The Moiré Effect

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Quick outline

- What are moiré patterns?
- How do you make them?
- Some simulations.
- Moiré in real life: problems and applications
- Future research and thoughts.

“moiré”

- latin? → arabic? → english → french → **english**

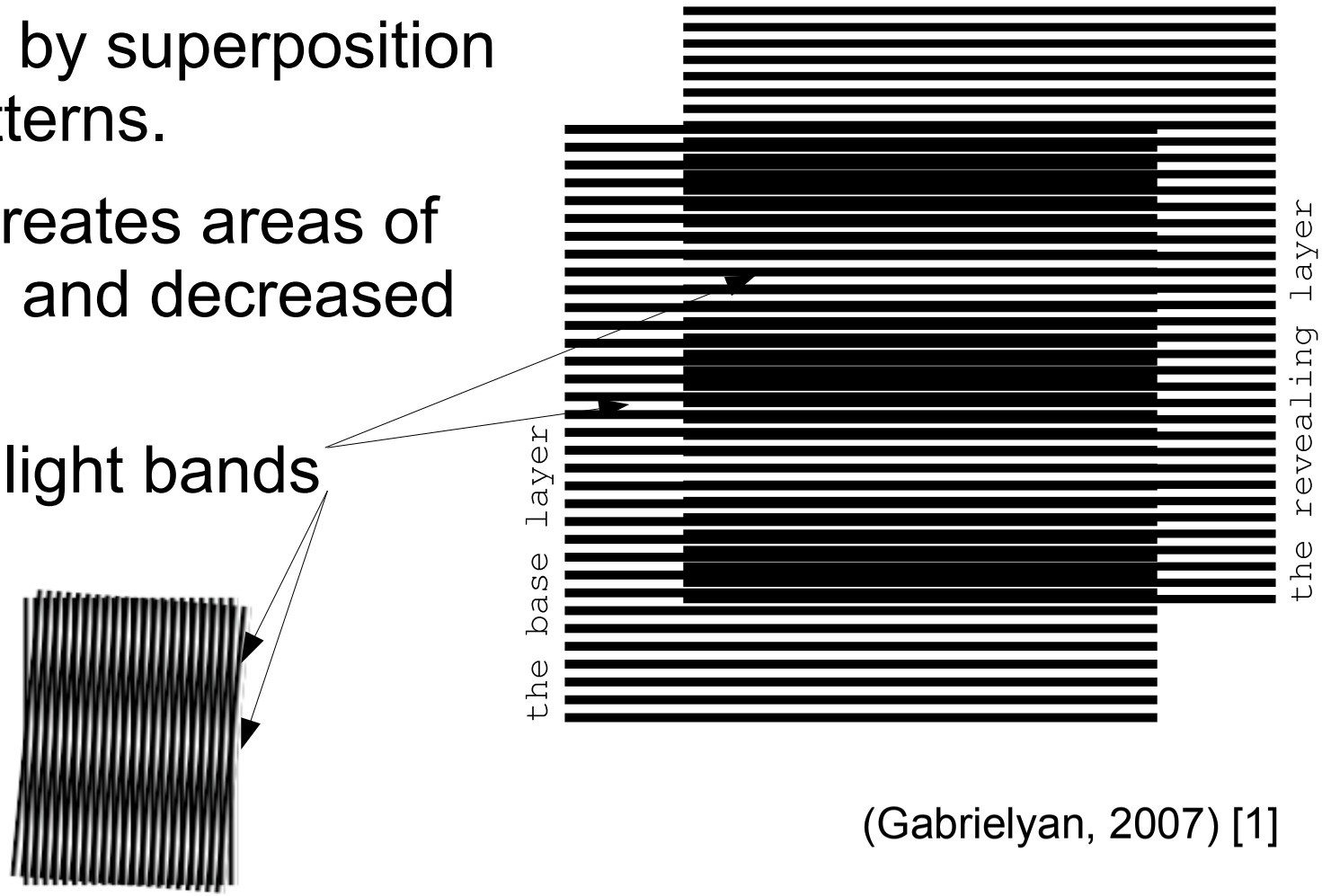
Moiré in art

Peter I of Russia
1838, Paul Delaroche

Porträt der Prinzessin di Sant' Antimo
1840-1844, Francesco Hayez

How they are produced

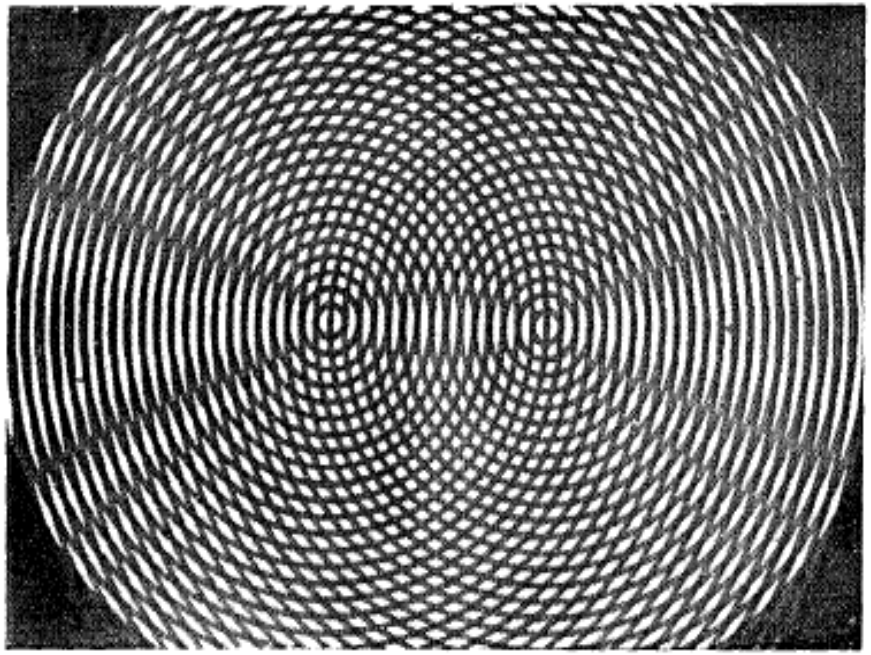
- Produced by superposition of two patterns.
- Overlap creates areas of increased and decreased intensity.
- Dark and light bands



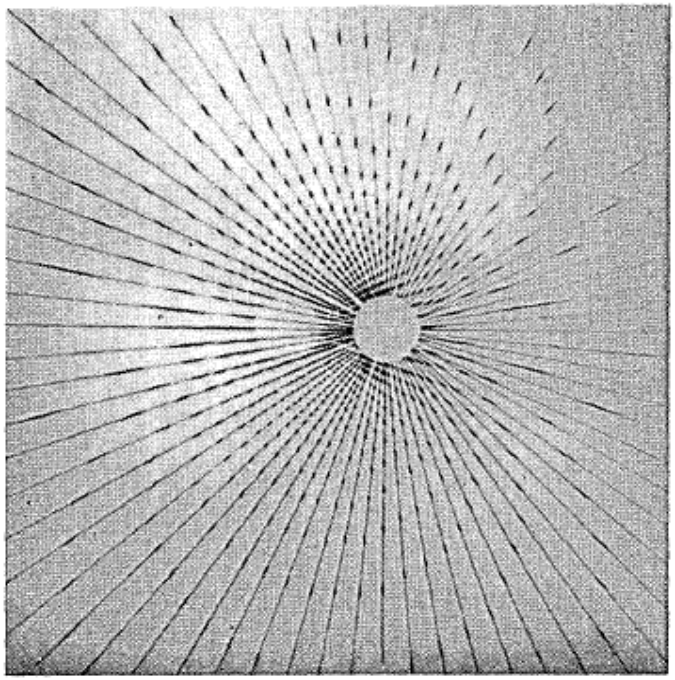
(Gabrielyan, 2007) [1]

Grating types

- Simplest is linear. However, not limited:

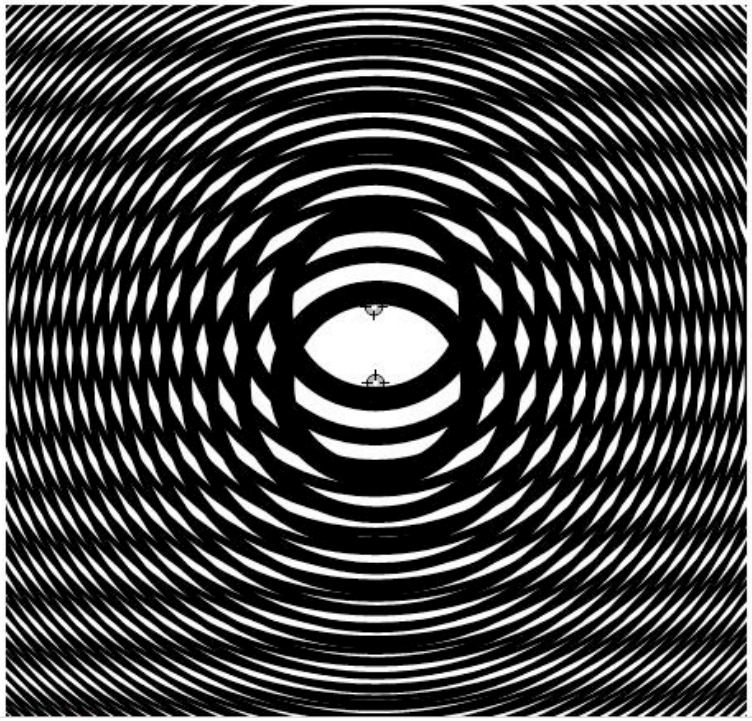


Circular

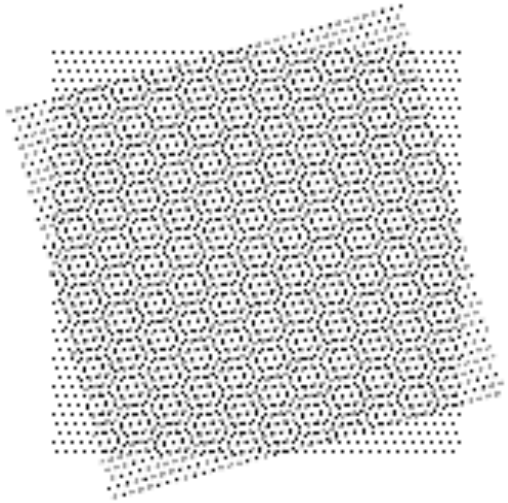


Radial

(Oster *et al.*, 1964) [2]



Fresnel Zone Plate

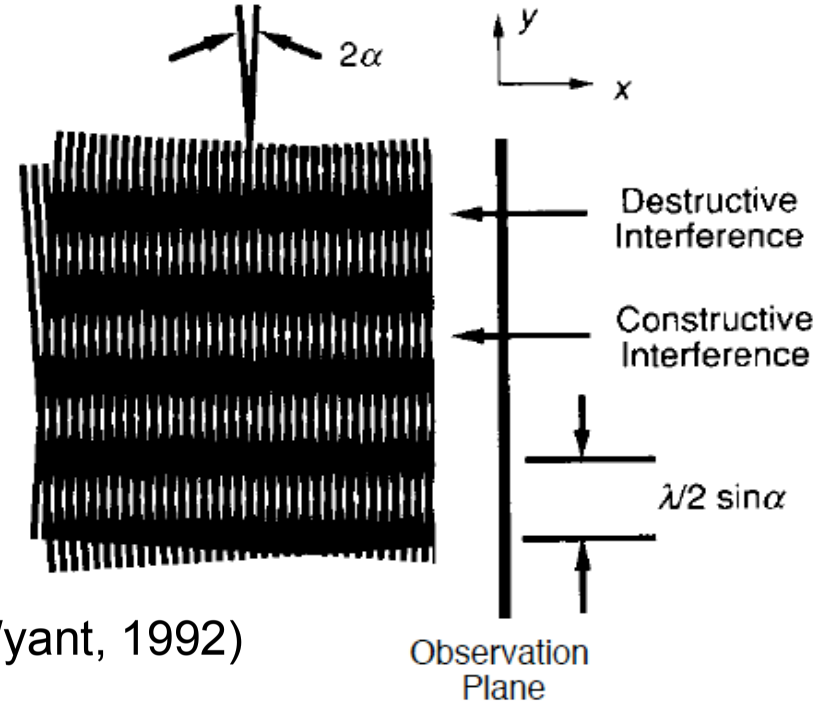


Grid

Random

(Glass, Pérez, 1973) [3]

