Creating optical vortex modes with a single cylinder lens

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ABSTRACT

Optical vortex (Laguerre-Gauss) modes can be created by introducing a $\pi/2$ phase shift between orthogonal components of Hermite-Gauss (HG) modes. The well-known astigmatic mode converter design described by M.W. Beijersbergen et al. [Optics Communications **96**, pgs. 123–132, 1993] achieves this condition by manipulating the differing Gouy phases along orthogonal axes between a matched pair of cylinder lenses. Apparently not well known is that quite useful mode conversions can easily be achieved with a single cylinder lens. We explain the operating principle of such a single lens mode converter, and describe and illustrate how to match the input HG mode to the required Rayleigh range $z_R = f_{cyl}$ with one additional spherical lens. Setting up and optimizing such a simplified mode converter is an excellent exercise for undergraduate students, and the resulting optical vortex beams can be used for a variety of instructional experiments.

Keywords: optical vortex, Laguerre-Gauss mode, astigmatic mode converter, mode matching

1. INTRODUCTION

Optical vortex laser beams and the applications of such beams have provided a rich field of study for the last two decades.[1] In particular, the orbital angular momentum (OAM) carried by an optical vortex beam[2] allows many new possibilities in micro-manipulation, free-space communication, quantum information, and cold atom interactions.[3–6] As discussed by Padgett et al., Galvez, and others, optical vortices and related topics in the wave optics of laser light fields also have considerable pedagogical interest and value.[7–11]

The work we report here had its origin in a student project[12,13] the overall goal of which was to create an optical vortex laser tweezers that could rotate microscopic particles through the transfer of orbital angular momentum.[14–16] The intention was to create the required Laguerre-Gauss (LG) optical vortex beam from a synthetic Hermite-Gauss (HG) mode[17] in an astigmatic mode converter of the type described by Beijersbergen et al.,[18] which employs a matched pair of cylinder lenses. While setting up such a converter, one of us (HS) noticed that an optical vortex beam could be produced under certain conditions with the second cylinder lens removed. This chance observation prompted the current investigation of the mechanism behind such a 'single lens mode converter,' and methods to quickly and effectively implement one.

After our study had been completed we discovered that Abramochkin and Volostnikov employed essentially the same method based on a single cylinder lens to record the experimental mode transformations included with their comprehensive theoretical analysis of beam transformations under the influence of astigmatism.[19] Unfortunately, however, their lengthy and complex work is not well known and their presentation doesn't emphasize the essential simplicity and practical utility of the single cylinder lens method, as we do here.

Sections 2 and 3 below summarize past work related to astigmatic mode converters and our experimental methods, respectively. Sections 4 and 5 describe the basic operating principles of the single lens mode converter, and explain how to set one up. Section 6 shows experimental HG → LG mode conversions we obtained.

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2. BACKGROUND

The Hermite-Gauss (HG) and Laguerre-Gauss (LG) modes are two independent basis sets of solutions to the paraxial wave equation that describes a laser beam propagating at a small angle to some axis z.[7,8,20] This fact immediately implies that a mode of either type can always be expanded in a series of modes of the other type. Fortunately, the number of terms required is limited by the condition that the order $N = n + m$ of the modes in the expansion match the order of the mode being expanded.[18] Furthermore a diagonal HG mode oriented at 45° to the x and y axes can be expressed in a similar way in terms of HG modes oriented along those axes. These mathematical relationships make it possible to interconvert HG and LG modes by creating a $\pi/2$ relative Gouy phase shift in one or more astigmatic elements (cylinder lenses), a concept that was introduced by Tamm and Weiss (1990)[21] and Abramochkin and Volostnikov (1991).[19]

In the first of two closely-related papers from the Woerdman group at Leiden, Allen et al. (1992) discussed such mode conversions in the context of the orbital angular momentum contained in LG modes, and generalized the earlier ideas to include HG modes of any order.[22] Beijersbergen et al. (1993) subsequently presented a more complete analysis of the generalized mode converter concept, gave a detailed design for such a device, and demonstrated its operation.[18] Their design[18] localizes the astigmatic region between two aligned cylinder lenses of focal length f spaced a precise distance ($\sqrt{2}f$) apart. The two cylinder lenses are preceded by one or more spherical mode-matching lenses to produce an input waist of Rayleigh range $z_R = (1 + 1/\sqrt{2})f$ midway between them. The incident HG mode is oriented at 45° to the axes of the cylinder lenses. Under these conditions the Gouy phase shift of the beam component along the active axis of the cylinder lenses is three times that of the orthogonal component and the net Gouy phase difference between the two axes is $\pi/2$.

In principle an astigmatic mode converter of this type can create a perfect LG mode with 100% efficiency, from a perfect HG mode. Courtial and Padgett (1999)[23] have discussed the performance of the Beijersbergen et al. design[18] with respect to various 'imperfections' in its construction or use. They found, for example, that errors of up to 5% in beam focusing (input waist size) or cylinder lens focal length have a minimal effect on the final mode purity – over 97% of the beam energy is still contained in the principal mode.

