Universal Quantum Gates for Order N=1 OAM States

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Abstract: Universal single-qubit quantum logic gates acting on order N=1 orbital angular momentum states, based on SU(2) Euler rotations and spatial and polarization gate analogues, are investigated. The universal polarization gate analogue was constructed and analyzed.

OCIS Codes: (270.5585) Quantum information and processing

1. Mode-Converters as Rotation Operators, Wave Plate analogues

The most arbitrary Astigmatic Mode-Converter (AMC), used to transform orbital angular momentum (OAM) modes of light, has two degrees of freedom – the angular setting α of its primary axis in the plane normal to the light's direction of propagation, and the relative Gouy phase χ [1] inserted between the two orthogonal Hermite-Gauss eigenmodes of that rotated coordinate system. By having the angular setting α set to 0 ($\pi/2$) from vertical, and varying the phase χ, the $\boldsymbol{R}_Z(\chi)=\boldsymbol{e}^{-i\sigma_Z\chi/2}$ ($\boldsymbol{R}_X(\chi)=\boldsymbol{e}^{-i\sigma_X\chi/2}$) rotation operator can be made. Also, an AMC with χ-setting of $\pi/2$ (π) and at variable transverse angle α, transforms the Hermite-Gauss eigenmodes HG_{10} and HG_{01} in a fashion exactly analogous to how the vertical and horizontal polarization states are transformed by a variable Quarter-Wave Plates (QWPs) (Half-Wave Plates (HWPs)).

2. Quantum Logic Gate Design, Construction, and Analysis

With the previously mentioned forms of AMCs we can make various combinations which are able to perform arbitrary state transformations for all order N=n+m=2p+|l|=1 OAM states, based on SU(2) Euler rotations for the rotation operators $R_{x,y,z}$, and based on the analogous QWP-HWP-QWP polarization gate for the forms similar to waveplates [2]. In this experiment the latter was constructed, with the choice based on the dearth of optical elements necessary and the flexibility of changing the gate settings. Shown in Fig. 1, this universal quantum gate based on polarization consists of two $\pi/2$ Mode-Converters with primary axes rotated at variable transverse angles sandwiching a single variable-angle π Mode-Converter (realized in this experiment by a Dove prism). With the input state to our quantum gate being the HG_{10} mode, we observed state transformations to HG_{01}, the rotated Hermite-Gauss eigenmodes $\boldsymbol{HG}_{10}^{+45/+135}=\frac{1}{\sqrt{2}}\left(\left|\boldsymbol{HG}_{10}^{0}\right\rangle \mp \left|\boldsymbol{HG}_{01}^{0}\right\rangle\right)$, the Laguerre Gauss eigenmodes $\boldsymbol{LG}_{0}^{\pm l}=\frac{1}{\sqrt{2}}\left(\left|\boldsymbol{HG}\right\rangle \mp \left|\boldsymbol{HG}_{01}\right\rangle\right)$, and a fourth basis set as well.

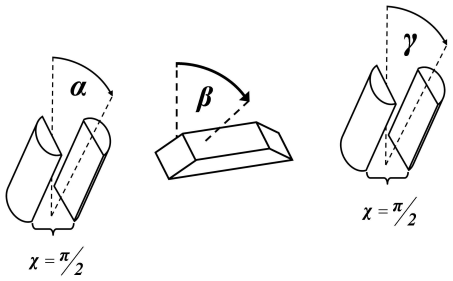
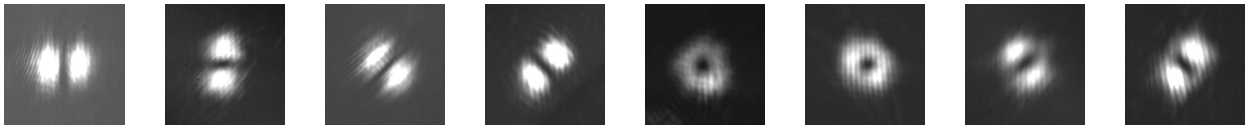


Fig. 1. Physical design for a universal quantum logic gate acting on the bipartite order N=1 orbital angular momentum (OAM) states of photons, based on the universal polarization gate consisting of two variable Quarter-Wave Plates sandwiching a variable Half-Wave Plate. In the OAM analogue, two $\pi/2$ Mode-Converters (represented by the cylindrical lenses of an Astigmatic Mode-Converter, with mode-matching spherical lenses not shown) with primary axes rotated at variable transverse angles α and γ are sandwiching a single π Mode-Converter rotated at a variable angle, which is shown here as the equivalent variable Dove prism.

The intensity profiles of these modes and the associated operation matrices of the logic gate are shown below.



$\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$ $\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$ $\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ -1 & 1\end{pmatrix}$ $\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}$ $\frac{1}{\sqrt{2}}\begin{pmatrix}1 & -i\\ -i & 1\end{pmatrix}$ $\frac{1}{\sqrt{2}}\begin{pmatrix}1 & i\\ i & 1\end{pmatrix}$ $\frac{1}{\sqrt{2}}\begin{pmatrix}1 & -i\\ -i & 1\end{pmatrix}$ $\frac{1}{\sqrt{2}}\begin{pmatrix}1 & i\\ i & 1\end{pmatrix}$

3. Summary

We constructed and analyzed the performance of a universal single-qubit quantum logic gate acting on order N=1 OAM states of light, based on the analogous polarization gate consisting of a QWP-HWP-QWP sandwich.

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