Some mathematics: Linear Grating



(Creath, Wyant, 1992) [4]

$$\lambda_{beat} = \frac{\lambda_1 \lambda_2}{\lambda_2 - \lambda_1}.$$

Beat wavelength

$$\phi_1(x, y) - \phi_2(x, y) = \frac{2\pi}{\lambda_{beat}} x \cos \alpha + \frac{4\pi}{\lambda} y \sin \alpha,$$

Special case #1

- If $\lambda_1 = \lambda_2 = \lambda$, the first term of the equation vanishes:

$$\phi_1(x, y) - \phi_2(x, y) = \frac{2\pi}{\lambda_{\text{beat}}} x \cos \alpha + \frac{4\pi}{\lambda} y \sin \alpha,$$

Special case #1

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Special case #1

- If $\lambda_1 = \lambda_2 = \lambda$, the first term of the equation vanishes:

$$\phi_1(x, y) - \phi_2(x, y) = \frac{4\pi}{\lambda} y \sin \alpha,$$

- This can be rewritten as: $\boxed{M\lambda = 2y \sin \alpha,}$

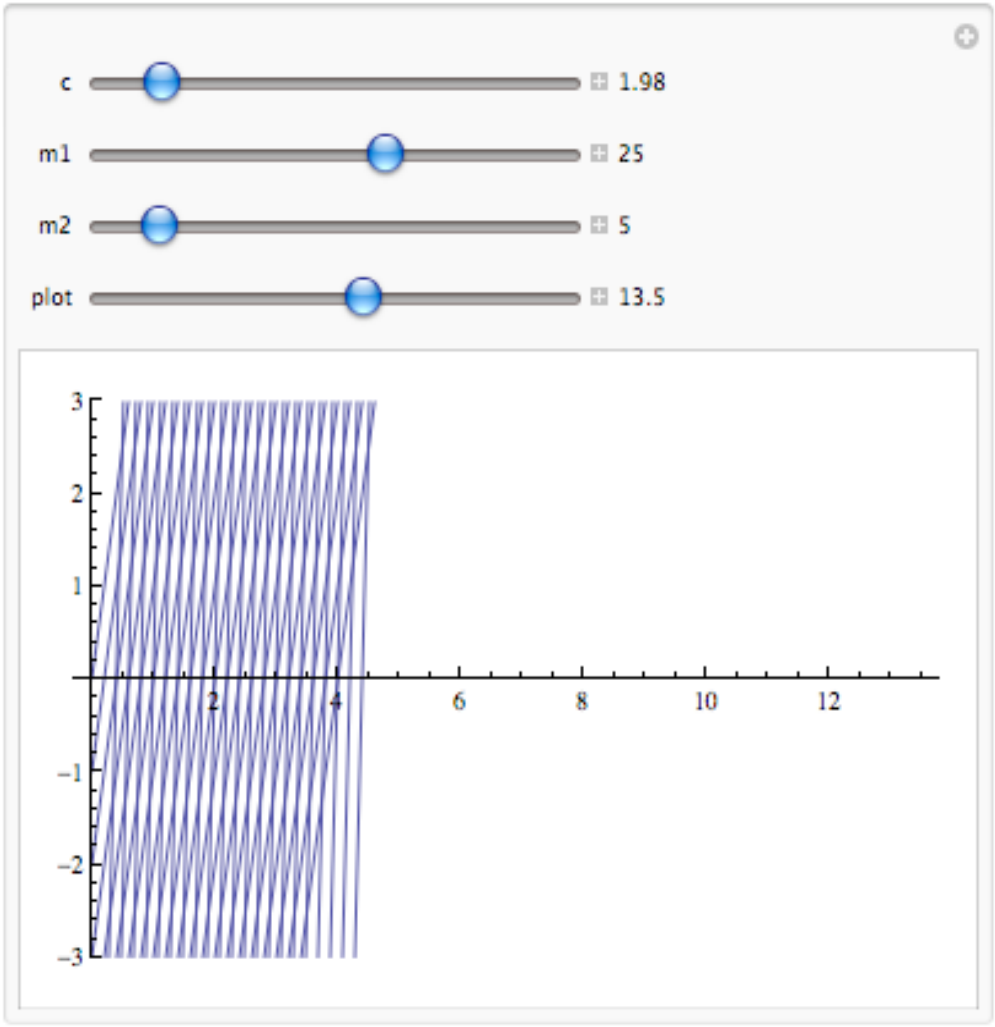
M: fringe order

y: distance between two fringes

α: half of angle between gratings

← My model of case #1

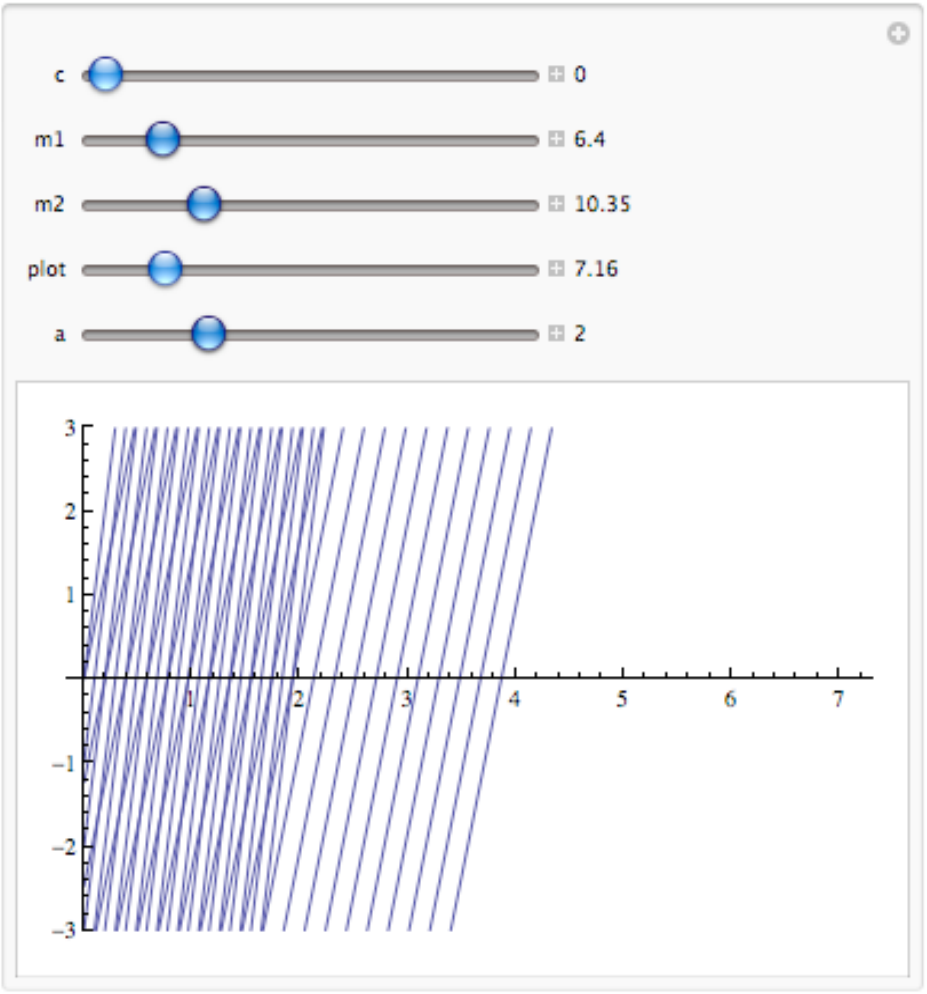
To find the angle between the two gratings,
use the following formula: tan(x) = (m2-m1)/(1+m1m2)

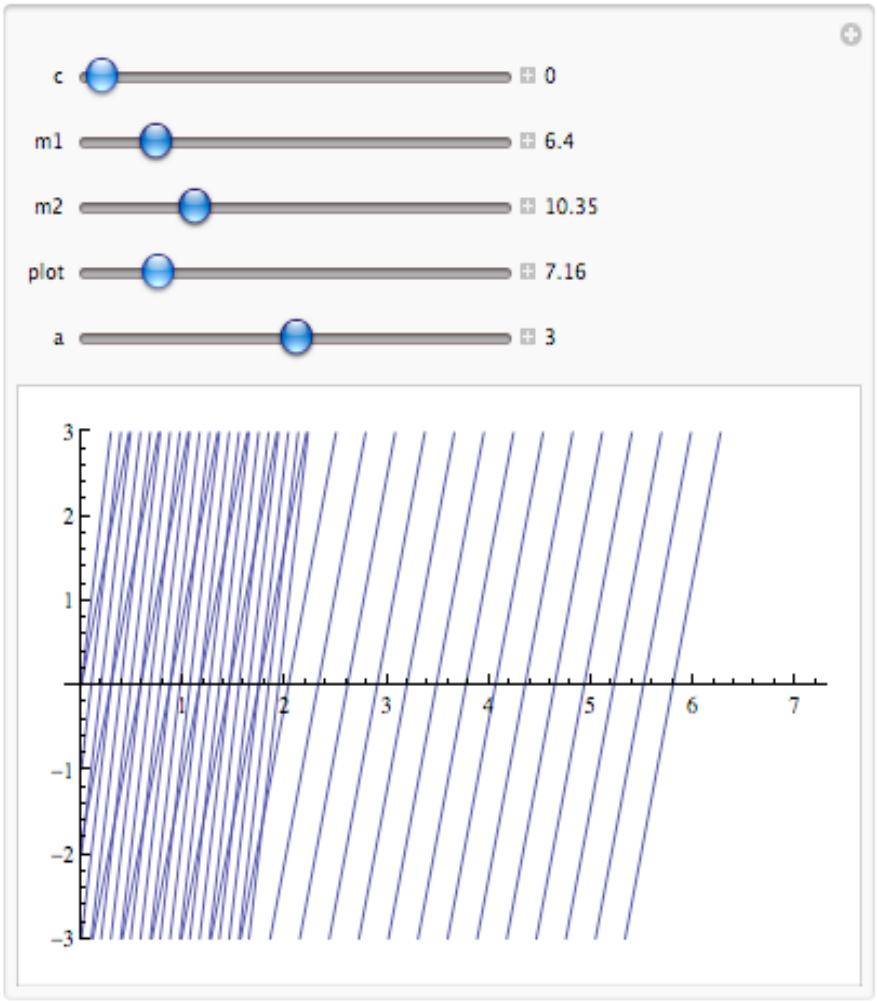


- If the periods are equal, the fringes are horizontal.
- If one period is an integer multiple of the other, fringes are still horizontal!
- Unexpected, because the x dependence does not vanish in that case.

```
Plot[Table[{y = m1 x - (m1/m2)*(k + c),
y = m2 x - k}, {k, 0, 20}]
```

