John Galletta 11/6/20

The Intensity Profile of a Gaussian Beam

Intro:

Before the seventeenth century, the primary way to model light was by applying geometrical transformations to rays passing through space. This method became antiquated in the nineteenth century when James Clerk Maxwell theorized that light propagates as electromagnetic waves and that an electromagnetic vector field exists at all points of the universe. To model the electromagnetic field, Maxwell devised four differential equations which show that light travels not as rays, but as waves (Maxwell, 460)**.** A by-product of these equations was the fact that light waves do not propagate in a parallel manner, meaning it would be impossible to accurately model light as rays or beams as was done by the geometrical models, thus, light began to be modeled almost exclusively as waves. These equations were tedious to solve and difficult to comprehend intuitively, however. To counteract this, Hermann von Helmholtz made a number of simplifications to Maxwell's equations. The most important of these simplifications were the time-independency of the Helmholtz equation and the introduction of a paraxial approximation which assumes that all angles are substantially smaller than one radian. From these simplifications, the Helmholtz equation provided a more streamlined and scientifically practical model for modelling light under parameters which are nearly always present. Despite this, light could still not be modelled accurately as rays or beams, but a means of collimating light developed over the succeeding centuries to allow for a method of making light propagate as nearly a beam, but in a slightly non-parallel manner. This manner of collimation was derived from Carl Frederick Gauss's normal distribution curve and the paraxial approximation (Gauss, 367). Light collimated in this manner was named a Gaussian beam and has a number of special properties unique to Gaussians. These properties include a waist where the beam diameter is narrowest, a symmetrically expanding beam diameter, a rayleigh length where the beam diameter is twice that of the waist, but most importantly to this experiment, an intensity profile which follows a normal distribution in every direction, radially symmetrical to the center of the beam. When lasers were developed in the twentieth century, Gaussian beams became more prevalent in optics studies, as most lasers produce a Gaussian beam. A whole subset of optics, called Gaussian Optics has been dedicated to the study of Gauusian beams and their applications reach even outside the realm of optics into quantum physics, especially the study of superposition in photons (Kogelnik and Li, 1565).

A defining property of a Gaussian beam is its intensity profile, which as mentioned above, follows a Gaussian distribution which is depicted by Figure 1. This means that the vast majority of lasers also have an intensity of a Gaussian distribution.What's more is that when the cavity distance of a laser is altered, the intensity profile of the beam begins to form a Gaussian mode which can be achieved by performing a series transformations on the Gaussian equation. A great deal of research involving electron excitement requires that Gaussian beams have an intensity profile of a Gaussian distribution, and excited electrons are integral parts of many studies in the fields of quantum entanglement and superposition (McLaren, 23589). Because of this, it is of paramount importance to experimentally prove that Gaussian beams have an intensity profile of a Gaussian distribution.

Therefore, the current study endeavors to experimentally prove the theoretical intensity of a Gaussian beam using a method prescribed by the physics 300 course as taught by Dr. Dominik Scheble at Stony Brook University. This will be done by measuring the current created by the intensity of a Gaussian laser as an increasing portion of the beam is blocked, decreasing the current output of the beam's intensity by an amount equal to the blocked portion of the beam. As a result of this, as a Gaussian beam is blocked incrementally, the plot of the current output of the beam should match the integral of a Gaussian distribution. This is how the intensity of a Gaussian beam will be proven. I will find the intensity of a Gaussian beam by incrementally occluding the beam and comparing the plot of this method to the integral of a Gaussian distribution. If there is little variance between the integral of the Gaussian distribution and the current output of the beam's intensity, then the beam does have a Gaussian intensity distribution.