Yoshikawa and Sasada (2002) created the equivalent of a symmetrical two-lens converter[18] by utilizing a single cylinder lens in a reflective geometry.[17] Some quite different designs for $\pi/2$ astigmatic mode converters have been proposed, but these require additional or highly specialized optical elements. For example, Malyutin's 'tunable' converter (2004) consists of a spherical lens centered between two 'quadrupole' cylinder lenses.[24]

Astigmatic mode converters of the classic type[18] have been used to create optical vortex beams for applications such as atom trapping[25] and laser tweezers.[14,16] Very recently, picosecond optical vortices up to order 9 have been obtained by conversion of HG modes from a mode-locked laser,[26] opening up the possibility for further novel applications. In recent years however alternative means of creating optical vortex beams such as spatial light modulators and micro-fabricated phase plates have become commercially available and are increasingly popular. Such devices are relatively expensive and/or specialized however, and the very simple readily-implemented method discussed in this paper remains attractive for teaching laboratories in particular.

3. METHODS

Mode conversions were studied experimentally with Hermite-Gauss modes obtained from a 633 nm open-cavity HeNe laser similar to that described by Padgett et al.[7] Our laser employs a 24 cm long Melles-Griot 05-LHB-570 laser tube, which is sealed with a Brewster window at one end and an $R = 60$ cm high reflectivity mirror at the other. The plane output coupler mirror is located 10–15 cm from the Brewster window. At the time of this work we used a single thin ($\sim 50\,\mu$m) human hair inside the cavity to force specific HG modes. The hair could be rotated to any angle with respect to the vertical and translated horizontally. Modes up to order three were routinely achieved. The Rayleigh range of both HG(1,0) and HG(0,0) modes was determined to be ~ 300 mm from measurements of the beam radius $w(z)$ at several distances z.

4. OPERATING PRINCIPLE

Figure 1 below shows the essential features of a single-lens $\pi/2$ astigmatic mode converter. A spherical matching lens (focal length f_m) produces a waist with Rayleigh range z_R a distance z_R ahead of a cylinder lens whose focal length f_{cyl} is equal to z_R. As discussed in the Appendix, under these conditions the active axis of the cylinder lens symmetrically transfers the active plane of the beam to a second waist with the same Rayleigh range z_R over an additional distance $2\,z_R$. The inactive axis of the cylinder lens is assumed to not affect the beam (thin lens approximation), and therefore this axis maintains the Rayleigh range z_R of the first waist. As shown in Fig. 1, the net result is that the beam downstream of the cylinder lens has an equal Rayleigh range z_R along both axes. The two orthogonal beam components will therefore diverge at the same rate and the beam will be restored to a circular profile in the far field. The ellipticity (transverse diameter ratio) at a distance z from the cylinder lens varies with the relative distance z/z_R and is 1.22 at $z/z_R = 10$ and 1.02 at $z/z_R = 100$.

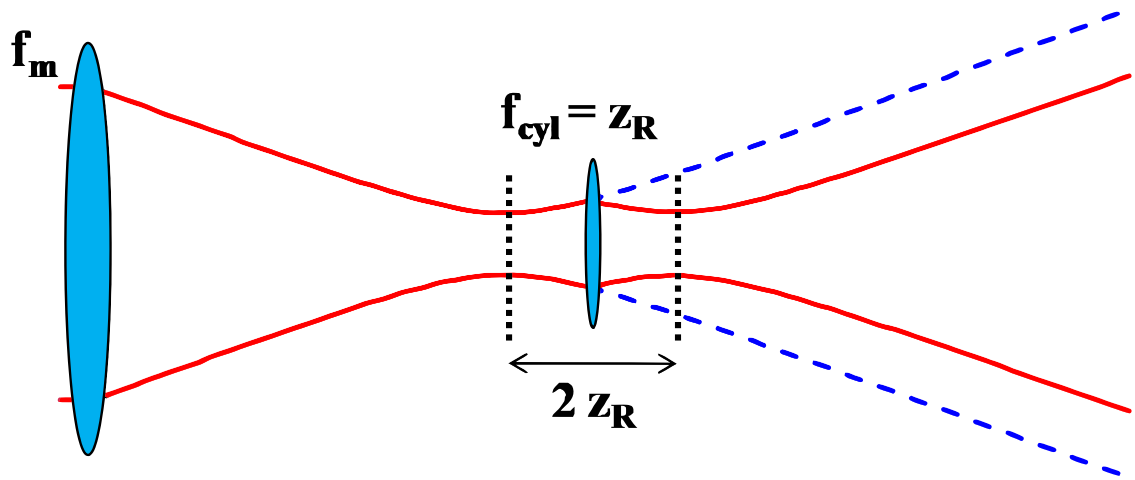

Figure 1. Essential features of the single lens mode converter. The two vertical dashed lines show the positions of the waists produced by the spherical matching lens (focal length f_m) and by the active axis of the cylinder lens (focal length f_{cyl}).