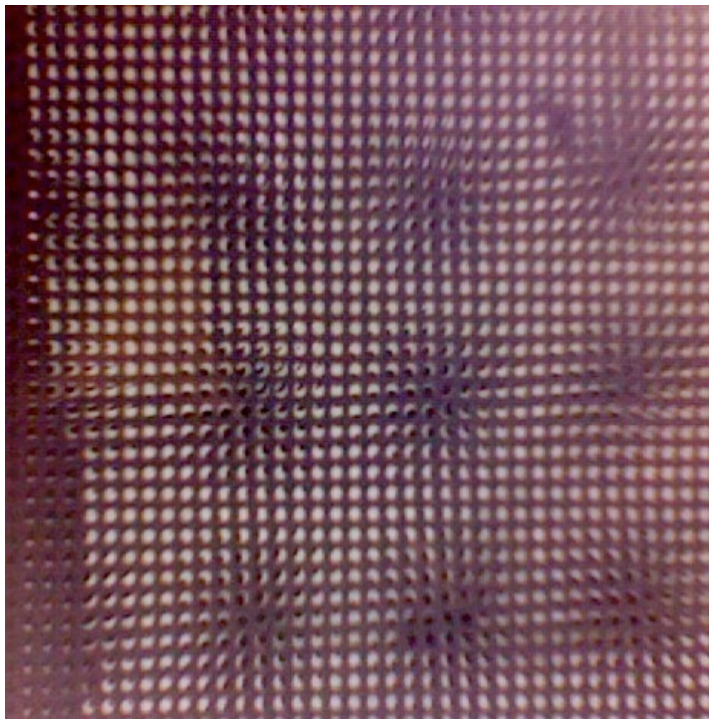
When $\lambda_1 = k\lambda_2$ k in {Z+}



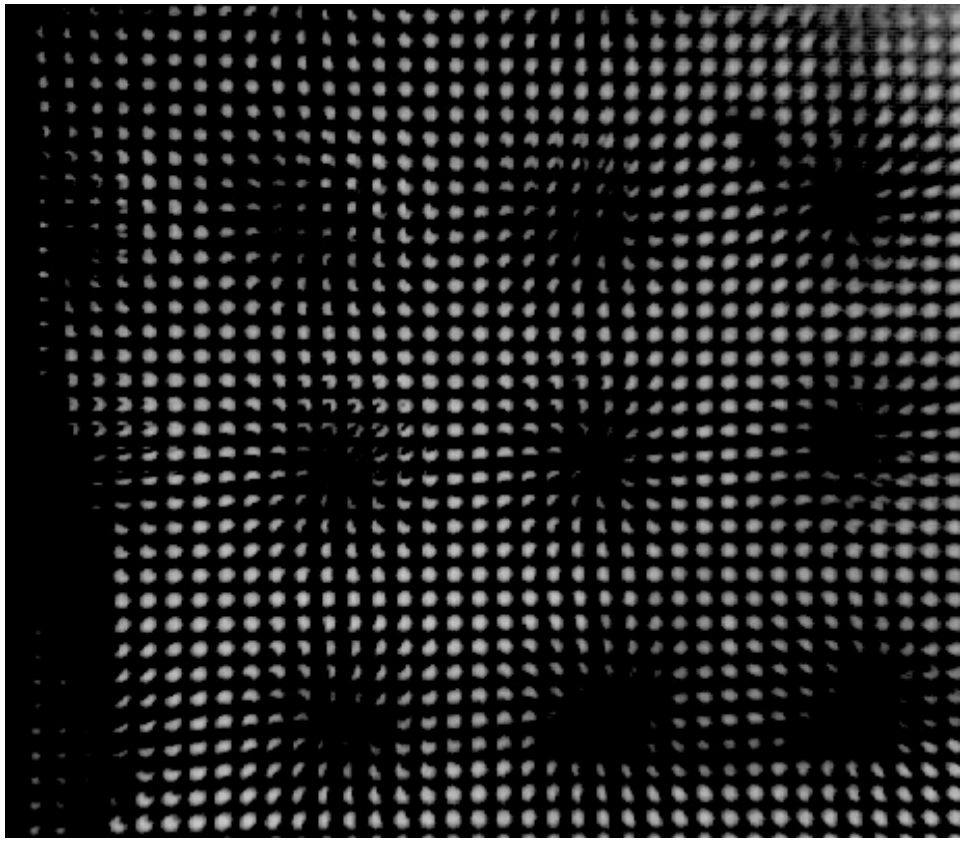


All controls same as previous, **a is scale factor of gratings.**

Dot patterns

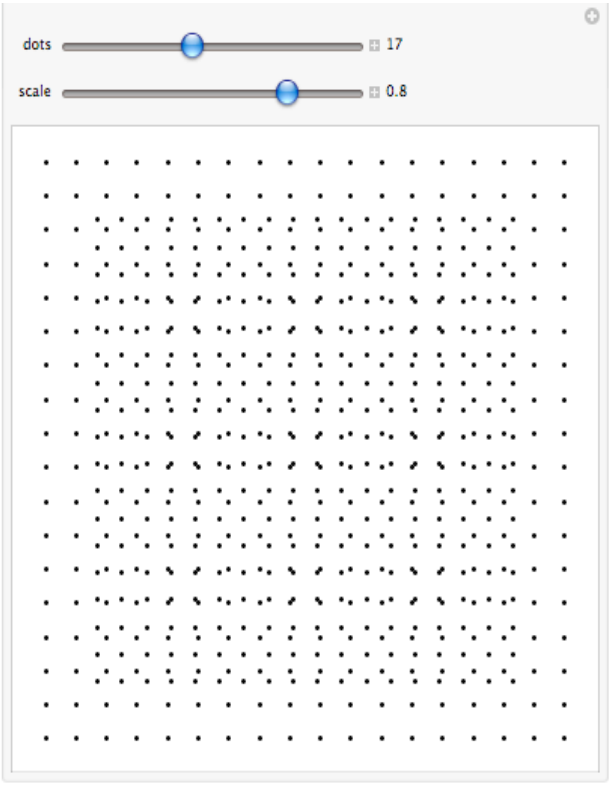


Enhanced contrast

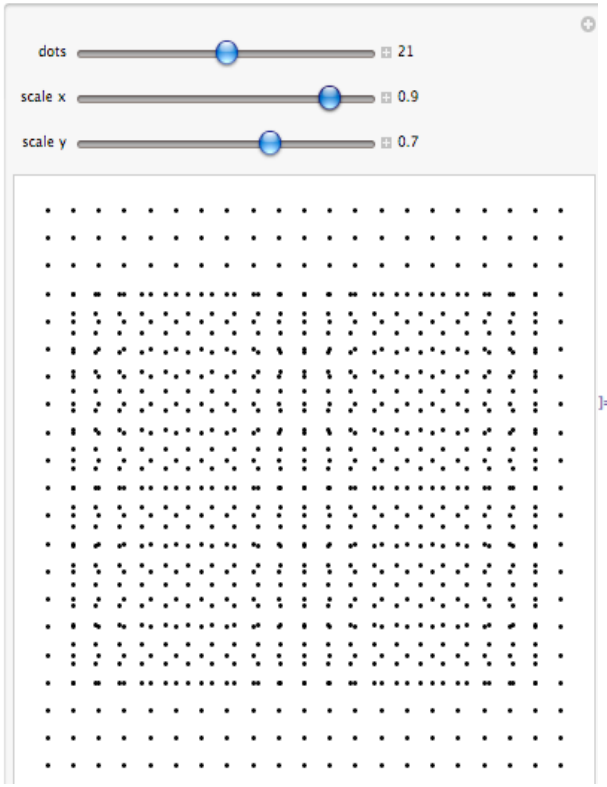


Moiré effect produced with grids of equal period. Grating is magnified.

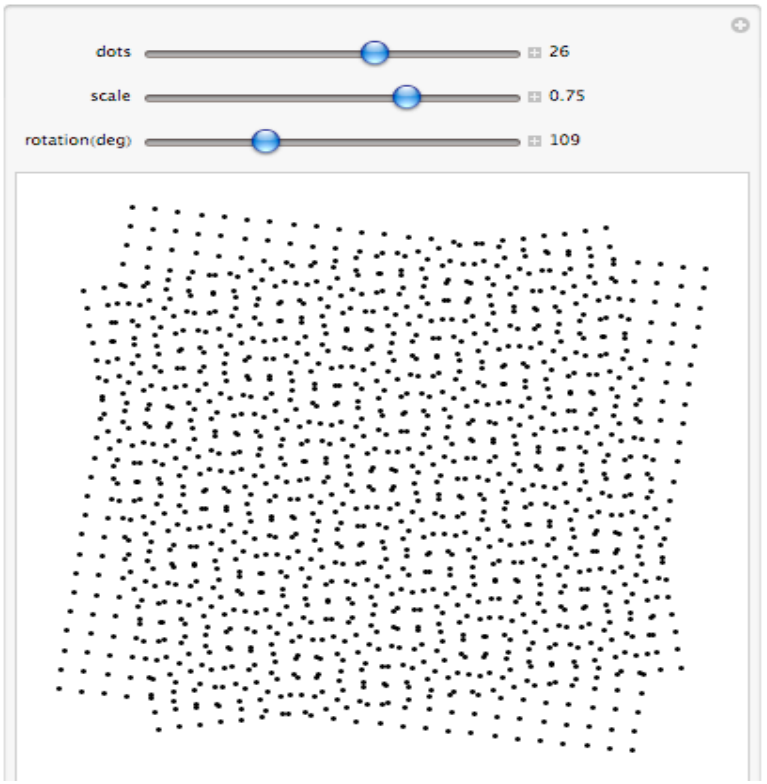
More dot patterns



Compression:
(x, y) → (0.8x, 0.8y)

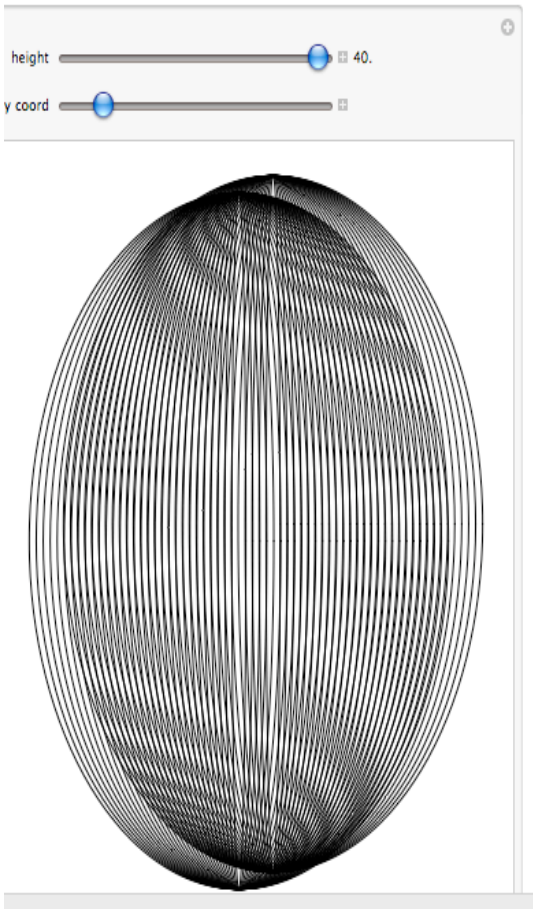


Compression:
(x, y) → (0.9x, 0.7y)