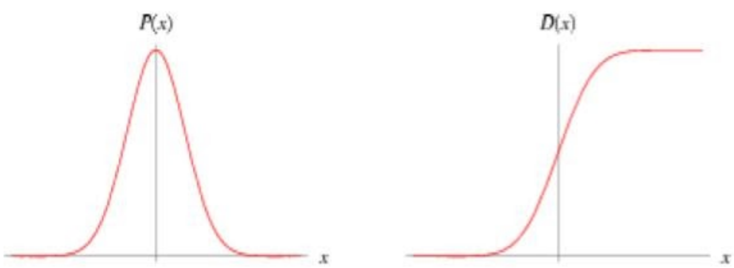


Figure 1: A Gaussian distribution (left) is famously used in statistics to analyse the distribution of points in a data set relative to their mean and standard deviations which is theoretically the intensity of a Gaussian beam. This will be compared to the derivative of the average of the data taken. The integral of that curve (right) will be compared to the data by its derivative' comparison to the derivative of the data.(Wolfram, 1).

Methods:

The primary components of this experiment are a 635 nm Helium-Neon laser diode (HeNe), converging lenses, a photodiode, a razor blade, and a translation stage with a micrometer. These lenses can be of any focal distance, but the lenses used in this experiment are 70 mm and 35 mm converging lenses. As seen in Figure 2, the lenses should be mounted on separate lens mounts, the HeNe and photodiode should be mounted on separate mounts and bases, and the razor should be mounted on the translation stage such that it is able to pass through the optical axis and the blade is perpendicular to the table. Once the HeNe, the lenses, the photodiode, and the translation stage with the razor blade are aligned properly, mount all of them to the optics table or securely tape them to a regular table to avoid unwanted misalignment and the consequent experimental error. Give power to the HeNe by plugging it into its power supply powered by a 115 V outlet, and connect the photodiode to a multimeter measuring mA. At this point the HeNe, photodiode, and multimeter should be turned on, the lenses should be aligned, the translation stage should be calibrated such that the razor does not yet cross through the optical axis, and the lenses should be positioned such that the beam diverges after the first lens, so the razor can cut it, and then reconverges into the photodiode. From this point on, the lens situated closest to the laser will be referred to as lens 1 and the lens nearest the photodiode will be referred to as lens 2. The distance between components of this experiment are contingent on the types of lenses used, and they can be calculated using the equations for the waist of a Gaussian and by the thin lens equation after its modification to fit for a Gaussian beam. Because in this experiment, lens 1 was of focal distance 70 mm and lens 2 was of focal distance 35 mm, lens 1 and two were 196 mm apart, the razor blade is 115 mm from the first lens, and the second lens was 28 mm from the photodiode. Once all these components are aligned in accordance with Figures 2 and 3, the translation stage should be moved forward in even increments such that the razor blade passes through the blade entirely after twenty to thirty increments as seen in Figure 3. At each incremental movement, the translation stage position and the multimeter reading in milliAmps should be recorded. The above process should be repeated as many times as desired with all measurements recorded as shown in Figure 4. In this experiment, five trials of 28 increments of .004 inches were conducted.

In python, a series of graphs were generated, five each representing the points generated by each trial and a scatterplot of all the points on one graph as seen in Figure 5. The average of all the trials was then calculated and plotted with plus/minus 2 standard error bars in the y-direction but plus/minus 1 standard error in the x-direction. The derivative of that curve was also taken and plotted separately. Superimposed onto this graph was a uniformly scaled down Gausian curve to compare the average results to the known intensity of a Gaussian beam as seen in Figure 6. Finally, the difference between the trials and the mean of the trials was plotted to best depict the variability of the trials which is also depicted by Figure 5.

