Abstract

We consider the design and performance of an astigmatic mode converter as a method of generating optical vortices, or Laguerre-Gaussian (LG) laser modes. An open-cavity Helium-Neon (HeNe) laser is used to generate Hermite-Gaussian (HG) modes, and the mode converter exploits the Guoy phase to alter the superposition of the component HG modes to form a LG mode.

The analysis of the mode converter's performance is conducted with the aid of a charge-coupled device (CCD) camera, connected to a computer. On the computer, the CCD images are analyzed with the aid of Scion Image, and percentages of mode conversion are calculated.

1 Choice of Project

Ever since learning about the operating principles of a laser, I have been fascinated with the conceptual beauty of the device. When I started researching lasers in more detail in preparation for a research project, I came across the idea of studying transverse modes. I had never known a laser could have a transverse shape other than a central intensity spot, and thought it was a cool novelty to have rectangular grids of spots, and especially hollow beams.

Only when I read about the special properties and applications of optical vortices, or Laguerre-Gaussian (LG) modes, did I see the potential for a research paper. In reading about the di erent generation methods, I found astigmatic mode converters to be the most versatile, which I consider the most important feature of any component in any system, including an optical system. In my research, I saw no comprehensive paper describing all aspects of their design and performance, and that was when I knew I had found my topic.

2 Purpose

Laguerre-Gaussian (LG) modes, also known as optical vortices, are useful in several applications. Due to an azimuthal phase dependence, proportional to $\exp(il\ )$, where l is one of the indices of the mode, LG modes of order $N > 0$ have an annular pro le [?]. It has been shown that in addition to a spin angular momentum associated with the polarization state [?], each photon has an orbital angular momentum of lh [?]. This angular momentum has been experimentally transferred to a birefringent plate [?], allowing a measurement of the torque due to this angular momentum. The orbital angular momentum has also been transferred to microscopic particles, with the aid of an optical tweezing device [?]. Besides the ability to spin particles, the annular pro le of LG modes is also important to optical tweezers. Since they draw their trapping power from the transverse component of the momentum of tightly focused light, greater intensity towards the edge of the objective lens should give greater trapping power. In addition, intensity in the center of the objective works against the tweezers. The annular profile of LG modes makes them ideal for use in such a device.

Another relatively new application of LG beams is the generation of arbitrary high-order Bessel beams. A Bessel beam is a non-diffracting, or propagation invariant, solution to the paraxial wave equation, as originally predicted by Durnin [?]. They can be generated by focusing a LG beam through an axicon [?], which is a conical lens.

For the generation of LG modes, the three well-known methods are intra-cavity circular absorbers [?], computer-generated holograms [?], and astigmatic mode converters. Intra-cavity circular absorbers face two major problems. First, they give a sizeable cut to the intensity of the laser beam, which is unacceptable for certain applications. Also, their use requires access to the laser cavity, which is often impossible and/or impractical. Computer-generated holograms are time-consuming and difficult to produce and use. In addition, LG modes with index $p > 0$ cannot be created by this method. Instead, the resultant beams are Gegenbauer-Gaussian modes [?], which are best expressed as superpositions of several LG modes. These modes, upon far-field propagation, will eventually give the desired intensity distribution. Astigmatic mode converters provide an extra-cavity solution for producing pure LG modes of arbitrary indices [?]. The only major limiting condition is the need for an incident high-order HG mode of a certain index, depending on the desired LG beam.

In this paper, the design of an astigmatic mode converter is covered in detail. All relevant calculations leading up to design parameters are included, as well as the ones that were experimentally realized. The beams generated by the device are subjected to analysis in order to determine performance and performance factors.

3 Theory

This section is a review of scientific literature pertinent to this research. It begins with a subsection on transverse laser modes, and then goes on to describe the use of Gaussian beam optics, specifically for the purpose of mode-matching. The following subsection analyzes the operating principles of an astigmatic mode converter, and goes on to apply the knowledge of the preceding subsections to the design of the mode converter.

3.1 Transverse Laser Modes

3.1.1 The Fundamental Gaussian

The geometry of a laser resonator cavity will determine what portions of the gain material contribute to the laser beam. The patterns that develop in the resultant beam are modes that are solutions to the complex wave equation:

$$\nabla^2 U - \frac{1}{c^2}\frac{\partial^2 U}{\partial t^2} = 0$$

In a typical laser cavity, several modes will oscillate and compete for the gain material. Most commercially designed lasers are designed to eliminate all except the fundamental Gaussian mode, in order to minimize the beam radius and hence improve the beam quality.