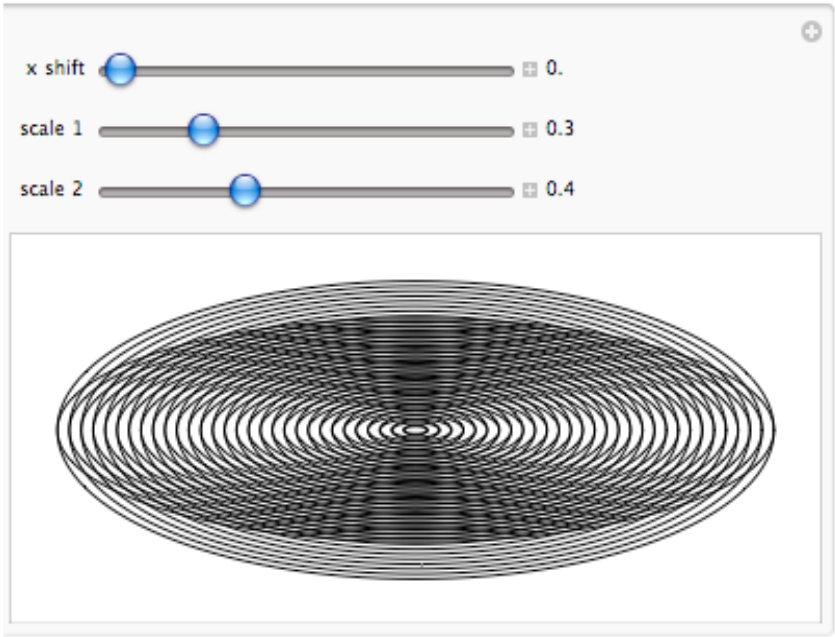
Rotation:
Unknown-- Mathematica trouble

Ellipses



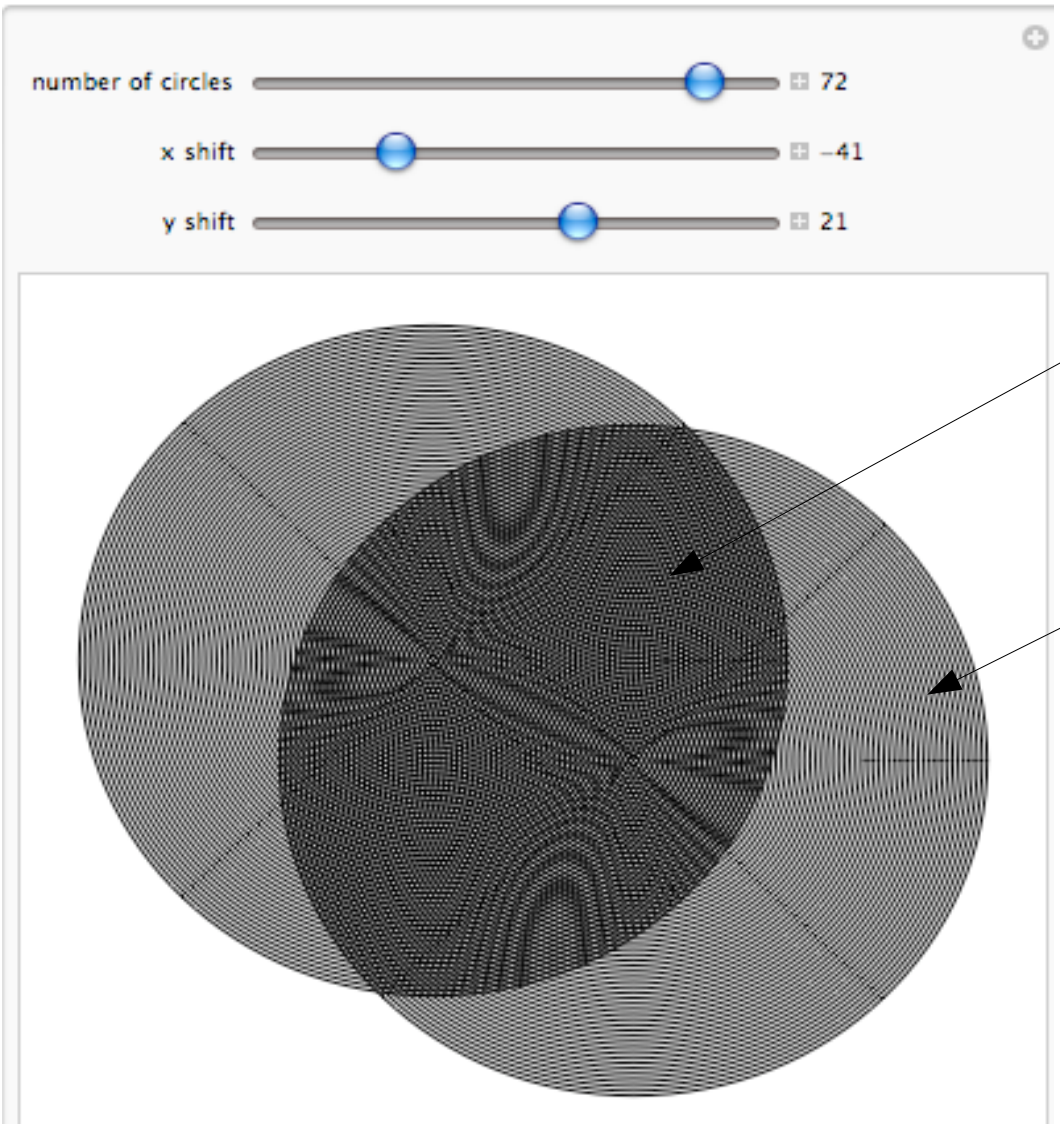
a=constant ;
b = changing

Measure eccentricity?



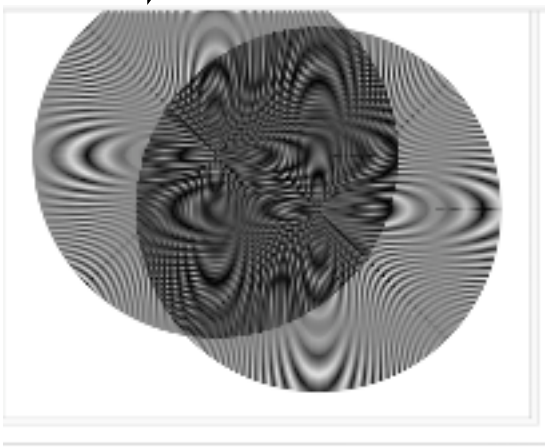
a, b changing in fixed ratio

Another simulation



moiré

not moiré



Aliasing

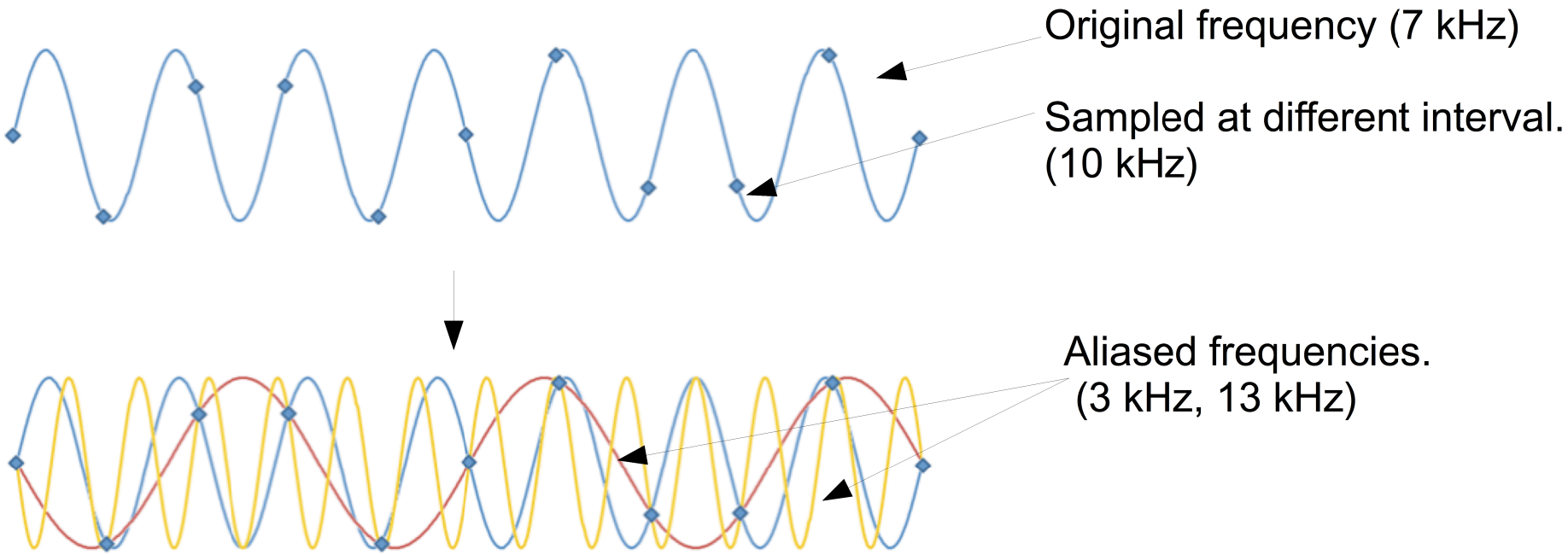
- Detail of pattern is more than computer (or camera) can resolve with pixels.
- So, it only samples once every couple of pixels--gives us incomplete, false image!
- Not really moiré in the previous sense, but looks like it.

Temporal aliasing-- images

- “Wagon-wheel effect” in old movies.
- Due to aliasing-- image of wheel is sampled at different frequency than that of wheel's rotation.
- Wheels appear to spin backwards, slowly, or not at all.
- Example of the “Stroboscopic effect.”

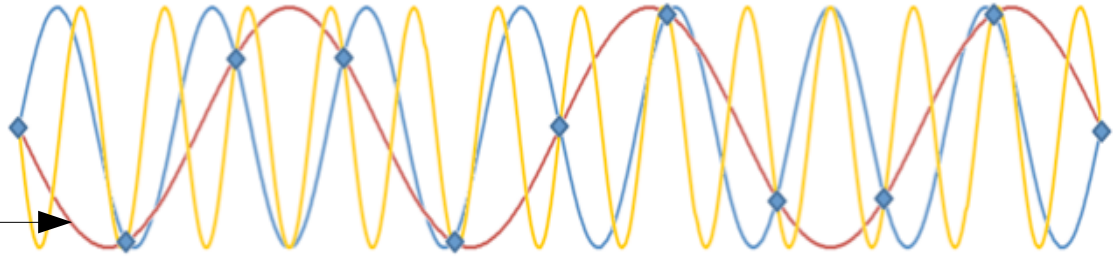
Temporal aliasing-- sound

- Similar effect happens in recording.



The Nyquist frequency

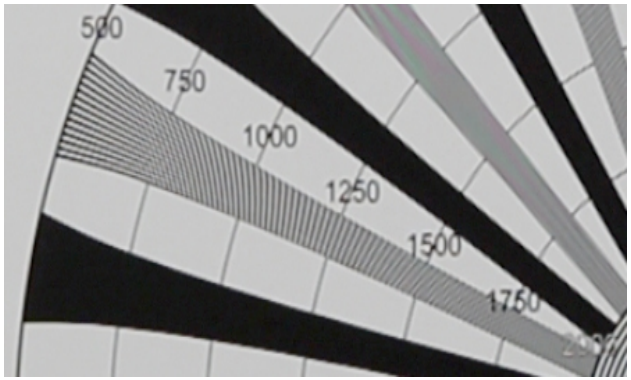
However, there is only one frequency that is between 0 and half the sampling frequency (5 kHz) -- the 3 kHz wave.



- No lower frequency wave can be constructed-- 3 kHz wave is free from aliasing.
- *Nyquist frequency* is sampling frequency / 2.
- All waves with a frequency less than the Nyquist frequency are free from aliasing.

