My Foray into Moiré: Exploring the Mathematics of Visual Overlap Patterns

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Background

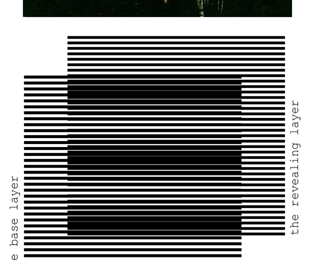
My interest in moiré effects dates from a group lunch at the Simons Center Cafe, where my mentor pointed out some strange, wavy lines on the screens covering a heater. The patterns shifted dramatically with small changes in the viewing angle, creating an ethereal appearance. I soon learned that those lines are moiré patterns.

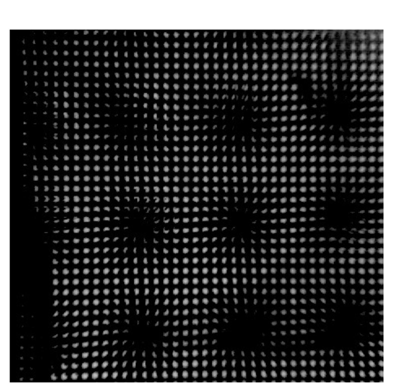
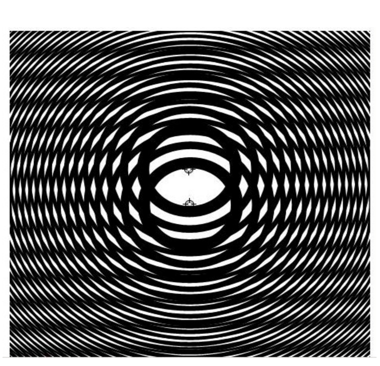
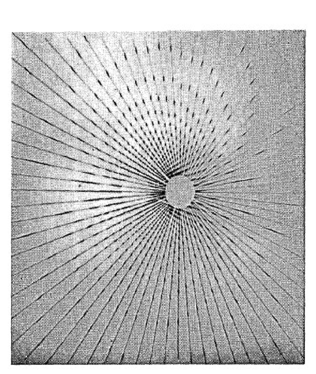

Moiré patterns are observed when two objects with partially-transparent patterns, such as window screens or chain-link fences, overlap. The surreal hazy lines that appear result from the differing amounts of visual overlap (transparency or opacity) of the two patterns when one is tilted with respect to the other or has a different period. Moiré-like effects have been known since the Middle Ages, when fabrics such as watered silk were revered for the rippled "watermarks" created by their unique weave.

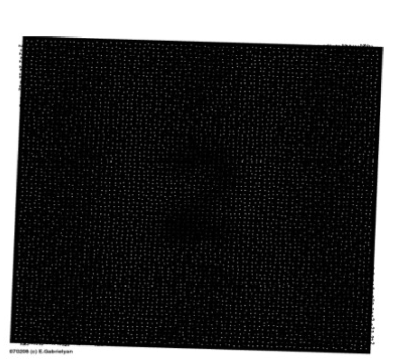
Examples

Dot grating: Squares of metal with holes punched out in a periodic manner.

Varied gratings: Moiré can be observed with many types of gratings.


Linear (Gabrielyan, 2007)



Fresnel Zone Plate

Radial
(Oster *et al.*, 1964)


Random moiré in the shape of a "2" (Gabrielyan, 2007)

Random moiré: Periodicity is not a requirement for the moiré effect. *Glass* patterns can be made to have any shape.