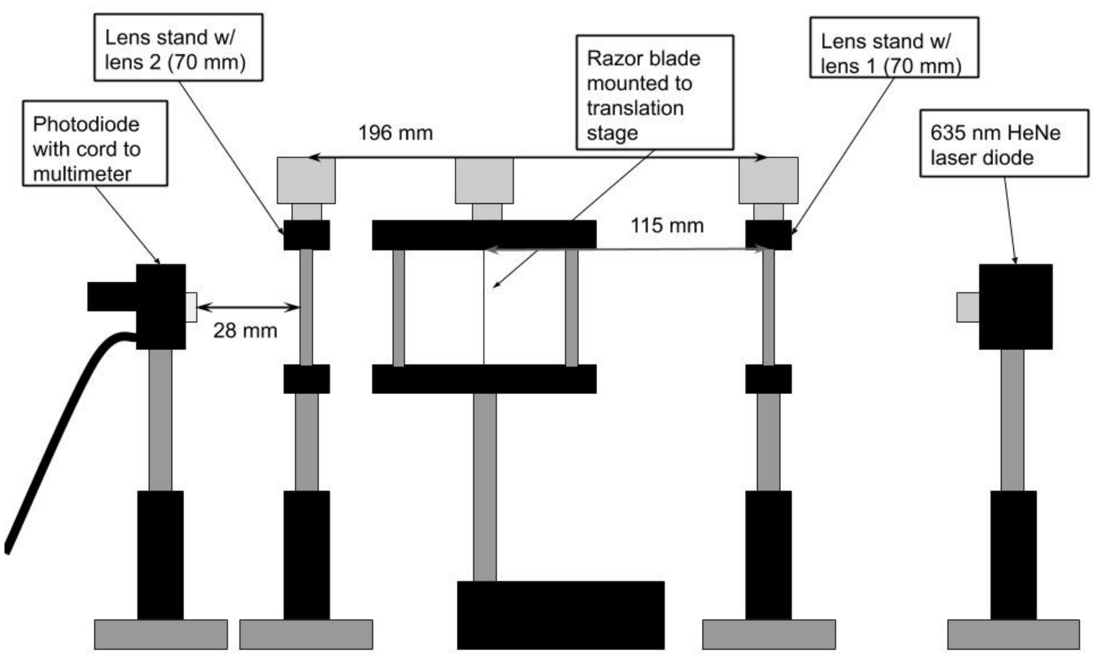


Figure 2: Depicted above is the side view of the experimental setup. The laser beam passes through lens 1 which diverges it to be cut by the razor, and then reconverges into the photodiode to generate current which can be viewed via the multimeter. All dimensions are contingent on the lenses used and should be altered in accordance with the lenses used by the utilization of the thin lens equation with its modifications for a Gaussian beam.

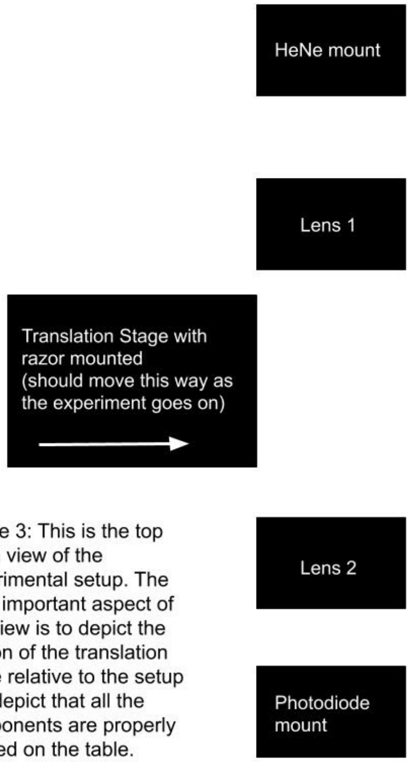


Figure 3: This is the top down view of the experimental setup. The most important aspect of this view is to depict the motion of the translation stage relative to the setup and depict that all the components are properly aligned on the table.