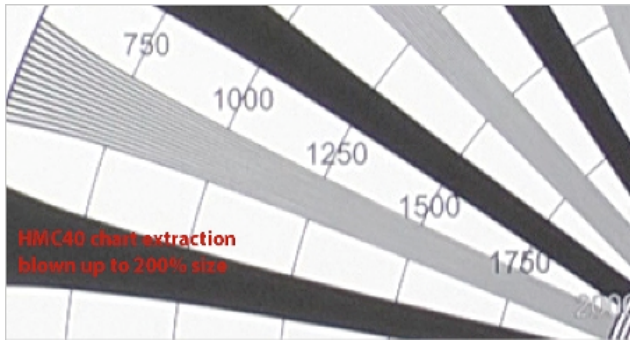
Spatial aliasing

- Spatial aliasing is a similar phenomenon. Imagine a zebra against a picket fence.
- Cameras with “high resolution” try to resolve intricate patterns. Result = aliasing.



High res. → aliased

Computer generated image



Conventional video camera
→ blur, but no alias

Anti-aliasing filters

- AA filters reduce resolution of camera so aliasing is mostly eliminated.
- Some cameras don't have the filters, so they offer higher (false) resolution.


```
drwxr-xr-x@  15 ariana  staff    510 Apr 13 09:18 CTY-Calculus
drwx------+   8 ariana  staff    272 Mar  4 16:58 Music
drwxr-xr-x   12 ariana  staff    408 Jan 28 15:08 HTML
drwxr-xr-x   11 ariana  staff    374 Dec 18  2011 Buzzer_Live
drwxr-xr-x+   5 ariana  staff    170 Sep 10  2011 Public
drwxr-xr-x+   5 ariana  staff    170 Sep 10  2011 Sites
ariana-rays-macbook:~ ariana$
```

```
drwxr-xr-x@  15 ariana  staff    510 Apr 13 09:18 CTY-Calculus
drwx------+   8 ariana  staff    272 Mar  4 16:58 Music
drwxr-xr-x   12 ariana  staff    408 Jan 28 15:08 HTML
drwxr-xr-x   11 ariana  staff    374 Dec 18  2011 Buzzer_Live
drwxr-xr-x+   5 ariana  staff    170 Sep 10  2011 Public
drwxr-xr-x+   5 ariana  staff    170 Sep 10  2011 Sites
ariana-rays-macbook:~ ariana$
```

Not all bad...

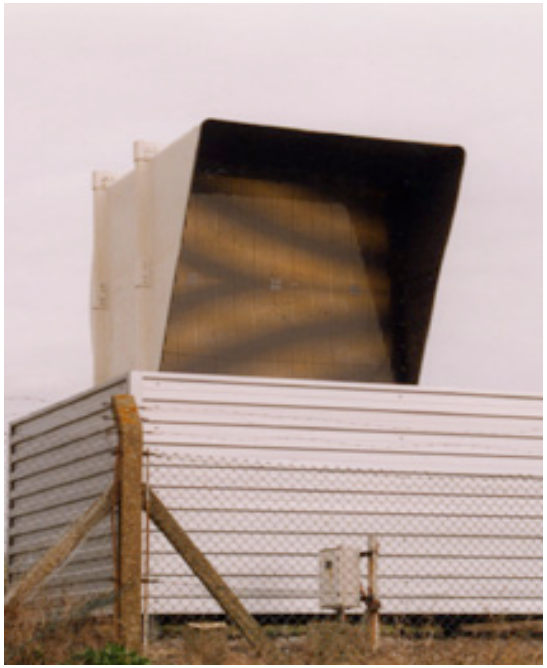
- Moiré patterns have many useful applications. Here are some:
  - Strain measurement (topography)

    (Takasaki, 1970)
  - Currency (anti-counterfeit)
  - **Collimation testing**

    (Kothiyal and Sirohi, 1987)
  - Navigation
  - **Steganography**

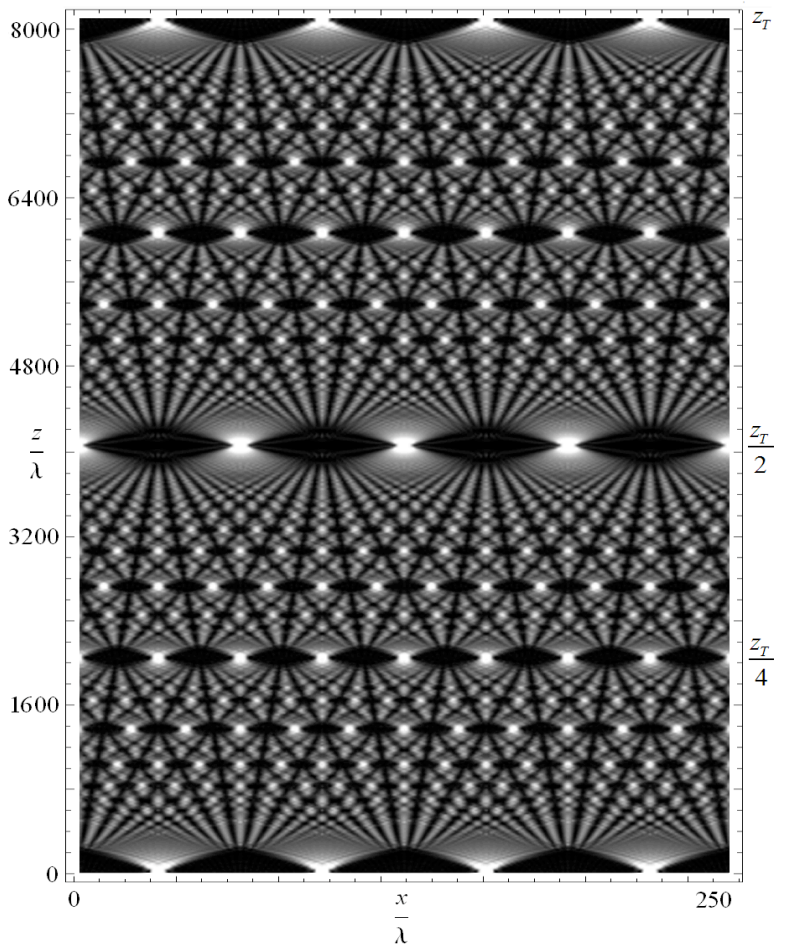
    (Ragulskis *et al.*, 2009)
  - Find focal length of lens

    (Nakano and Murata, 1985)



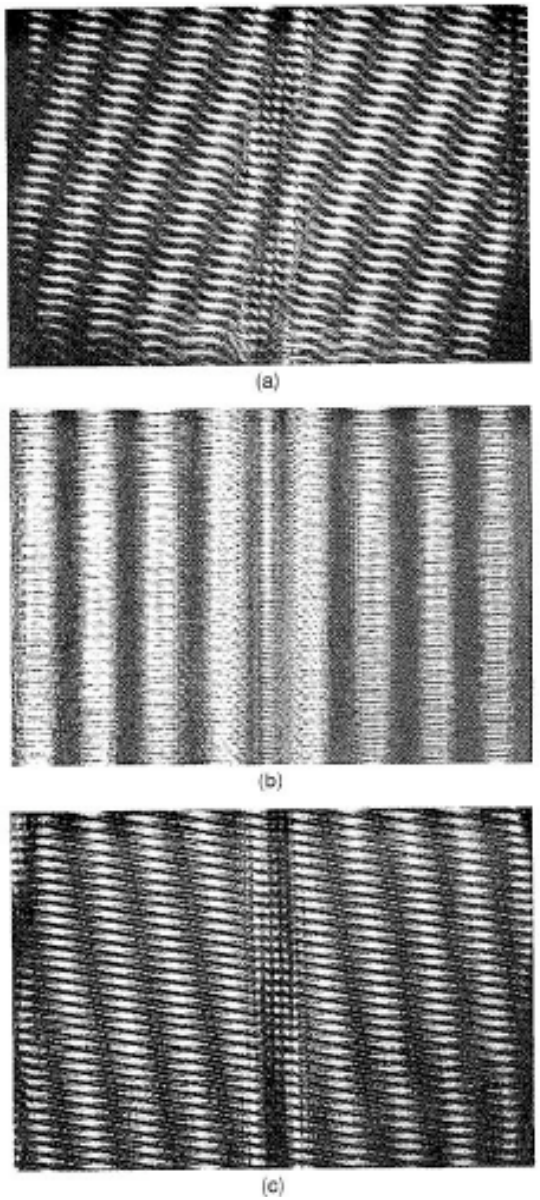
Collimation testing with moiré (and Talbot)

- Talbot effect: near-field diffraction effect. Light shines through a periodic grating; at certain distances (Talbot length), the image of the grating is reproduced.
- At half Talbot length, grating is shifted by ½ grating period.



Continued

- Light shone through periodic grating, grating moved to Talbot length to make self-image.
- Identical grating placed on top of image of grating, so moiré patterns are observed.
- If the light is collimated, moiré patterns will be parallel to grating. If light is convergent, moiré patterns will tilt one way; if divergent, patterns will tilt the other way.



Divergent

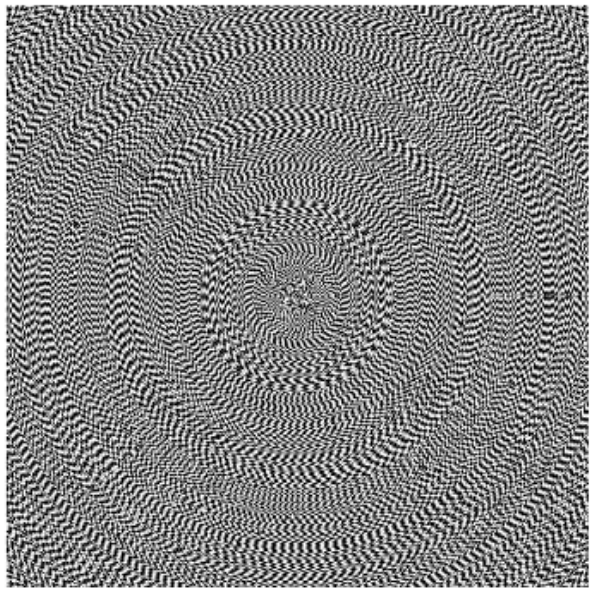
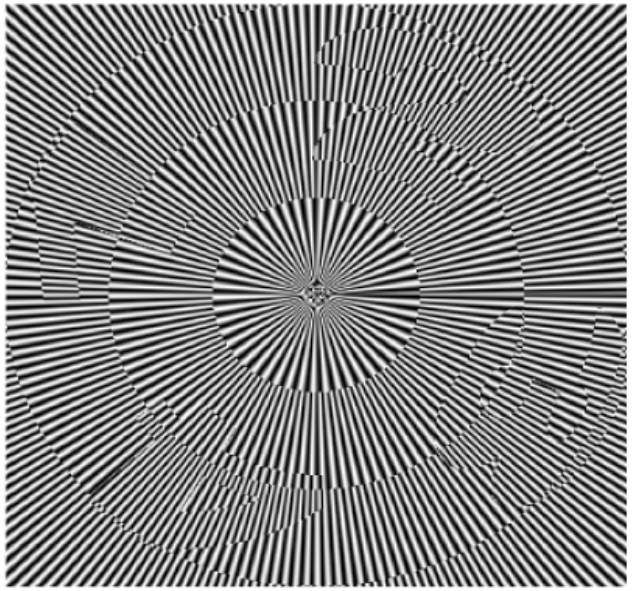
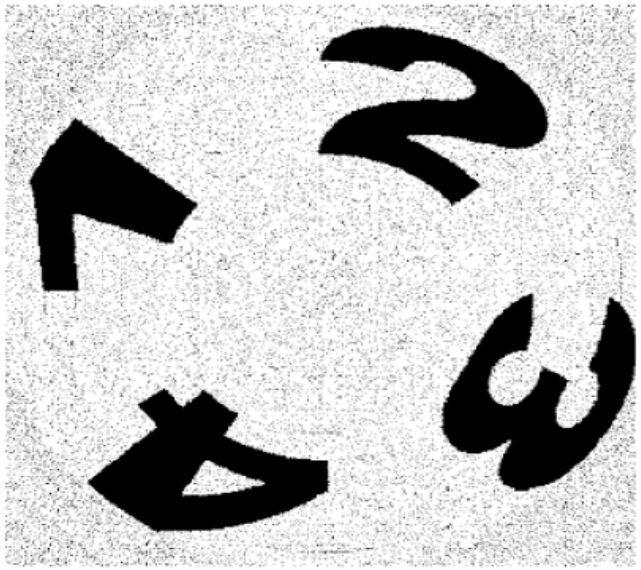
Collimated