The fundamental mode can be written:

$$U_0(r;z) = \frac{1}{w}\exp\left[-\frac{ikr^2}{2R} - \frac{r^2}{w^2} - i\phi\right]$$

It is presented in this form so that the propagation properties of the Gaussian beam may be more easily analyzed. The symbols will all be explained in the next subsection, with the exception of $\phi(z)$, the Guoy phase, which will be explained in the subsection after that.

Figure 1: A CCD image of the fundamental Gaussian mode.

3.1.2 The Hermite-Gaussian

However, the fundamental Gaussian mode is rarely found alone, except when other modes are selectively blocked. In order to fully describe a certain mode, a basis set is required. One specific basis set is called the Hermite-Gaussian (HG) family of modes. HG modes have rectangular symmetry, and are fairly easy to isolate, by introducing a crosswire into an open-cavity laser. This will be elaborated on later. The function for an HG mode is:

$$U^{HG}_{m,n}(x,y,z) = C^{HG}_{m,n}\frac{1}{w}\exp\left[-\frac{ik(x^2+y^2)}{2R} - \frac{x^2+y^2}{w^2} - i(m+n+1)\psi\right] H_m\left(\frac{x\sqrt{2}}{w}\right) H_n\left(\frac{y\sqrt{2}}{w}\right)$$

Where H_m is a Hermite polynomial, defined by:

$$H_m(u) = (-1)^m e^{u^2}\frac{d^m}{du^m}e^{-u^2}$$

The HG mode can be normalized by multiplying by the normalization factor:

$$C^{HG}_{m,n} = \left(\frac{2}{\pi m!n!}\right)^{1/2} 2^{-N/2}$$

Where $N = m + n$ is the order of the mode. These HG modes are used as inputs to the astigmatic mode converter to produce pure modes of another basis set, the Laguerre-Gaussian.

3.1.3 The Laguerre-Gaussian

The Laguerre-Gaussian is another basis set of modes. They are circularly symmetric, and like the HG modes, reduce to the fundamental Gaussian for the zeroth order.

$$U^{LG}_{l,p}(r,\phi,z) = C^{LG}_{l,p}\frac{1}{w}\exp\left[-\frac{ikr^2}{2R} - \frac{r^2}{w^2} - i(l+p+1)\psi - i(l-p)\phi\right] (-1)^{\min(l,p)}\left(\frac{r\sqrt{2}}{w}\right)^{|l-p|} L^{|l-p|}_{\min(l,p)}\left(\frac{2r^2}{w^w}\right)$$

Where L^k_n is a generalized Laguerre polynomial:

$$L^k_n(x) = \sum_{m=0}^{n}(-1)^m\frac{(n+k)!}{(n-m)!(k+m)!m!}x^m$$

Just like the HG mode, the LG can be normalized with the constant:

$$C^{LG}_{l,p} = \left(\frac{2}{\pi l!p!}\right)^{1/2}\min(l,p)!$$

Since both the HG and LG form basis sets for transverse laser modes, it must be possible to express a mode of one as a sum of modes of the other. It is:

$$U^{LG}_{l,p}(x,y,x) = \sum_{k=0}^{N} -^k b(l,p,k)\, U^{HG}_{N-k,k}(x,y,z)$$

$$\left(b(n,m,k) = \left(\frac{(N-k)!k!}{2^N n! m!} \right)^{1/2} \frac{1}{k!} \frac{d^k}{dt^k} \left[(1-t)^n (1+t)^m \right] \Big|_{t=0} \right)$$

3.2 Gaussian Beam Optics

The propogation of Gaussian mode laser beams is described by a mathematical formalism called Gaussian beam optics [?]. According to this formalism, the beam parameters $w(z)$ and $R(z)$ are given by:

$$w(z) = w_0 \sqrt{1 + (z/z_0)^2}$$

$$R(z) = z \sqrt{1 + (z/z_0)^2}$$

Here w represents the radius of the beam's cross section, and r represents its radius of curvature. If the beam diameter is graphed against propagation distance starting from the waist, it is seen that the profile of the beam waist is hyperbolic in shape.

There are several ways to express the relations of Gaussian beam optics, but the most useful and well-known is the ABCD Law [?]. The only two relations necessary to calculate the effects of an arbitrary paraxial system are:

$$\frac{1}{q(z)} = \frac{1}{R(z)} - \frac{}{w(z)^2}$$

$$q_2 = \frac{Aq_1 + B}{Cq_2 + D}$$

q is called the complex beam parameter, and it contains all necessary information about a Gaussian beam. A, B, C, and D are real parameters determined by the particular system in question. It is worth noting that the parameters happen to be the same A, B, C, and D as would be found in the ray optics matrix description of the same system.