Micrometer Position (inches)	Set 1 (mA)	Set 2 (mA)	Set 3 (mA)	Set 4 (mA)	Set 5 (mA)
0.5	0.32	0.32	0.32	0.32	0.32
0.496	0.32	0.32	0.319	0.32	0.32
0.492	0.32	0.32	0.319	0.32	0.32
0.488	0.319	0.32	0.319	0.32	0.32
0.484	0.319	0.32	0.319	0.319	0.319
0.48	0.319	0.319	0.319	0.319	0.319
0.476	0.319	0.319	0.319	0.319	0.318
0.472	0.318	0.318	0.317	0.318	0.315
0.468	0.314	0.318	0.307	0.315	0.305
0.464	0.302	0.309	0.289	0.301	0.292
0.46	0.286	0.297	0.27	0.285	0.276
0.456	0.27	0.282	0.244	0.258	0.263
0.452	0.242	0.262	0.217	0.231	0.243
0.448	0.225	0.243	0.196	0.207	0.216
0.444	0.194	0.218	0.164	0.182	0.192
0.44	0.166	0.178	0.135	0.146	0.174
0.436	0.148	0.155	0.109	0.121	0.141
0.432	0.122	0.126	0.078	0.095	0.114
0.428	0.095	0.104	0.055	0.072	0.087
0.424	0.073	0.076	0.035	0.05	0.057
0.42	0.05	0.055	0.016	0.03	0.039
0.416	0.027	0.032	0.005	0.013	0.022
0.412	0.01	0.02	0.003	0.004	0.007
0.408	0.002	0.005	0.002	0.002	0.003
0.404	0.002	0.002	0.002	0.002	0.002
0.4	0.002	0.002	0.002	0.002	0.002
0.396	0.001	0.001	0.001	0.001	0.001
0.392	0.001	0.001	0.001	0.001	0.001

Figure 4: On this table are the numerical recordings of the five datasets taken. As the micrometer position decreases, the blade cuts further into the beam until it fully occludes the beam.

Results:

As seen in Figure 4, the datasets follow a similar pattern of a slow decrease in amplitude which accelerates until just after midway through the graphs at which point the rate at which the amplitude decreases decelerates until a baseline of .001mA is reached which is equivalent to current generated by the ambient light in the very dark room in which the experiment was conducted. The point where the rate at which the amplitude decreases varies between datasets which could be attributed to the backlash of the translation stage micrometer. The scatter plot of all the data with their average indicates that there is relatively little variability, but it is difficult to tell from the tight arrangement of the data what the exact variability is.