Convergent

(Kothiyal and Sirohi, 1987) [5]

Steganography

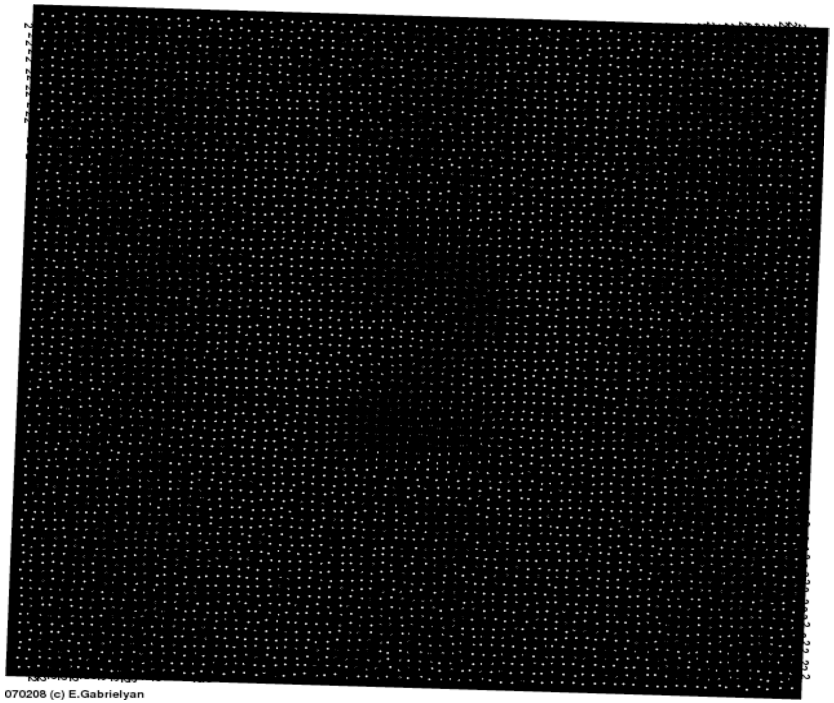
- Moiré patterns can also be used in steganography, or image-hiding.
- When the “wheels” are oscillated at the correct frequency, images appear.

(Ragulskis *et al.*, 2009) [6]

Something else really awesome

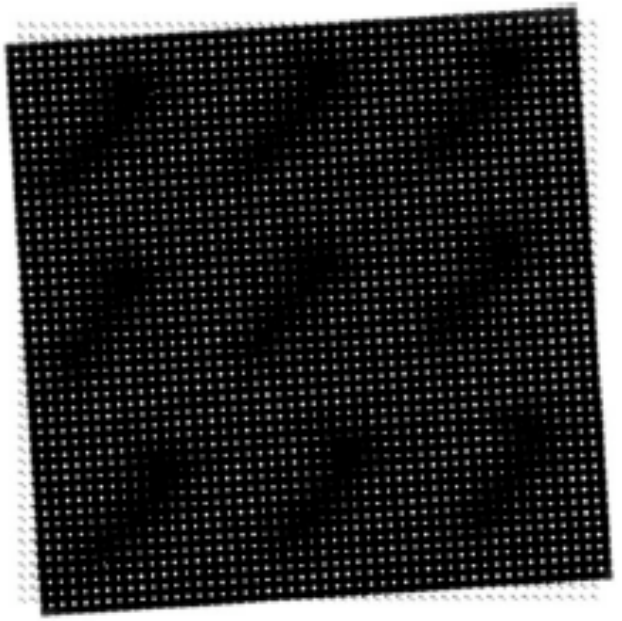
- A Glass pattern is the superposition of two identical, random dot patterns.
- It was discovered in 2003 that Glass patterns can be made to have any shape and intensity profile!



(Gabrielyan, 2007) [7]

Random

Periodic



(Amidror, 2003) [8]

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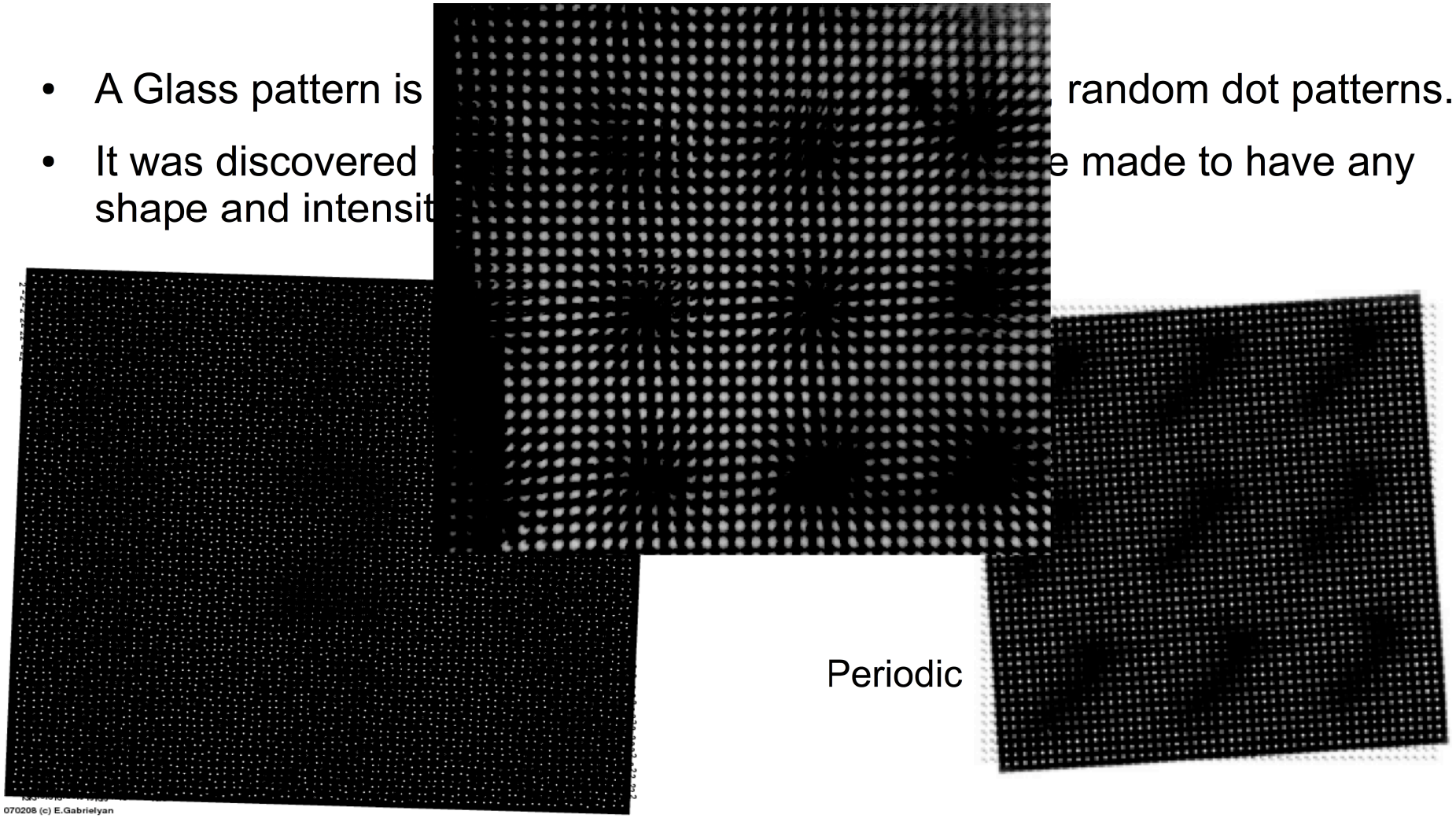


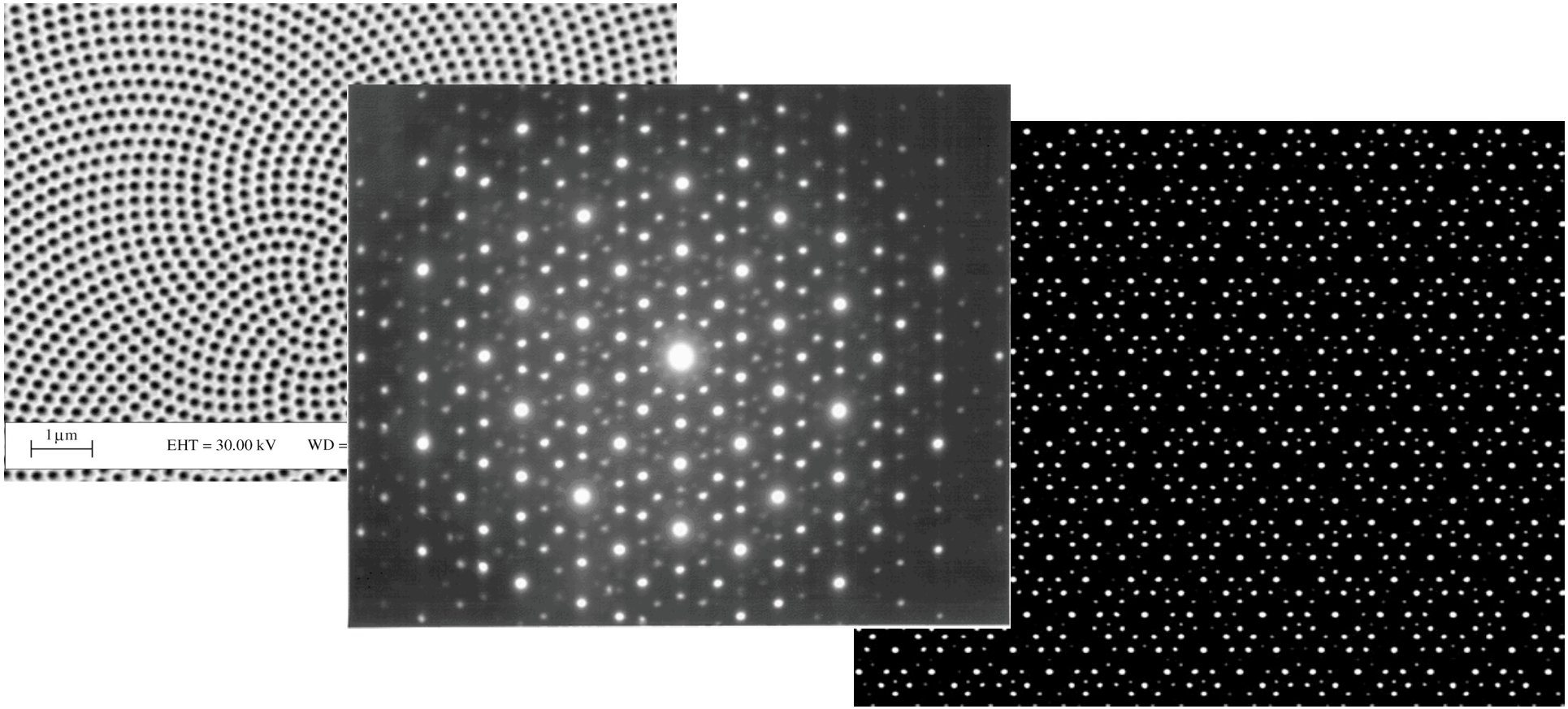
Periodic

(Gabrielyan, 2007) [7]

(Amidror, 2003) [8]

Quasiperiodic?