In the design of an astigmatic mode converter, the topic of mode-matching comes up. Mode-matching is the process of manipulating a Gaussian beam to a speci c waist location and spot size. If one knows these, and the wavelength of the light, one can also determine the Rayleigh range (z_0) and the divergence ( ):

$$z_0 = \quad \frac{\ }{\ } \quad w_0^2$$

$$= \frac{\ }{w_0}$$

The Rayleigh range is the distance from the waist at which the beam will grow to double the transverse area at the waist. The divergence is the angle between the far- eld asymptote of the graph of $w(z)$ and the horizontal (z) axis of the graph.

The radius of curvature of the beam is always zero at the beam waist. To mode-match, a converging lens is used. If one has a beam with waist radius w_0 and desires a beam with waist w_0', the minimum focal length of the mode-matching lens is given by the relation

$$f_0 = \frac{w_0 w_0'}{\ }$$

The distance from the initial beam waist to the lens (of focal length f) and the distance from the lens to the new waist, d_1 and d_2 respectively, are given by:

$$d_1 = f \quad \frac{w_0}{w_0'} \sqrt{f^2 \quad f_0^2}$$

$$d_2 = f \quad \frac{w_0'}{w_0} \sqrt{f^2 \quad f_0^2}$$

Either the positive or negative of the second term of each equation may be used, but both must be chosen to be the same. If one term ends up negative, it obviously must be rejected, as it has no physical signi cance.

One more thing worth noting, since it applies to the experimental setup used, is the beam parameters as determined by a laser resonator consisting of a spherical mirror and a plane mirror output coupler. The waist radius of such a setup is:

$$w_0 = \sqrt{d(R - d)}$$

Where d is the distance between the two mirrors, and R is the radius of curvature of the spherical mirror. As long as $d < R$, the above relation holds true, and the waist is located at the plane mirror output coupler.

3.3 Theory of an Astigmatic Mode Converter

To begin the discussion of the theory behind an astigmatic mode converter, we return to the definition of the Gaussian family of modes. The symbol previously left undefined, $= \tan^{-1}\left(\frac{z}{z_0}\right)$, is multiplied by $(m + n + 1)$ for HG modes or $(l + 2p + 1)$ for LG modes to form the Guoy phase. It is the exploitation of this phase that allows the mode conversion. This gains relevance in the superposition of modes. For instance, the superposition of the $HG_{0;1}$ and $HG_{1;0}$ modes is a $HG_{0;1}$ mode, rotated $\frac{\pi}{4}$ from the axis of the component modes. But if one of the component modes is $\frac{\pi}{2}$ out of phase with the other, the superposition is no longer a HG mode. Rather, it is a $LG_{1;0}$ mode. This is a result of the mathematical relation between the HG and LG basis sets, as described above in the subsection "Transverse Laser Modes." This out-of-phase superposition is the operating principle of the astigmatic mode converter.

Now we are faced with the problem of how to create this phase shift. It is possible to split a HG beam, geometrically rotate one by $\frac{\pi}{2}$ with respect to the other, adjust the path lengths to achieve the required phase shift, and then superimpose them, making sure to match the beam radii of both size and curvature. As if this weren't hard enough, any higher order modes would require more than two input modes, and the whole process would be prohibitively difficult and complex.

Instead, the phase shift is achieved by rotating the input HG mode by $\frac{\pi}{4}$ with respect to the focusing axis of two identical cylindrical lenses. Since the cylindrical lenses only focus in one direction, this creates a path length difference between the components of the HG mode. In other words, the beam is made astigmatic. The second lens serves to remove the astigmatism, so that the LG beam formed by the mode converter can continue to propagate without astigmatism. It can be shown [?] that in order for this to work, the cylindrical lenses must be separated by a definite distance, determined by their focal lengths, which must be equal. This distance is:

$$d_3 = f_{cyl}\sqrt{2}$$

The distance is subscripted as 3 for later convenience. The other requirement of the mode converter is that the input HG mode must have a waist halfway between the two cylindrical lenses. In addition, this waist must be mode-matched so that its Rayleigh range is:

$$z'_0 = \left(1 + \frac{1}{\sqrt{2}}\right) f_{cyl}$$

Once again, the primed value is for later convenience. Of course, the waist radius of the beam is easily calculated from this Rayleigh range. This mode-matching is performed as described in the previous subsection, "Gaussian Beam Optics."