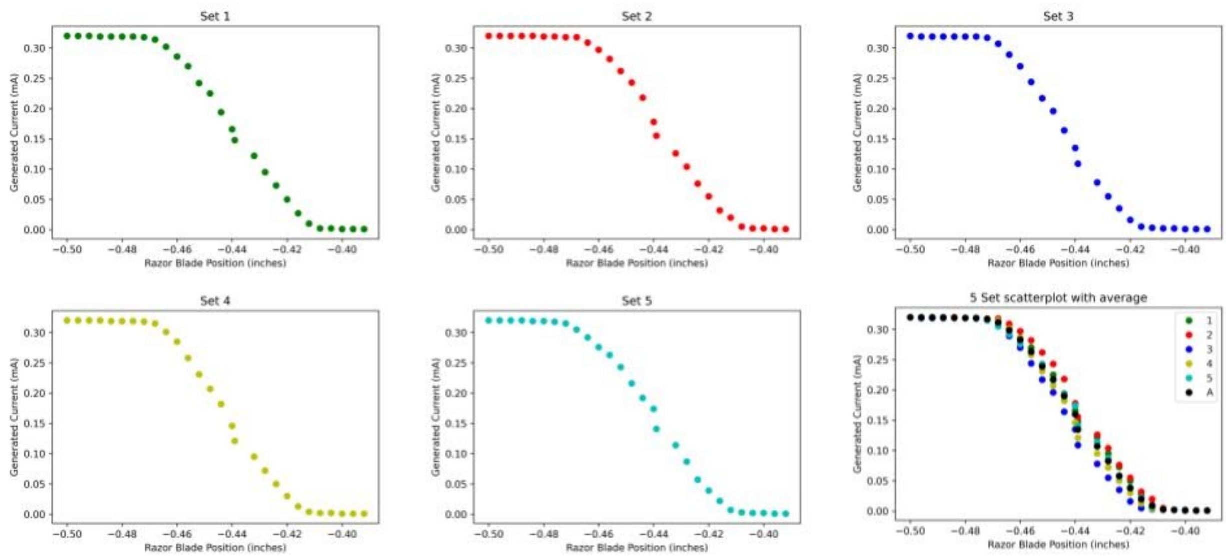
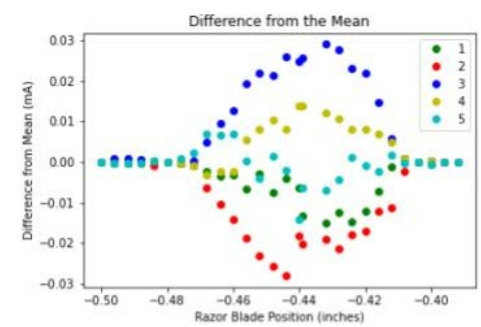
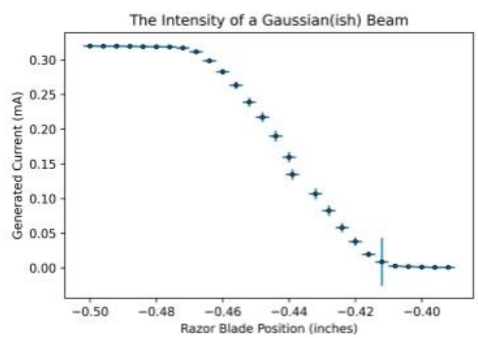


Figure 5: Depicted here are five trials where the beam was cut into by a laser in increments of .004 inches and the current output of the beam's intensity in mA as the beam was occluded incrementally by the blade. As the x-axis intervals increase, the blade was moved further into the beam. The mean of these datasets is also plotted along with all the datasets on one scatter plot to visually show the variation between the sets.

To better understand the data collected, Figure 6 shows that the maximum deviation from the mean is .003 mA which is a relatively small interval in the scale of the experiment and could very well be attributed to the backlash of the micrometer which moves the translation stage along with millimeter sized movements in any of the optical instruments. The average of the data alone was also plotted with error bars in the x direction which reflect the error of the micrometer which is measured to be .001 inches and with error bars in the y direction which reflect the standard error of the data collected at each respective micrometer measurement. From this, it can be seen that the most statistically significant changes in amplitude occurred at or about the midpoints of the data. Equally significant is Figure 6's depiction of the numeric derivative of the average of the datasets which is plotted in red as it compares to the Gaussian distribution which is plotted in blue. The Gaussian distribution was fit and scaled to the size of the data, and by this it can be seen that the data's rate of change accelerates more abruptly, followed by a more gradual deceleration at the vertex, and an even more gradual acceleration in the negative direction which is followed by an abrupt deceleration in the rate of change in the final data points when compared to the Gaussian distribution. These slight differences can most probably be attributed to an aperture which was built into the laser diode used in this experiment. This aperture, despite making the beam profile very radially symmetrical, causes the beam to diffract slightly and alter its natural Gaussian state.





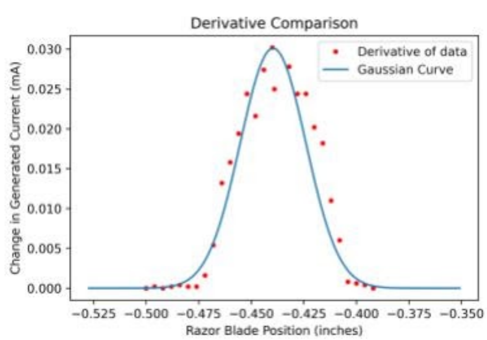


Figure 6: The difference from the mean of all the datasets was plotted (top left).The average of the 5 datasets was plotted with x error bars corresponding with the error of the micrometer of the translation stage od plus/minus .001 inches and y error bars derived from the standard deviation of the data sets from their mean at each point (top right). The derivative of this data was then taken numerically and then plotted with a Gaussian distribution fitted to the data (bottom).