- What would moiré patterns / Glass patterns look like when the dots are quasiperiodic?



Sunflowers and the Fibonacci sequence

- In 2004, H. Vogel came up with an improved formula for the pattern that sunflower florets make.

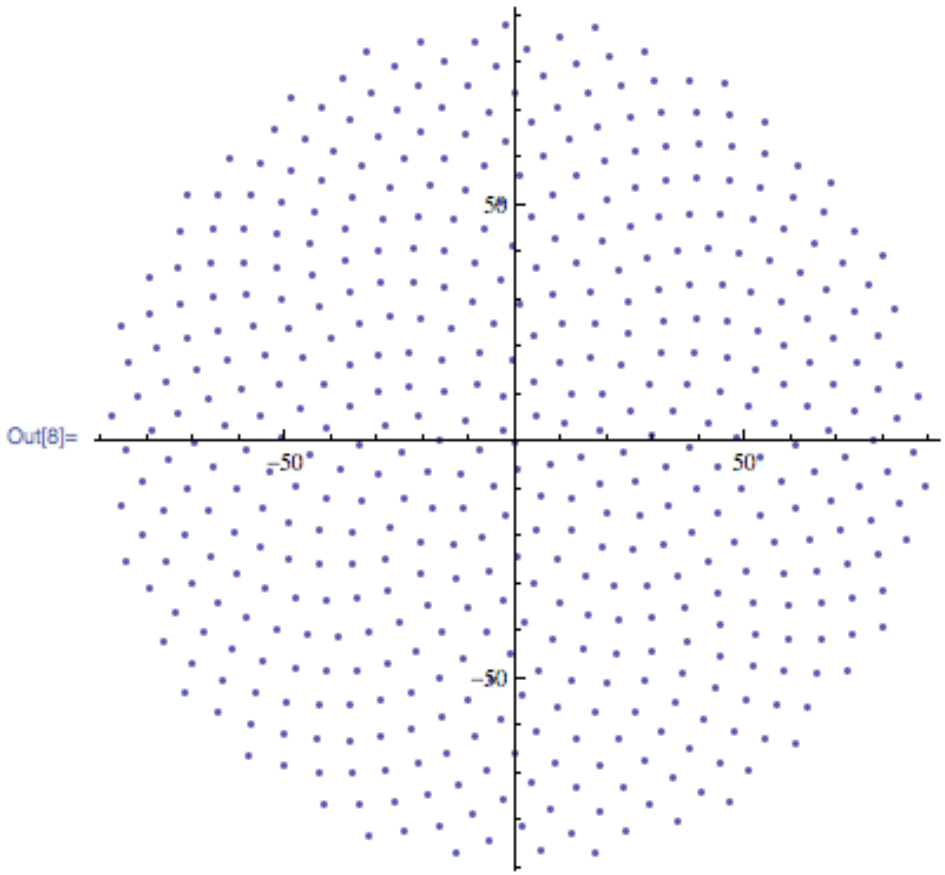
•This formula was based on the Fibonacci sequence.

•Fibonacci numbers often appear in nature.

My model of sunflower florets

In[5]:=
```
c = 4
theta = 2 Pi * n / 2.61803
r = c * Sqrt[n]
ListPolarPlot[Table[{theta, r}, {n, 0, 500}]]
```

Out[8]=

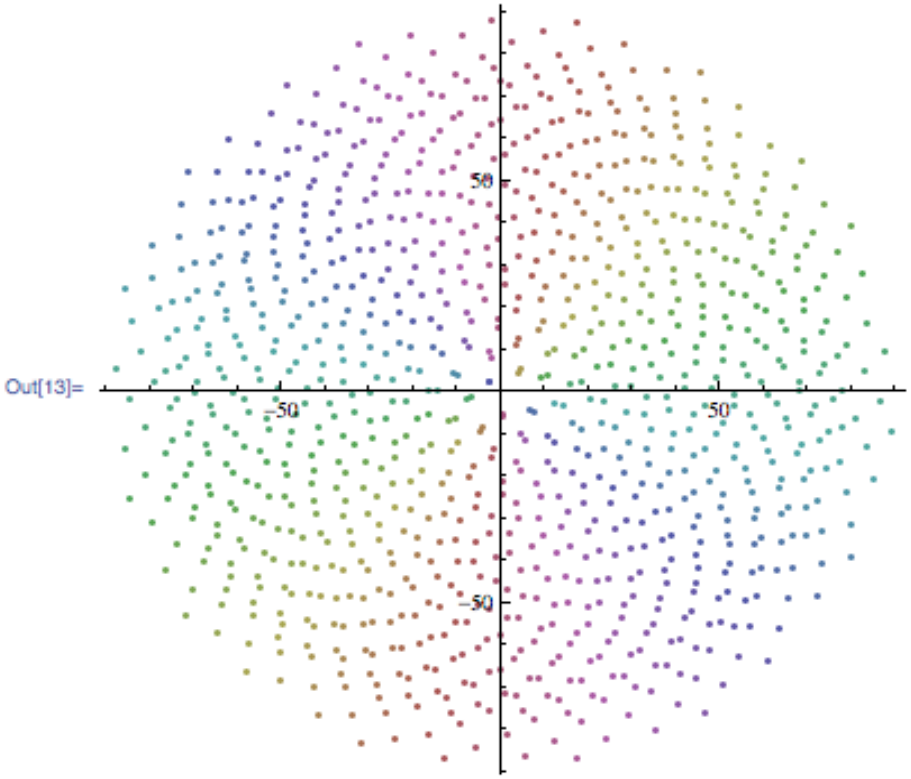


Vogel's formula:

