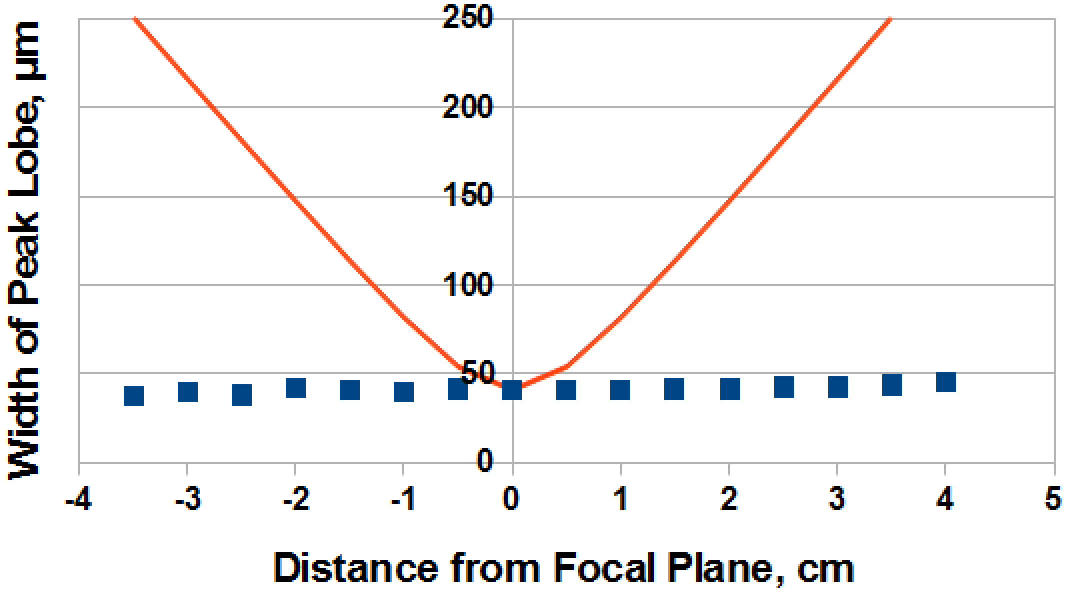


Figure 12: Actual Airy peak lobe widths (blue points) and theoretical Gaussian beam width (red line)

### 6.3 Parabolic Deflection

As the Airy beam propagates, it exhibits a parabolic deflection. This can be calculated using Equation 7:

$$\delta(z) = 0.037\lambda^2 z^2 / W_A^3 \tag{21}$$

where $\delta$ is the transverse deflection, $z$ is distance in the direction of the propagation of the beam, and $W_A$ is the width (FWHM) of the primary lobe of the Airy beam. In Fig 13, the deflection of the peak lobe of our Airy beam is compared to the theoretical path of a peak lobe of an Airy beam of the same minimum width.

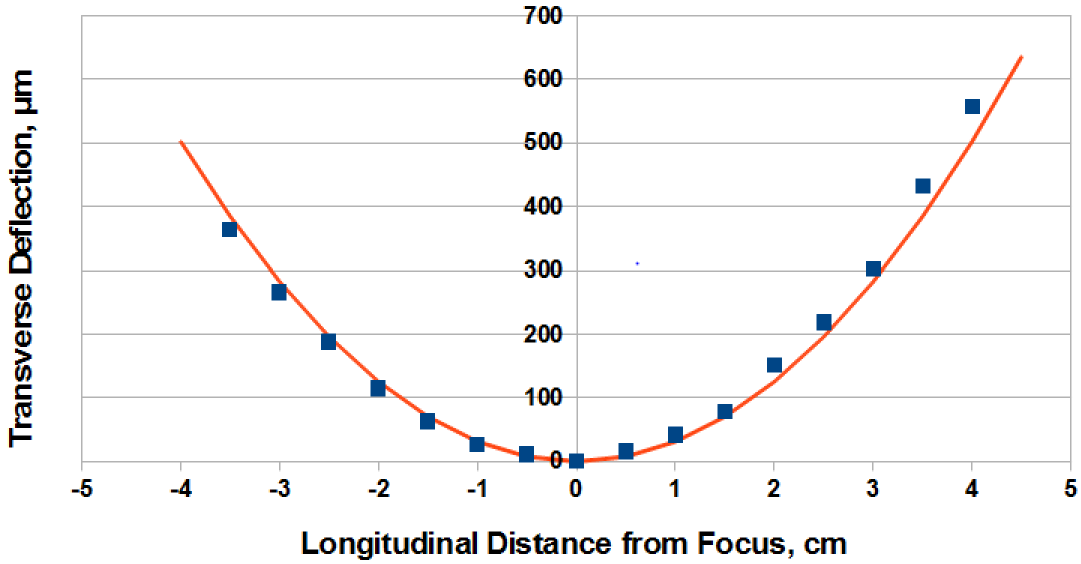


Figure 13: Actual transverse deflection of the beam (blue points) and predicted deflection of the beam (red line)

The slight discrepancy between the parabolic prediction and our data points is a linear term, likely caused by a small angular offset between the propagation path of the beam and the linear rail upon which the camera was mounted.

# 7 Ongoing and Future Work

We will continue our work on Airy beams, using our existing setup, in order to investigate the reconstructing property of our Airy beams by blocking the peak lobe with a small obstruction. To our knowledge, Airy beam self-healing has never been demonstrated with a one-dimensional Airy beam. We additionally plan to use mirrors in place of lenses in a similar set up. Although we were unable to purchase stock cylindrical mirrors, we were able to coat stock lenses with a reflective gold film. Airy beams generated in this manner could be generated using a much greater spectrum of electromagnetic radiation than the with the lens or spatial light modulator approach. Lastly, we hope to develop a model of the wavefront, allowing greater ease in understanding and visualizing the Airy beam as it is created from a Gaussian beam.

# 8 Conclusions

In our research, we were able to create high quality Airy beams and demonstrate their various properties. The results we achieved in a modest teaching laboratory with low-cost equipment were spectacular. Airy beams have a variety of application that are currently being explored. Their curving nature makes them useful in micro particle manipulation [2], plasma channel generation [1], in Terahertz generation [3], and remote sensing. They are even proposed to be able to trigger lightning [23]. Our simple setup can be used to continue researching the variety of applications of Airy beams. The rich history and physics of the Airy beam makes it of great interest to study, and gives the simplified method we used a great pedagogical value. Finally, by using mirrors, we hope to bring bring to life another simple setup which could greatly expand the range of applications of the Airy beam, as a much broader range of wavelengths could be used.

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