Conclusion:

Gaussian beams and their intensity profiles are of utmost importance to the studies of optics. The study of Gaussian beams has an entire subset of optics allocated to it, and their intensity is a major area of those studies. Many studies of Gaussian beams require the analysis of the intensity profile of a Gaussian beam or a Gaussian mode of that beam which still has an intensity which is derived from the Gaussian distribution (Zhang, 14407). Because of this, it is incredibly important to verify experimentally that lasers and Gaussian beams do, in fact, have an intensity profile of a Gaussian distribution.

The results of this experiment demonstrate that lasers do generally have an intensity profile of a Gaussian distribution. The derivative of the average of the datasets fits well with the Gaussian distribution, following the same general trends of the Gaussian distribution. The noise in the derivative is likely generated by the space between the data points along with the slightly non-Gaussian nature of the beam created by an aperture embedded in the HeNe laser diode. It is likely that a combination of smaller intervals at which the micrometer is moved as well as an increase in the amount of datasets taken would mitigate this effect, but not entirely while a full HeNe laser is not being used. Despite these slight experimental errors, the data does correspond nicely to the Gaussian distribution; thus, this experiment has shown that lasers and Gaussian Beams have an intensity profile which follows a Gaussian distribution.

The results of this study corroborate both the theoretical behavior of a Gaussian Beam and previous experimental studies of Gaussian Beams. Very early studies with lasers found that they have intensity profiles which follow a Gaussian distribution (G.D. Boyd, D.A. Klienman, 3606). Boyd and Klienman also discovered that lasers collimate light into a Gaussian beam with an intensity profile which follows a Gaussian distribution. This information was integral to many experiments regarding Gaussian optics and their applications in quantum optics. The intensity of entangled Gaussian and Bessel Beams was studied to observe the effects of entangling collimated beam photons (McLaren, 23589). In this way, the intensity profile of a Gaussian beam is a necessary piece of scientific information to continue research in the field of Gaussian optics as well as quantum mechanics.

Bibliography

Boyd, G. D., and D. A. Klienman, (1968) Parametric Interaction of Focused Gaussian Light Beams. *Journal of Applied Physics*. 39, 3597-3639. https://aip.scitation.org/doi/10.1063/1.1656831. Accessed 6 November 2020.

Clerk Maxwell, James, (1864) A Dynamical Theory of the Electromagnetic Field. *The Royal Society*. 8, 459-512.

Gauss, Carl F. (1801) *Disquisitiones Arithmeticae*. Lipsiae.

Kogelnik, H. and Li, T. (1966) Laser Beams and Resonators. *The Optical Society*. 5, 1550-1567. https://www.osapublishing.org/ao/fulltext.cfm?uri=ao-5-10-1550&id=14408.

McLaren, Melanie, et al, (2012) Entangled Bessel-Gaussian beams. *The Optical Society*. 20, 23589-23597. https://www.osapublishing.org/oe/fulltext.cfm?uri=oe-20-21-23589&id=242670. Accessed 6 November 2020.

Wolfram, Stephen. (2020) Normal Distribution. *Wolfram MathWorld*. https://mathworld.wolfram.com/NormalDistribution.html.

Zhang, Lifu, et al. (2016) Propagation dynamics of super-Gaussian beams in fractional Schrödinger equation: from linear to nonlinear regimes. *The Optical Society.* 24, 14406-14418. https://www.osapublishing.org/oe/fulltext.cfm?uri=oe-24-13-14406&id=344871.