It is also easy to see why the arrangement shown in Fig. 1 produces the $\pi/2$ net Gouy phase difference needed for an HG $\rightarrow$ LG mode converter. As shown in Eq. 22 of Ref. 18, the Gouy phase difference between successive terms in the HG series expansion of a 45° diagonal HG mode is dependent only on the difference in $\arctan(z/z_R)$ for the x and y directions, and is independent of the input HG mode indices m and n. Therefore we need only consider the $\arctan(z/z_R)$ function, which reaches $\pi/4$ at $z = z_R$ and approaches $\pi/2$ as $z/z_R \rightarrow \infty$. In other words, starting from a beam waist, one half of the maximum possible $\pi/2$ Gouy phase shift is acquired in the first Rayleigh range, and the remainder from that point on into the far field.

Now consider the active beam component shown by the solid red line in Fig. 1, starting from the cylinder lens. This beam component gains a phase of $\pi/4 + \pi/2$ in moving from the lens through a second waist into the far field. On the other hand, the inactive beam component (blue dashed line) does not pass through a second waist. It has already acquired a phase of $\pi/4$ at the lens and gains an additional $\pi/4$ as it moves into the far field. The difference of the final phase shifts in the two beam components is thus the required $\pi/2$.

In the classic converter[18] the beam becomes elliptical in the first cylinder lens and is restored to its initial isotropic state by the second cylinder lens. Similarly the entire $\pi/2$ Gouy phase shift difference is developed in the limited region between the two lenses. In the single lens converter the Gouy phase difference and the beam shape gradually evolve as the beam moves into the far field. Since the relevant distance parameter is z/z_R for both processes, it is desirable to use a cylinder lens with the shortest practical focal length. However, even in the far field the beam will retain a longitudinal astigmatism (source point separation) of $2\,z_R$, which will affect the shape of the beam if it is brought to a subsequent focus.

5. PRACTICAL CONSIDERATIONS

We now consider some practical issues related to creating an optical vortex beam with a single lens mode converter. They mainly involve creating a suitable HG input mode and matching the characteristics of this mode to the requirements of the converter. Similar issues are involved if an LG mode is to be converted to the corresponding HG mode, or with mode conversions in the traditional two cylinder lens converter.[7,18]

5.1 Creating a Suitable HG Input Mode

The HG input mode is best obtained directly from a laser with an accessible cavity, as described in Sect. 3 and Ref. 7, and this approach can provide modes of several orders. If no such laser is available an approximate 'synthetic' HG(1,0) mode is easily created by phase shifting one-half of a normal (fundamental) Gaussian mode by $\pi/2$.[13,17] An alternative method involves more optical elements but avoids edge-diffraction effects.[28] Whatever the source of the mode it must be oriented at 45° to the axis of the cylinder lens; a dove prism in a rotatable mount could be useful for this.

Mode matching requires knowledge of the Rayleigh range of the input mode, which can be obtained from scans of the transverse beam profile at one or more distances from the laser.[27] One distance is sufficient if the position of the laser beam waist is already known. While it's convenient to scan a fundamental mode, any higher-order mode will have the same Rayleigh range (but a larger overall size) and can be used if it is suitably oriented and its profile is fitted to the appropriate HG function.[8] If a commercial laser is used to generate a synthetic mode its Rayleigh range can be calculated from its specified output beam diameter or divergence.

5.2 Mode Matching with a Single Spherical Lens

The goal of mode matching is to transform the input HG mode with a given initial Rayleigh range and waist position into a beam with a Rayleigh range $z_{\rm R}$ equal to the focal length $f_{\rm cyl}$ of the cylinder lens employed. (As noted in Sect. 4, it is best to keep $f_{\rm cyl}$ relatively short, say 25 mm or less. This minimizes both the physical distance to the far field and the distance required for mode matching.) Mode matching is always possible with a single spherical lens if the position of the second waist is not limited in any way. One of the advantages of the single lens mode converter is that translating a single cylinder lens is more convenient than moving two lenses while maintaining their precise spacing, unless these lenses have been incorporated into a custom-built mount.

The focal length $f_{\rm m}$ of the matching lens for a given laser–lens distance, or the placement of a given matching lens, can be calculated with the q-parameter methods summarized in the Appendix. Let $z_{\rm L}$ be the Rayleigh range of the laser, and d_1 and d_2 the distance from the laser's beam waist to the matching lens and from there to the second waist, respectively. With these defintions the Rayleigh range $z_{\rm R}$ after the matching lens (which must equal the focal length $f_{\rm cyl}$ of the cylinder lens) is given by

$$z_{\rm R} = {\rm Im}(q_2) = \frac{f_{\rm m}^2 z_{\rm L}}{z_{\rm L}^2 + (f_{\rm m} - d_1)^2} = f_{\rm cyl} \quad .$$

This expression can be solved for either $f_{\rm m}$ or d_1 to give

$$f_{\rm m} = \frac{-f_{\rm cyl} d_1 + \sqrt{z_{\rm L}^3 f_{\rm cyl} - z_{\rm L}^2 f_{\rm cyl}^2 + z_{\rm R} f_{\rm cyl} d_1^2}}{z_{\rm L} - f_{\rm cyl}} \quad \text{or} \quad d_1 = f_{\rm m} + \sqrt{f_{\rm m}^2 z_{\rm L}/f_{\rm cyl} - z_{\rm L}^2} \quad .$$

The resulting (positive) distance d_2 to the second beam waist is given by

$$-d_2 = {\rm Re}(q_2) = \frac{f_m(-z_{\rm L}^2 + f_m d_1 - d_1^2)}{z_{\rm L}^2 + (f_m - d_1)^2} \quad .$$

The spacing between the mode-matching lens and the cylinder lens is just $d_2 + f_{\rm cyl}$.