Figure 2: The elliptical LG21 mode above is an example of what happens if the selection rules for the mode converter are not precisely obeyed. While the mode conversion is virtually full, the beam shape is incorrect.

The end result of the astigmatic mode converter is that a mode $HG_{m;n}$ is converted into a mode $LG_{l;p}$ following the change-of-parameter rules:

$$l = |m - n| \qquad p = \min(m; n)$$

4 Experimental Procedure

4.1 Materials

The most important component of the experiment was the open-cavity helium-neon (HeNe) laser that was used, which will be described in its own following subsection. Other than the laser, the materials that comprised the astigmatic mode converter were a f = 300 mm converging lens, two f = 150 mm cylindrical lenses, and several di erent converging and diverging lenses placed after the cylindrical lenses, depending on what came after the mode converter in the optical system.

To make accurate images of laser beams, a charge-coupled device (CCD) camera was used. The particular model was EDC-1000N, from Electrim. It was hooked up to a PC running Windows 95, and communicated via software that came with the camera. The images were manipulated with Corel Photo-Paint, and analyzed with Scion Image. Some minor image editing was also done with The GIMP, available from GNU, and included in most Linux systems.

Experiments involving the mode converter also made use of a cooling fan from Radio Shack, model 273-241C, a digital oscilloscope from Tektronix, model TDS 2012, a photodetector from ThorLabs, model DET-110, and various digital multimeters. In some cases, polarizers and neutral density lters of various strengths were used, all from ThorLabs. They were primarily to reduce optical intensity, so that CCD images would not saturate.

4.2 The Laser

The laser used in the experiment was an open-cavity HeNe laser, operating at 632.8 nm. The tube was model 05-LHB-570 from Melles-Griot, which had a pre-mounted spherical mirror at one end, with a radius of curvature of 600 mm, and a built-in Brewster window. It was mounted into a metal frame on a piece of optical breadboard, upon which the output coupler, a plane mirror 99 % reflective at 633 nm, was mounted. Also added within the cavity, between the Brewster window and the output coupler, was a translation stage aligned to translate a human hair across the beam. The human hair was mounted on a rotation stage to allow for different mode angles, although for the experiment, only $\frac{}{4}$ was used. By diffraction methods, the human hair was determined to be approximately 50 m in diameter. Its presence and position cut out parts of the gain material, which selected certain transverse modes of oscillation, which in most cases were high-order HG modes.

Figure 3: Two images of the mode converter apparatus and laser.

HG modes achieved were $\boldsymbol{HG}_{0;0}$ (fundamental Gaussian), $\boldsymbol{HG}_{0;1}$, $\boldsymbol{HG}_{0;2}$, $\boldsymbol{HG}_{0;3}$, $\boldsymbol{HG}_{1;1}$, $\boldsymbol{HG}_{1;2}$, and the opposites (switching $\boldsymbol{m}$ and $\boldsymbol{n}$) of these, by having the hair at an angle of $\frac{3}{4}$, as opposed to $\frac{}{4}$.

4.3 Parameters of the Mode Converter

With the formula for the waist of the beam coming from the laser, the waist was determined to be 246 μm, implying a Rayleigh range of 300 mm. As stated above, the mode-matching lens had focal length $f = 300$ mm, and the cylindrical lenses had equal focal lengths of $f = 150$ mm. From this, the distance from the output coupler beam waist to the mode-matching lens was found to be $d_1 = 513$ mm, and the distance from the mode-matching lens to the second waist between the cylindrical lenses was $d_2 = 481$ mm. Since the distance between the cylindrical lenses must be $d_3 = 212$ mm, due to one of the astigmatic mode converter conditions, the distance between the mode-matching lens and the first cylindrical lens was 269 mm. The waist radius at the second waist was $w_0' = 227$ μm, and the corresponding Rayleigh range was $z_0' = 256$ mm. Using the ABCD law and $w(z)$ and $R(z)$ relations from there, the beam can be calculated and controlled after that point.

Figure 4: Visual representation of the variable parameters of the astigmatic mode converter.

4.4 Determination of Beam Radius

Although it was possible to theoretically calculate the parameters of the beam at all points in the optical system, it was important to be confident in these calculations. In order to verify calculations and debug problems with the mode converter, a procedure to systematically measure the beam radius was developed.