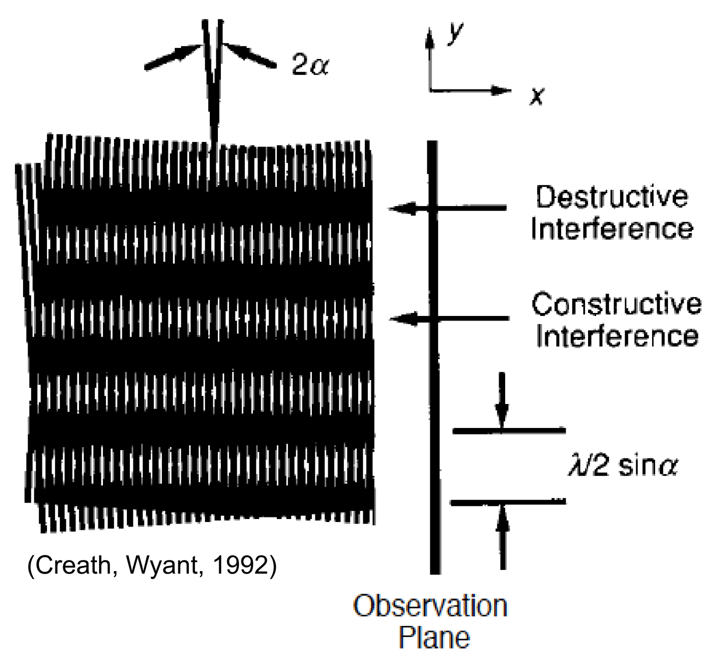

Glass pattern (Glass, Pérez, 1973)

Simulations

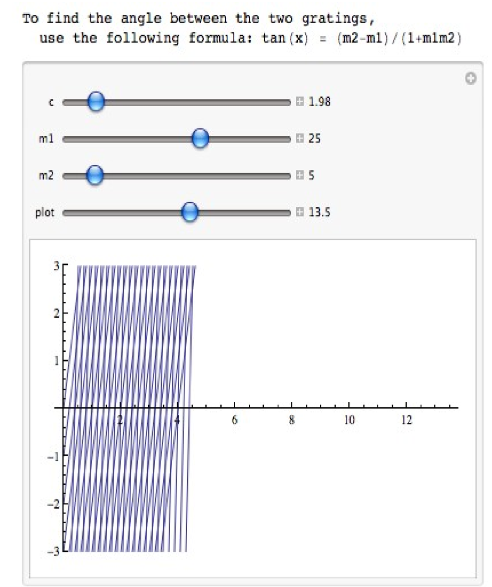
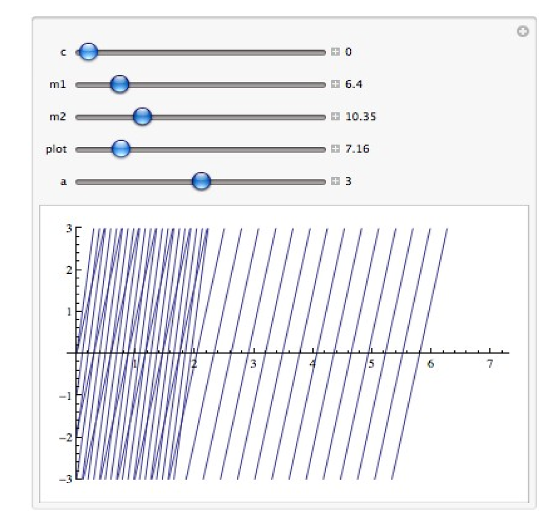
We explored a variety of moiré effects and ways to create them in *Mathematica*. Some gratings studied are not linear or strictly periodic (quasiperiodic). Periodic gratings include lines, concentric ellipses, concentric circles, and dot arrays. Models were animated using the *manipulate* tool to allow for user interaction. The resulting interactive simulations are good educational tools for teaching mathematical concepts.

Linear Model

$$\phi_1(x, y) - \phi_2(x, y) = \frac{2\pi}{\lambda_{beat}} x \cos\alpha + \frac{4\pi}{\lambda} y \sin\alpha \qquad \lambda_{beat} = \frac{\lambda_1\lambda_2}{\lambda_2 - \lambda_1}.$$



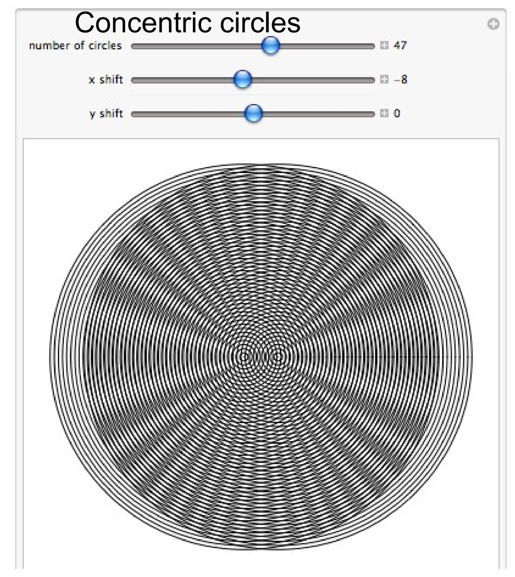
(Creath, Wyant, 1992)