Figures 2, 3 and 4 show examples relevant to our specific setup of how these formulae can be used to model the mode-matching arrangements, starting with a given cylinder lens.

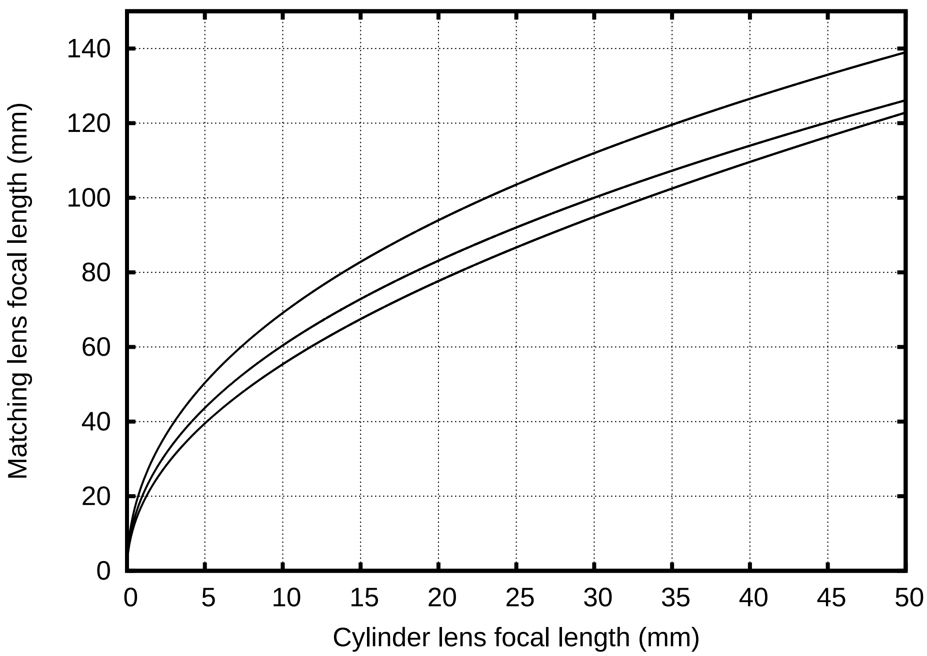

Figure 2. The appropriate mode-matching lens focal length for various cylinder lens focal lengths. The laser beam has an initial Rayleigh range of 300 mm, and the mode-matching lens is situated at a distance $d_1 =$ 100, 200 or 300 mm from the laser waist (bottom, middle and top curve, respectively).

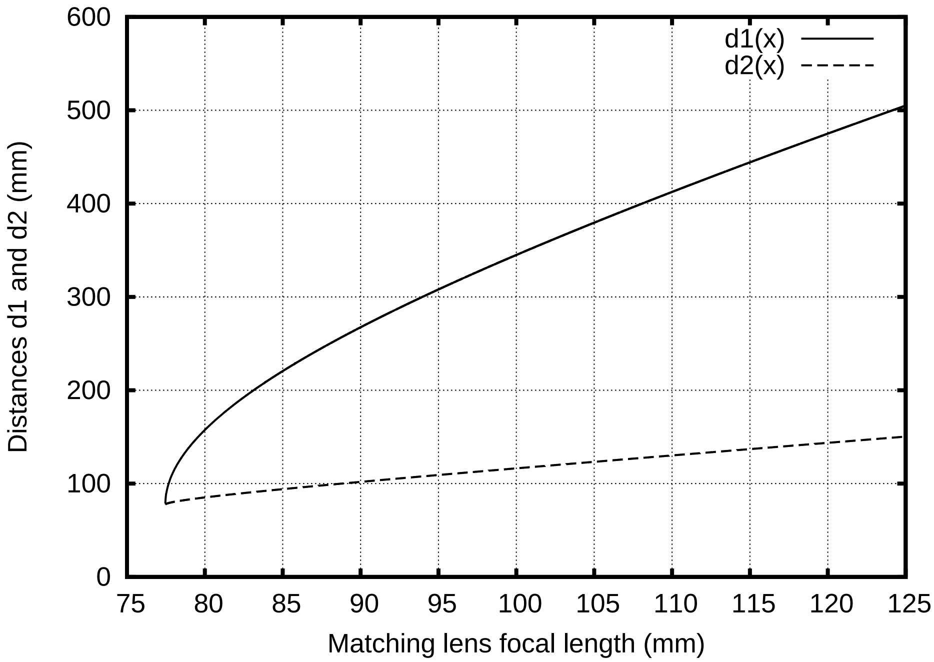

Figure 3. Waist distances d_1 and d_2 as a function of the mode-matching lens focal length f_m, for a laser Rayleigh range $z_\mathrm{L} = 300$ mm and a cylinder lens focal length $f_\mathrm{cyl} = 20$ mm. Note that there is a minimum possible f_m value and a corresponding minimum distance d_1, and that distance d_2 is slightly greater than the matching lens focal length, as expected.