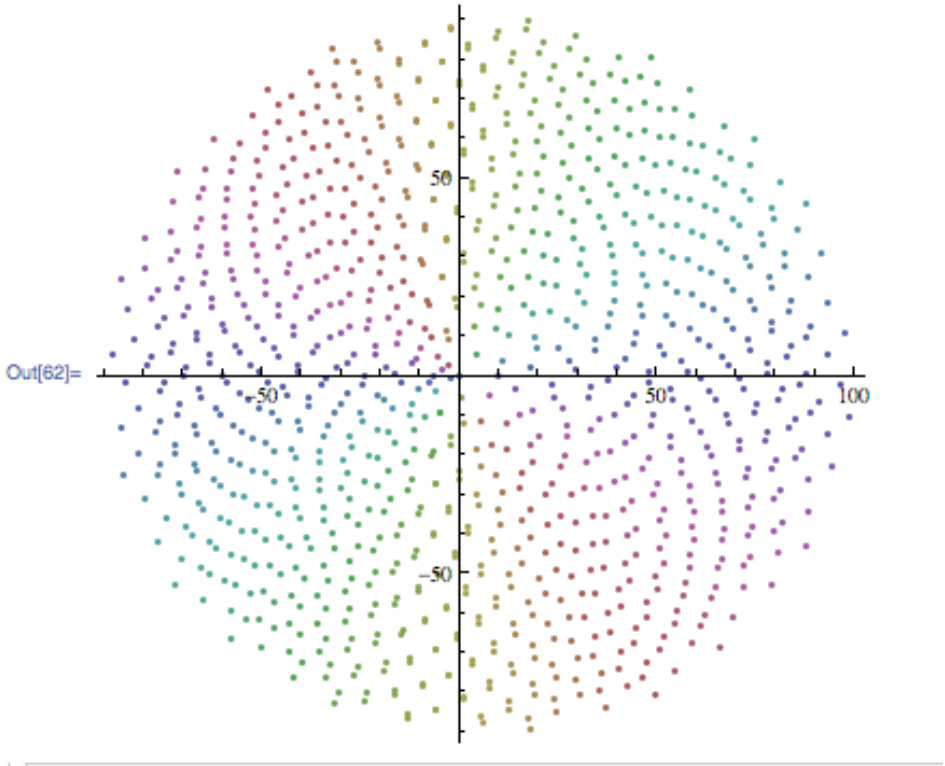
$$r = c\sqrt{n}$$

$$\Theta = \frac{2\Pi n}{\Phi^2} \approx 137.5n \quad \text{in degrees}$$

Quasiperiodic moiré

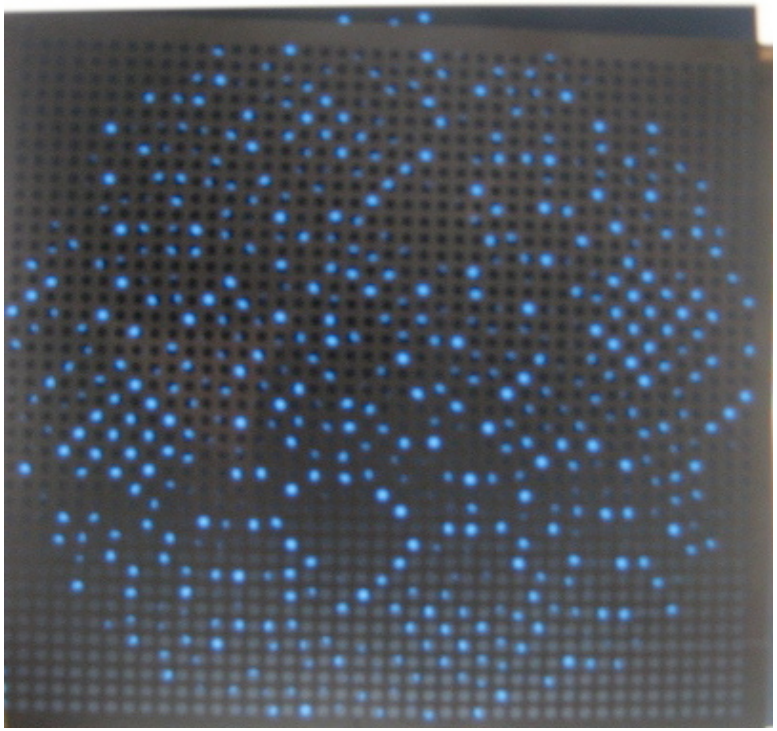


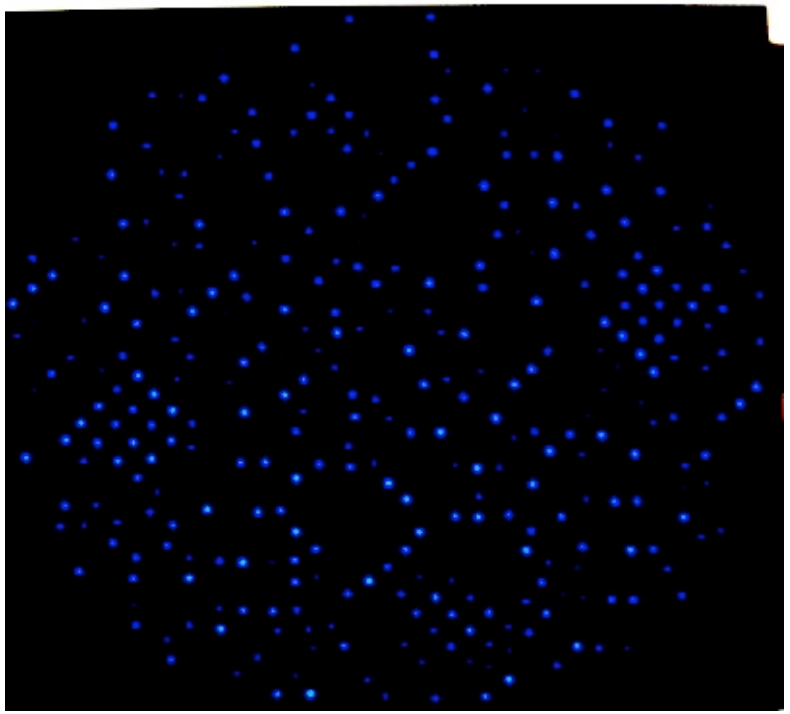
Dilation → 4 : 3.5



Shift → x + 10

Periodic dot grating and quasiperiodic pattern





Improved contrast

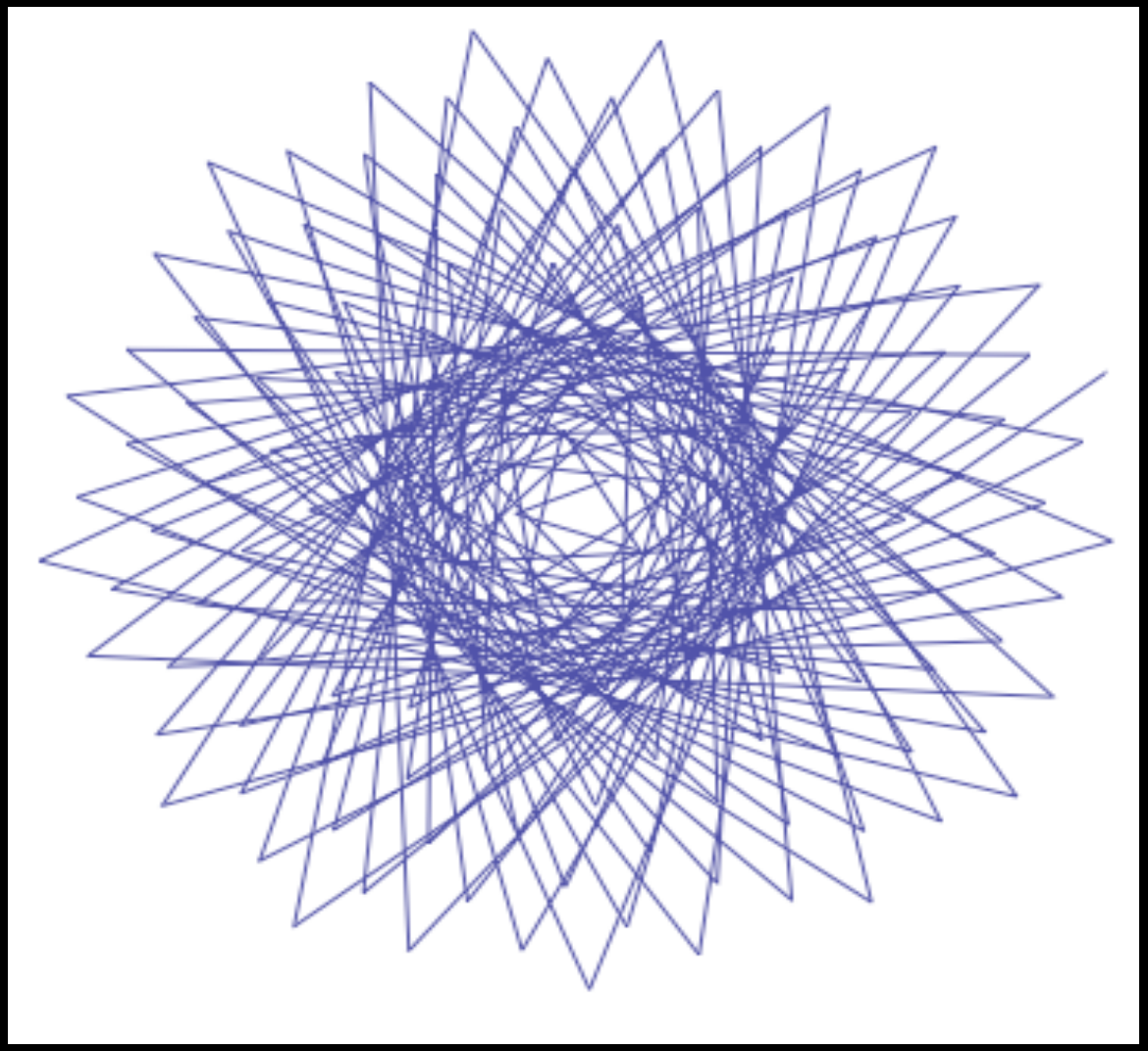
Future research

•Understand more of the math behind different kinds of gratings and moiré patterns.

•Explore quasiperiodic moiré-- maybe different ways to make quasiperiodic gratings. Also, figure out the math for that??

•More simulations!!

The end!



The end!

References

[1] Emin Gabrielyan, “The basics of line moiré and optical speedup”, arXiv, 9 pages, 8 March 2007.

[2] G. Oster, M. Wasserman, and C. Zwerling. “Theoretical interpretation of moiré patterns,” J. Opt. Soc. Am. 54, 169-175 (1964).

[3] L. Glass and R. Pérez, Nature 246, 360 (1973).

[4] K. Creath and J. C. Wyant, “Moiré and fringe projection techniques,” in Optical Shop Testing, D. Malacara, ed. (Wiley, 1992), pp. 653-660.

[5] M. P. Kothiyal and R. S. Sirohi. “Improved collimation testing using Talbot interferometry.” Appl. Opt. 26, “4056-4057” (1987).

[6] Ragulskis M. *et al.*, “Image hiding based on circular geometric moiré,” Recent Advances in Applied Mathematicas,137-142 (2009).

[7] Emin Gabrielyan, “The basics of line moiré and optical speedup”, arXiv, 9 pages, 8 March 2007.

[8] I. Amidror, "MoirÈ patterns between aperiodic layers: quantitative analysis and synthesis," J. Opt. Soc. Am. A 20, 1900-1919 (2003).

- Slide 3: http://switzernet.com/people/emin-gabrielyan/070306-optical-speedup/a5/a4.png
- Slide 2: http://anjouclothing.files.wordpress.com/2011/04/moire.jpg
- Slide 4: http://demonstrations.wolfram.com/MoirePatternOfTwoFresnelZonePlates/HTMLImages/index.en/popup_2.jpg
- Slide 5: http://www.oberonplace.com/dtp/moire/moire2.gif , http://demonstrations.wolfram.com/MoirePatternOfTwoFresnelZonePlates/HTMLImages/index.en/popup_2.jpg
- Slide 13: http://www.svi.nl/wikiimg/StFargeaux_kasteel_buiten1_aliased.jpg , http://www.dvxuser.com/articles/article.php/20
- Slide 15: http://blogs.msdn.com/b/audiofool/archive/2007/02/15/digital-audio-aliasing.aspx
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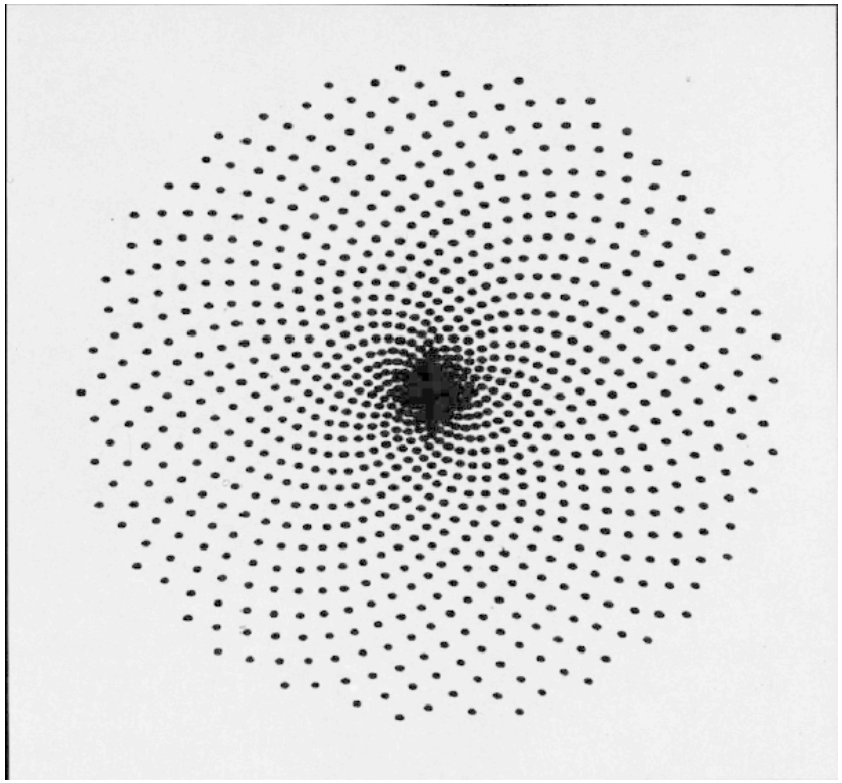
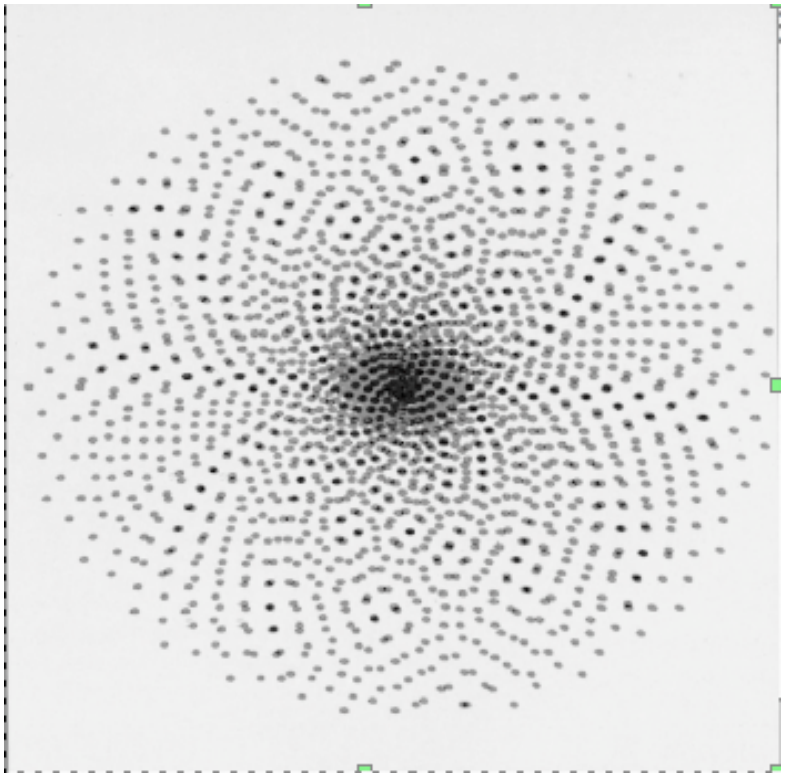
- Slide 19: http://www.trabas.de/html/enleuchtfeuer.html
- Slide 20: http://en.wikipedia.org/wiki/Talbot_effect
- Slide 23: http://switzernet.com/people/emin-gabrielyan/070212-random-moire/
- Slide 24: http://rsta.royalsocietypublishing.org/content/364/1838/189/F9.large.jpg ,http://spacecollective.org/userdata/bN7OUn0R/1270139627/diffraction.gif , http://theblackphoenix.files.wordpress.com/2011/03/quasicrystal-diffraction-pattern-by-michael-rule.jpg
- Slide 25:http://jwilson.coe.uga.edu/emat6680/parveen/fib_nature.htm, http://upload.wikimedia.org/wikipedia/commons/4/44/Helianthus_whorl.jpg
- Slide 26: http://www.maths.surrey.ac.uk/hosted-sites/R.Knott/Fibonacci/fibnat.html
- Slide : http://switzernet.com/people/emin-gabrielyan/070306-optical-speedup/

A different algorithm...





(Vogel, 2004) [9]

Even more dot patterns!