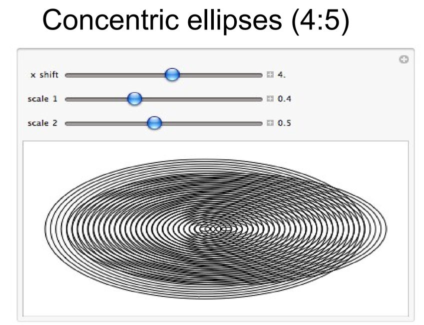



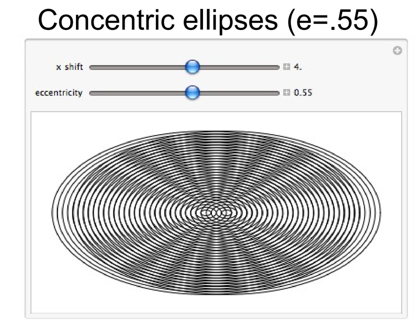
$\lambda_1 = \lambda_2 = \lambda$ $M\lambda = 2y\sin\alpha$ $\lambda_1 = 3\lambda_2$

```
Manipulate[Plot[Table[{y = m1 x - (m1/m2)*(k + c), y = m2 x - k}, {k, 0, 20}], {x, 0, plot}, PlotRange -> 3], {c, 0, 19,  Appearance -> "Labeled"},
{m1, 1, 40, Appearance -> "Labeled"}, {m2, 1, 40, Appearance -> "Labeled"}, {plot, 5, 20, Appearance -> "Labeled"}]
```

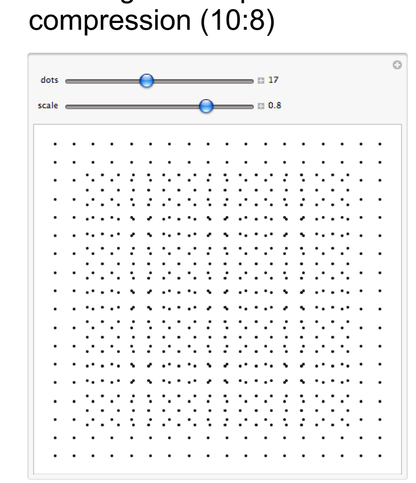
Other Models

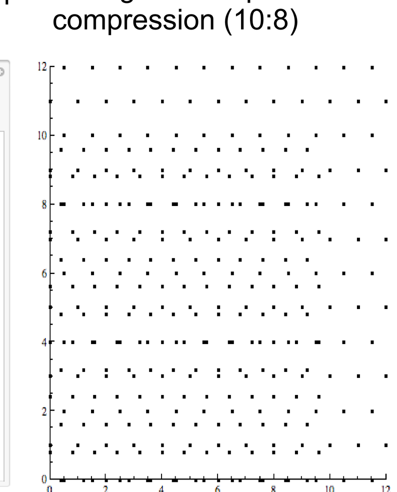
Concentric circles


Concentric ellipses (4:5)


Concentric ellipses (e=.55)


```
Manipulate[Graphics[Table[{Circle[{0, 0}, r1], Circle[{h, k}, r1]}, {r1, 0, n}]], {{n, 1, "number of circles"}, 1, 80, 1, Appearance -> "Labeled"}, {{h, 0, "x shift"}, -80, 80, 1, Appearance -> "Labeled"}, {{k, 0, "y shift"}, -80, 80, 1, Appearance -> "Labeled"}]
```

Rectangular dot pattern: even compression (10:8)


Hexagonal dot pattern: even compression (10:8)


```
Manipulate[Graphics[Table[Point[{{x, y}, {k x + (1 - k)*(a - 1)/2, k y + (1 - k)*(a - 1)/2}}], {x, 0, a - 1}, {y, 0, a - 1}]], {{a, 1, "dots"}, 1, 41, 1, Appearance -> "Labeled"}, {{k, .1, "scale"}, .1, 1, .05, Appearance -> "Labeled"}]
```

```
a= 1
Graphics[Table[Point[{{a n1/2, 2 a m1}, {a n2, 2 a m2}, {.8 a n1/2, .8*2 a m1}, {.8 a n2, .8*2 a m2}}], {n1, 1, 24, 2}, {m1, 0, 6, 1}, {n2, 0, 12, 1}, {m2, 1/2, 6, 1}], Axes -> True, PlotRange -> {{0, 12}, {0, 12}}]
```