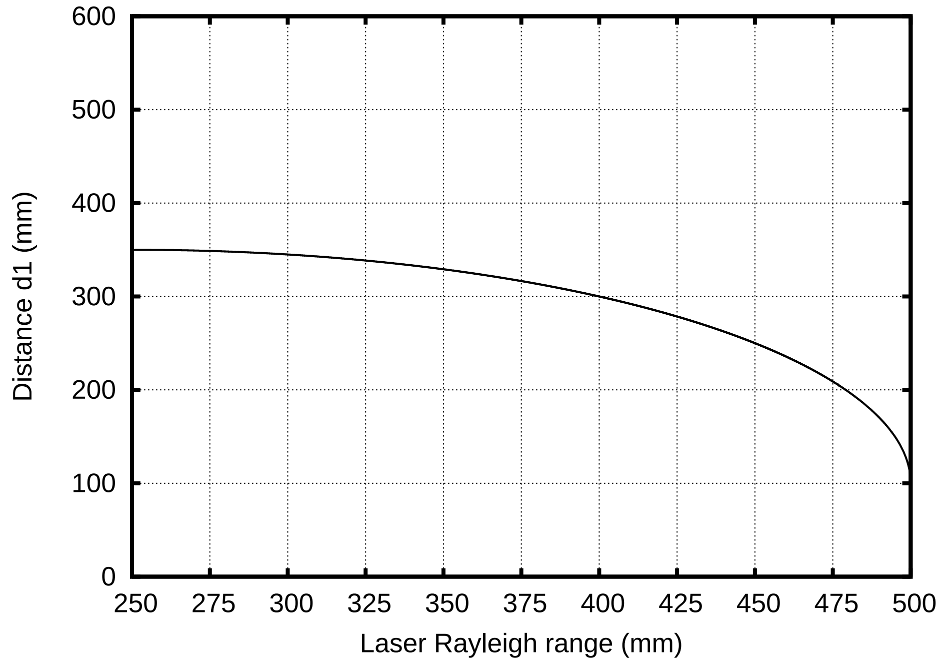

Figure 4. Distance d_1 between the laser waist and the mode-matching lens as a function of the laser Rayleigh range z_L, with the lenses we used (f_{cyl} = 20 mm, f_m = 100 mm). The Rayleigh range values shown are typical of low-power (few mW) HeNe lasers; our laser had $z_L \sim 300$ mm.

5.3 Alternative Mode Matching Arrangments

Abramochkin and Volostnikov[19] and Beijersbergen et al.[18] both employed a mode-matching telescope with two spherical lenses, for reasons they did not discuss. (Note that while Fig. 4 in Ref. 18 shows a single matching lens the text of the paper states that the authors "first tested the operation" of their converter "by means of two spherical lenses.") We used the formulae derived earlier in this section to estimate that the first authors[19] could have matched to their $f_{cyl} = 3.4$ mm cylinder lens with a single 70 mm matching lens at a reasonable distance d_1 between 600 and 800 mm, for any laser Rayleigh range z_L between 300 and 1000 mm.

In our initial work on this topic[13] an empirical iterative procedure of alternately tilting the matching lens and translating the cylinder lens was employed, and tilting was incorrectly believed to be essential for achieving a $\pi/2$ Gouy phase difference. (By 'tilting' we mean rotating the matching lens about an axis parallel to that of the cylinder lens. This was easily done in our setup, in which the cylinder lens axis was vertical.) While not essential, tilting the matching lens, which reduces its effective focal length, can be quite useful in practice. For example if one wished to minimize distance d_1 (and hence the overall converter length $d_1 + d_2 + f_{cyl}$) one would need to work near the cutoff f_m value of around 77 mm shown in Fig. 3. The d_1 curve is increasingly steep near cutoff and a change of just 1 mm (about 1%) in f_m could change d_1 by several cm or more. Such a change would require only a few degrees of tilt.[29] Tilting does of course introduce astigmatism – the change in effective focal length is predominantly (but not entirely) on the active axis perpendicular to the axis of rotation. The resulting separation of the x and y axis beam waists will result in unequal x and y Rayleigh ranges in the far field, but the net Gouy phase difference will remain essentially the same.

Finally, we note that the equivalent of a single spherical lens of some desired but unavailable focal length can often be easily created by placing two lenses of larger focal length in close proximity, where their reciprocal focal lengths are additive.

6. EXPERIMENTAL RESULTS

Figure 5 shows the basic equivalence of the HG → LG mode conversions achieved with single lens and two lens mode converters. In both cases the cylinder lens(es) had $f_{\rm cyl} = 20$ mm and the single spherical matching lens had $f_{\rm m} = 100$ mm. The incident HG modes (rows 1 and 3 in the figure) were obtained from an open-cavity laser, as described in Sect. 3. The resulting LG patterns (rows 2 and 4) were recorded on a screen about one meter from the converter.