To measure beam radius at a specific distance, a fan was placed in the path of the beam at that point. The beam was later focused into a photodetector, which was connected to a digital oscilloscope. Each time a blade of the fan traversed the beam, it would cut the intensity of the beam. The first necessary piece of data was the angular velocity of the fan. Since it had to be moved around a lot, sometimes next to things, that angular velocity changed with its surroundings, and therefore was measured at each position. To measure, the time scale is set so that several pulses resembling square waves are visible. The time for ve of the is recorded (the fan had ve blades), and the angular velocity would be $! = \frac{2}{t_5}$.

But to measure the beam diameter, the tangential velocity was needed, not the angular. So the radial position of the beam with respect to the fan was found, and the tangential velocity was then $v = !r$. Then the time scale was adjusted so that the ramping up of a single pulse lled the screen. This was the intensity cut moving across the beam. The approximate $\frac{1}{e^2}$ points were taken, and the beam radius was then given by $w = \frac{1}{2}v \;\; t_1$. The $\frac{1}{2}$ is there because without it, we would be measuring the diameter.

5 Beam Image Capture and Analysis

To capture an image of a beam, the beam was expanded and projected onto a piece of white paper. The lights were turned out, and the CCD camera was aimed at the beam. If on the computer the beam appeared as saturated (full white value in a region), the gain, bias, and exposure time parameters of the capture software were adjusted. If this still wasn't enough to darken it properly, a neutral density lter and/or polarizer(s) were introduced before the beam was expanded. All this trouble was taken to avoid saturation so that proper beam analysis could take place in Scion Image. The reason will now be apparent.

Given an image, Scion Image can perform an impressive array of tasks. For illustrative purposes, it color-codes and renders a 3D surface plot of the optical intensity of the beam. I used this for qualitative purposes only. Its 2D intensity vs. position plotting was a much more valuable tool to me, and was used quantitatively to nd mode compositions between input and output modes while adjusting the lenses for maximum mode conversion. For instance, a partial conversion of $\boldsymbol{HG}_{0;2}$ to $\boldsymbol{LG}_{2;0}$ would look like the proper LG mode, but with two brighter intensity spots across from each other on the ring.

Figure 5: At left is a CCD image of a LG10 mode. In the center and at right are two di erent surface plots generated by Scion Image, based on the CCD image.

To analyze this, two line selections were taken: rst, across the diameter connecting the two intensity spots, and second, the diameter perpendicular to this one. Subtracting out the background intensity from the second line selection, the height (intensity) of the LG mode can be obtained. Subtracting both the background and the LG height from the other diameter, the HG mode height is obtained. Dividing either height by the sum of both heights will give the percentage mode conversion. In summary:

$$h_{HG} = h_{D1} \quad h_{LG} \quad b = h_{D1} \quad h_{D2} \qquad h_{LG} = h_{D2} \quad b$$

$$\boldsymbol{P_{HG}} = \frac{\boldsymbol{h_{HG}}}{\boldsymbol{h_{HG}} + \boldsymbol{h_{HG}}} \qquad \boldsymbol{P_{LG}} = \frac{\boldsymbol{h_{LG}}}{\boldsymbol{h_{HG}} + \boldsymbol{h_{LG}}} = 1 \quad \boldsymbol{P_{HG}}$$

Where $\boldsymbol{h_{D1}}$ and $\boldsymbol{h_{D2}}$ are the heights of the maxima along the rst and second diameters as described above, $\boldsymbol{h_{HG}}$ and $\boldsymbol{h_{LG}}$ are the heights of the HG and LG mode in the composition, $\boldsymbol{P_{HG}}$ and $\boldsymbol{P_{LG}}$ are the percentage compositions of the HG and LG mode, and $\boldsymbol{b}$ is the height of the background light.

Figure 6: Complete versus incomplete mode conversion. At left is the correctly generated LG20 mode. At right is an incompletely generated LG20 mode, with strong remnants of the original HG02 mode visible.

5.1 Multimodes

The crosswire (human hair) in the laser cavity is used to select individual HG modes from a larger superposition of several modes, called a multimode. To nd a certain HG single mode, it is a good idea to have as varietal a multimode as possible. This means more active parts of the gain material, likely meaning a higher output power. I decided to test this hypothesis, since the ability to nd more HG single modes was vital to the testing of the astigmatic mode converter. So I took four di erent multimodes that had been useful to me, measured their average output power, and compared that to the number of modes they produced. The di erent multimodes were achieved by adjusting the angle of the output coupler.

Each single mode produced was classi ed as \good" or \bad." Don't let the names confuse you, however, since \bad" single modes were just distorted versions of the normal HG modes, implying a small superposition that works as a good approximation (unfortunately, only in terms of shape, not necessarily phase). So the \bad" modes imply more possible modes, rather than fewer.