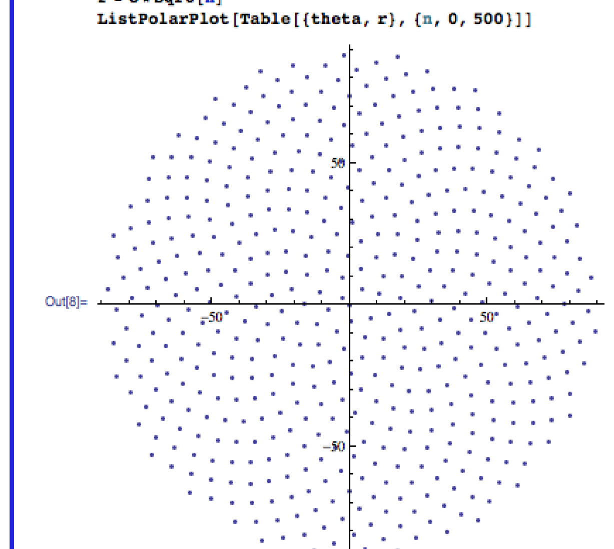
Quasiperiodic Model

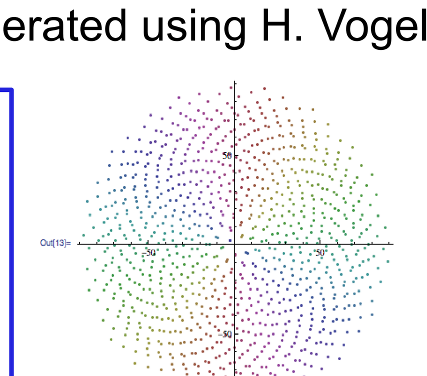
Perhaps the most famous example of a quasiperiodic structure is the Fibonacci sequence: the fruitlets of a pineapple, the scales of a pine cone, and the florets of a sunflower all follow the Fibonacci sequence (1, 1, 2, 3, 5, ...). Our quasiperiodic model consisted of an array of dots arranged like florets on a sunflower and was generated using H. Vogel's formula (2004).

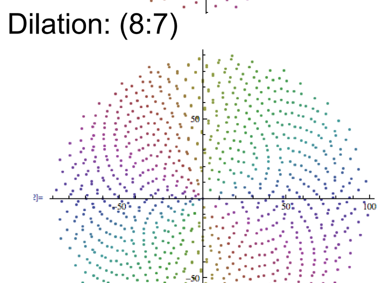
$$r = c\sqrt{n}$$

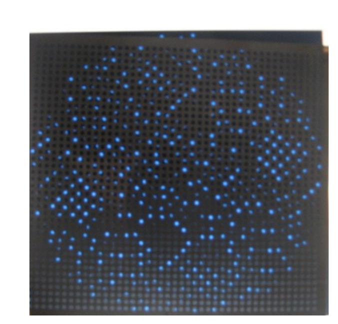
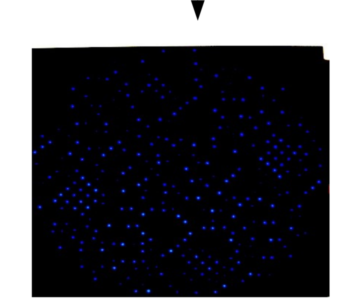
$$\Theta = \frac{2\Pi n}{\phi^2} \approx 137.5n \quad \text{in degrees}$$

```
In[5]:= c = 4
theta = 2 Pi*n/2.61803
r = c*Sqrt[n]
ListPolarPlot[Table[{theta, r}, {n, 0, 500}]]
```




Dilation: (8:7)