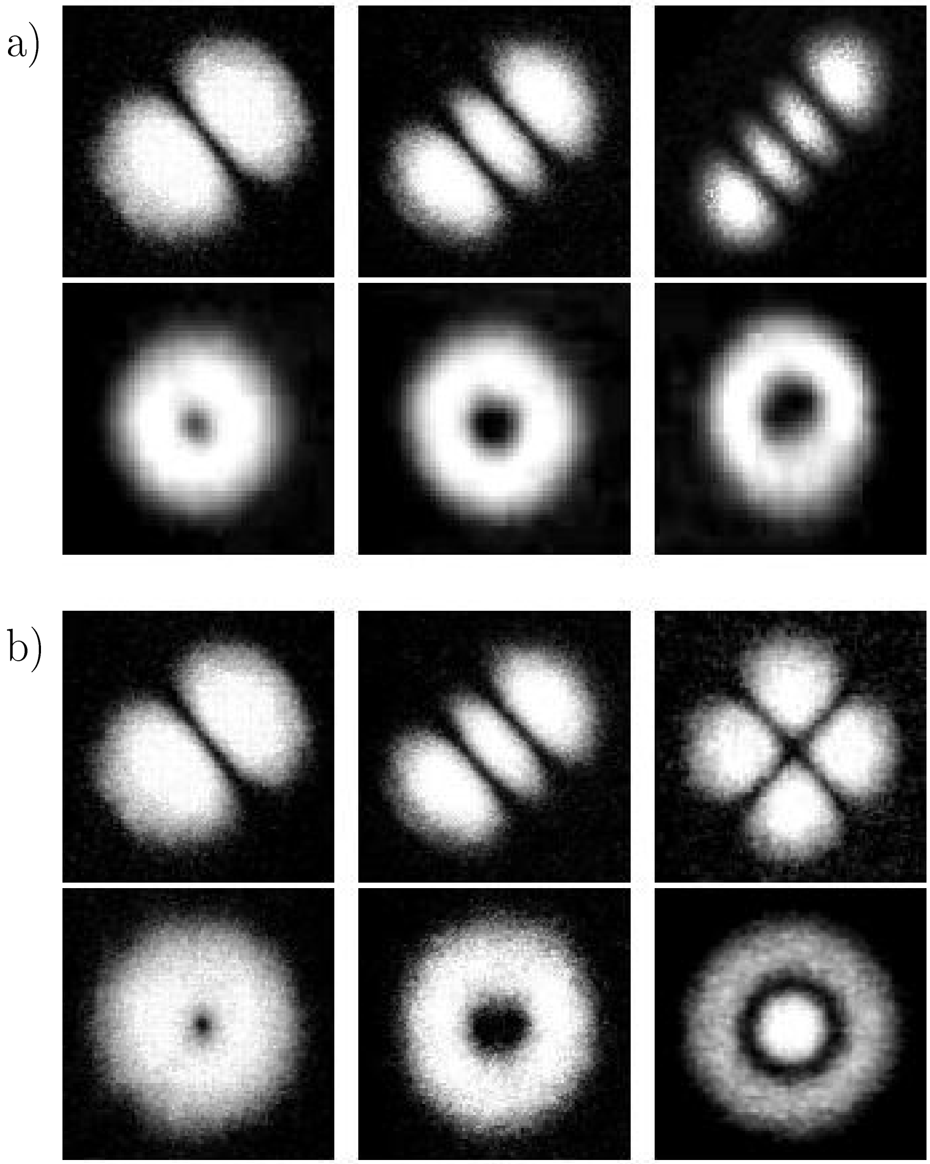

Fig. 5. Mode conversions achieved with a traditional two cylinder lens converter (a) and the simplified single lens converter (b). The pixelated appearance of the first three LG modes is because these particular images had to be greatly enlarged.

A single lens converter with $f_{\rm cyl} = 20$ mm was subsequently used in a student laser tweezers experiment.[13] The synthetic HG(1,0) mode[17] was created by inserting a tilted microscope cover slip into the HG(0,0) normal Gaussian beam of a 30 mW HeNe laser. This laser had $z_{\rm L} = 1000$ mm and the matching lens had $f_{\rm m} = 200$ mm.

7. CONCLUSIONS

We have shown that HG → LG mode conversions with a single cylinder lens can be easier to understand and implement than may be apparent from past publications. Matching the initial mode to the converter can be accomplished with a single spherical lens, whose focal length can be varied if necessary by tilting it.

A single lens mode converter has several practical advantages, the significance of which will depend on the situation. Compared to the traditional two lens design it is relatively easy to set up and align, requires no specialized cylinder lens mount, and has fewer optical surfaces. The optical vortex beam produced is elliptical but becomes increasingly circular as it propagates into the far field. The longitudinal astigmatism retained by the beam should only be significant in certain critical applications.

Creating an optical vortex beam using the methods described herein is an excellent open-ended student exercise for a teaching laboratory, that combines hands-on work with various aspects of the theory of Gaussian beams, and requires just two lenses in standard mounts. Suitable HeNe laser tubes to construct an open-cavity laser are available as surplus, or the input HG mode can be approximated by manipulating the phase profile of a normal Gaussian laser beam. The resulting optical vortex beams could be used for a variety of further experiments.