Shift

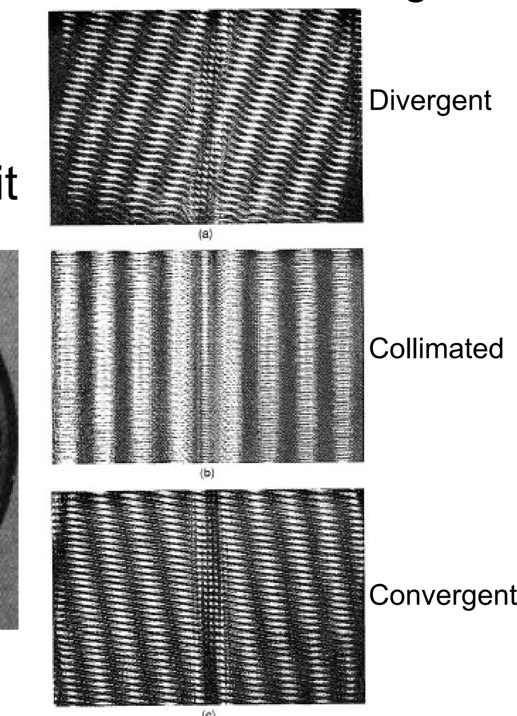
Applications

•Topography

•Focal length measurement

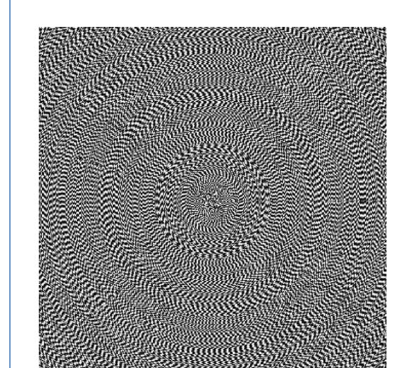
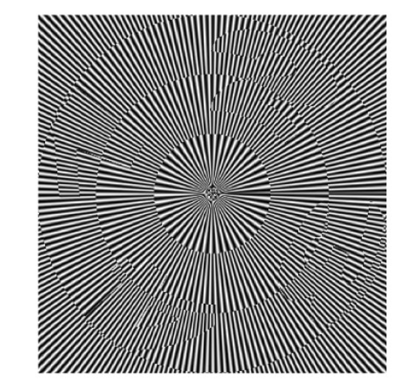
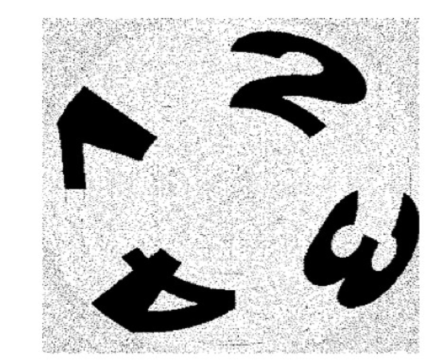
Navigation


Anti-counterfeit


Collimation testing


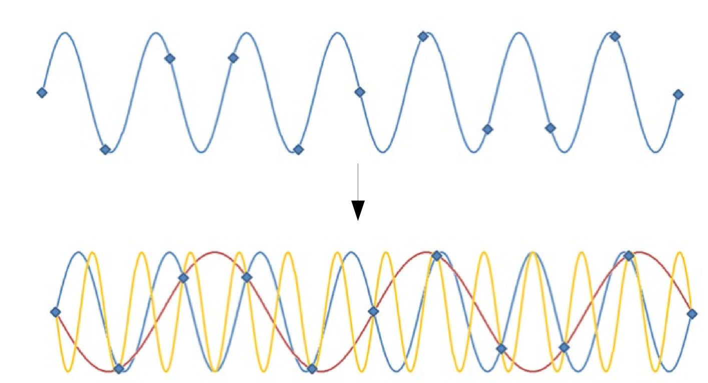
(Kothiyal and Sirohi, 1987)

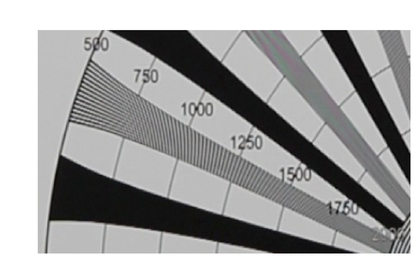
Steganography
(Ragulskis *et al.*, 2009)

Undesireable effects

Temporal Aliasing



Spatial Aliasing


Future Work

Creating these patterns by computer simulation is not the same as having a mathematical expression for them. In the future, we would like to better understand and extend previous work of this type, such as that of Oster *et al.* (1964) and Creath and Wyant (1992). We are especially interested in quasiperiodic gratings, such as the sunflower floret pattern we modeled.

References

[1] K. Creath and J. C. Wyant, "Moiré and fringe projection techniques," in Optical Shop Testing, D. Malacara, ed. (Wiley, 1992), pp. 653-660.

[2] G. Oster *et al.*, "Theoretical interpretation of moiré patterns," J. Opt. Soc. Am. **54**, 169-175 (1964).

[3] H. Vogel, "A better way to construct the sunflower head". Mathematical Biosciences **44**, 179-189 (1979).