APPENDIX A. WAIST-TO-WAIST TRANSFER

The propagation of Gaussian beams can be succintly described by the q-parameter formalism.[20,30,31] The complex parameter $q(z)$ is given by $z + iz_\mathrm{R}$, where z is the distance from the beam waist and z_R is the Rayleigh range. q also contains information about the wavefront radius of curvature $R(z)$ and the beam radius $w(z)$.

$$\frac{1}{q_1} = \frac{1}{R(z)} - \frac{2i}{kw(z)^2}$$

The q-parameters transform according to $q_2 = (Aq_1 + B)/(Cq_1 + D)$, where (A, B, C, D) are the elements of the usual 2×2 ray optics matrix. For example, over a drift space of distance d the initial $q_1 = z + iz_\mathrm{R}$ becomes $q_2 = (z + d) + iz_\mathrm{R}$. In passing through a thin lens, $q_1 = z + iz_\mathrm{R}$ becomes $q_2 = (z + iz_\mathrm{R})/(-z/f - iz_\mathrm{R}/f + 1)$. Note that distances z are postive moving away from a waist and negative when approaching a waist.

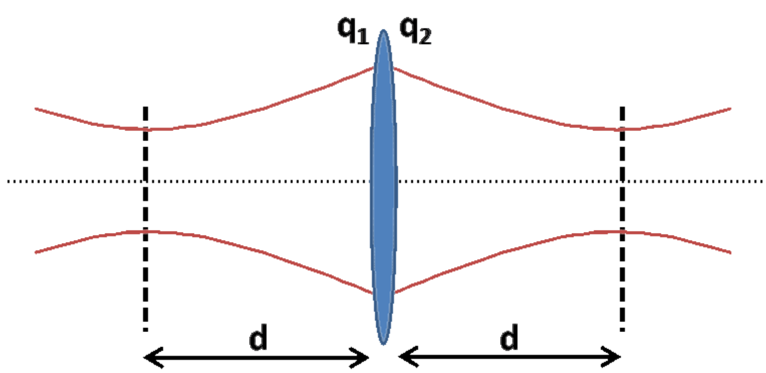

Figure 5. A thin lens displaces a beam waist by $2d$ and leaves the Rayleigh range z_R unchanged.

Now consider a symmetrical waist-to-waist transfer through a thin lens of focal length f over a total positive distance $2d$, as shown in Fig. 5 above. The Rayleigh range z_R remains the same and the initial waist distance $z = d$ at the lens becomes $z = -d$ after the lens:

$$q_1 = d + iz_\mathrm{R} \quad \Longrightarrow \quad q_2 = \frac{d + iz_\mathrm{R}}{-d/f + 1 - iz_\mathrm{R}/f} = -d + iz_\mathrm{R}$$

Equating the real and imaginary parts of the two expressions for q_2 gives:

$$\mathrm{Im}(q_2) = \frac{f^2 z_{\mathrm{R}}}{(-d+f)^2 + z_{\mathrm{R}}^2} = z_{\mathrm{R}} \quad \text{and} \quad \mathrm{Re}(q_2) = -\frac{f(d^2 - df + z_{\mathrm{R}}^2)}{(d-f)^2 + z_{\mathrm{R}}^2} = -d$$

whose common solution can be written

$$\frac{2d}{f} = \frac{4d^2}{d^2 + z_{\mathrm{R}}^2}$$

In ray optics ($z_{\mathrm{R}} = 0$) symmetrical point-to-point transfer is only possible over a total relative distance $d/f = 2$. The situation is very different for Gaussian beams ($z_{\mathrm{R}} > 0$), and there is no minimum relative distance d/f for a symmetrical waist-to-waist transfer. When $d/f = 1$ it follows that $d = z_{\mathrm{R}}$, which is the situation shown in Figure 1 (Section 4).

Equivalent results on symmetrical waist-to-waist transfer can be found in an article by S.A. Self, whose treatment extends the usual geometrical optics thin-lens formulae.[32] The "common point" shown on Fig. 4 of Self's paper corresponds to the present case: symmetrical transfer over a total distance $2f$.

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REFERENCES

[1] P. Coullet, L. Gil, and F. Rocca, "Optical vortices," Opt. Commun. **73**, 403–408, 1989.

[2] Leslie Allen, Stephen M. Barnett, and Miles J. Padgett, eds, *Optical Angular Momentum* (Institute of Physics Publishing, Bristol, 2003).

[3] K. Ladavac and D.G. Grier, "Microoptomechanical pumps assembled and driven by holographic optical vortex arrays," Opt. Express **12**, 1144–1149 (2004).

[4] Graham Gibson, Johannes Courtial, Miles J. Padgett, Mikhail Vasnetsov, Valeriy Pas'ko, Stephen M. Barnett and Sonja Franke-Arnold, "Free-space information transfer using light beams carrying orbital angular momentum," Opt. Express **12**, 5448–5456 (2004).

[5] A. Vaziri, J. W. Pan, T. Jennewein, G. Weihs, and A. Zeilinger, "Concentration of higher dimensional entanglement: qutrits of photon orbital angular momentum," Phys. Rev. Lett. **91**, 227902 (2003).

[6] L.S. Leslie, A. Hansen, K.C. Wright, B.M. Deutsch, and N.P. Bigelow, "Creation and detection of Skyrmions in a Bose-Einstein condensate," Phys. Rev. Lett. **103**, 250401 (2009).

[7] M. Padgett, J. Arlt, and N. Simpson and L. Allen, "An experiment to observe the intensity and phase structure of Laguerre-Gaussian laser modes," Am. J. Phys. **64**, 77–82 (1996).

[8] E.J. Galvez, "Gaussian beams in the optics course," Am. J. Phys. **74**, 355–361 (2006).

[9] E.J. Galvez and Nikolay Zhelev, "Orbital angular momentum of light in optics instruction," Proceedings 2007 Education and Training in Optics and Photonics (ETOP) Conference. http://spie.org/etop/2007/etop07fundamentalsIII.pdf

[10] G.F. Brand, "Phase singularities in beams," Am. J. Phys. **67**, 55–60 (1999)

[11] J. Peatross and M.V. Pack, "Visual introduction to Gaussian beams using a single lens as an interferometer," Am. J. Phys. **69**, 1169–1172 (2001).

[12] H. Sridhar, "Development of an inverted optical tweezers with full motional control," International Science and Engineering Fair, May 2007 (unpublished report). http://laser.physics.sunysb.edu/∼hamsa/papers/

[13] H. Sridhar, "A novel astigmatic laser mode converter for an optical vortex tweezers," Intel Science Talent Search, November 2007 (unpublished report). http://laser.physics.sunysb.edu/∼hamsa/papers/

[14] H. He, M.E.J. Friese, N.R. Heckenberg, and H. Rubinsztein-Dunlop, "Direct observation of transfer of angular momentum to absorptive particles from a laser beam with a phase singularity," Phys. Rev. Lett. **75**, 826–829 (1995).

[15] M.E.J. Friese, J. Enger, H. Rubinsztein-Dunlop, and N.R. Heckenberg, "Optical angular-momentum transfer to trapped absorbing particles," Phys. Rev. A **54**, 1593–1596 (1996).

[16] N.B. Simpson, K. Dholakia, L. Allen, and M. J. Padgett, "Mechanical equivalence of spin and orbital angular momentum of light: an optical spanner," Opt. Lett. **22**, 52–54 (1997).

[17] Y. Yoshikawa and H. Sasada, "Versatile generation of optical vortices based on paraxial mode expansion," J. Opt. Soc. Am. A **19**, 2127–2133 (2002).

[18] M.W. Beijersbergen, L. Allen, H.E.L.O. van der Veen, and J.P. Woerdman, "Astigmatic laser mode converters and transfer of orbital angular momentum," Opt. Commun. **96**, 123–132 (1993).

[19] E. Abramochkin and V. Volostnikov, "Beam transformations and nontransformed beams," Opt. Commun. **83**, 123–135 (1991).

[20] A.E. Siegman, *Lasers*, (University Science Books, Mill Valley, California, 1986).

[21] Chr. Tamm and C.O. Weiss, "Bistability and optical switching of spatial patterns in a laser," J. Opt. Soc. Am. B **7**, 1034–1038 (1990).

[22] L. Allen, M.W. Beijersbergen, R.J.C. Spreeuw, and J.P. Woerdman, "Orbital angular-momentum of light and the transformation of laguerre-gaussian laser modes," Phys. Rev. A **45** 8185818 (1992).

[23] J. Courtial, M.J. Padgett, "Performance of a cylindrical lens mode converter for producing Laguerre-Gaussian laser modes," Opt. Commun. **159**, 13–18 (1999).

[24] A.A. Malyutin, "Tunable astigmatic $\pi/2$ mode converter," Quantum Electronics **34**, 172–174 (2004).

[25] T. Kuga, Y. Torii, N. Shiokawa, T. Hirano, Y. Shimizu, and H. Sasada, "Novel optical trap of atoms with a doughnut beam," Phys. Rev. Lett. **78**, 4713–4716 (1997).

[26] H.C. Liang, Y.J. Huang, Y.C. Lin, T.H. Lu, Y.F. Chen, and K.F. Huang, "Picosecond optical vortex converted from multigigahertz self-mode-locked high-order Hermite-Gaussian Nd:GdVO4 lasers," Opt. Lett. **34**, 3842–3844 (2009).

[27] Matthew Whitrock and John Noé, "Beam profile of a Helium Neon laser," August 2004 (unpublished report). http://laser.physics.sunysb.edu/∼matt/2004-05/beam-report/

[28] Dmitri V. Petrov, Fernando Canal, Lluis Torner, "A simple method to generate optical beams with a screw phase dislocation," Opt. Commun. **143**, 265–267 (1997).

[29] George S. Monk, *Light: Principles and Experiments*, (McGraw-Hill, New York,1937)

[30] H. Kogelnik and T. Li, "Laser beams and resonators," Appl. Opt. **5** 1550–1567 (1966).

[31] B.E.A. Saleh and M.C. Teich, *Fundamentals of Photonics*, (John Wiley, New York, 1991).

[32] Sidney A. Self, "Focusing of spherical Gaussian beams," Appl. Opt. **22**, 658–661 (1983).